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Almost diameter of a house-hole-free graph in linear time via LexBFS ☆

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Abstract

We show that the vertex visited last by a LexBFS has eccentricity at least $diam(G) - 2$ for house-hole-free graphs, at least $diam(G) - 1$ for house-hole-domino-free graphs, and equal to $diam(G)$ for house-hole-domino-free and AT-free graphs. To prove these results we use special metric properties of house-hole-free graphs with respect to LexBFS. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction and basic notions

All graphs $G=(V,E)$ in this paper are finite, undirected, connected and simple (i.e. without loops and multiple edges). The (*open*) *neighborhood* of a vertex v is the set $N(v)=\{u \in V\colon uv \in E\}$ and the *closed neighborhood* is $N[v]=N(v)\cup\{v\}$. A *path* is a sequence of vertices $(v_0 - \cdots - v_l)$ such that $v_iv_{i+1} \in E$ for $i=0,\ldots,l-1$; its *length* is l. An *induced path* is a path where $v_iv_j \in E$ iff $i=j-1$ and $j=1,\ldots,l$. A *k-cycle* C_k is a path $(v_0 - \cdots - v_k)$ such that $v_0=v_k$; its *length* is k. An *induced cycle* is a cycle where $v_iv_j \in E$ iff $|i-j|=1\,(\mathrm{modulo}\, k)$. A *hole* is an induced cycle of length at least five.

The *distance* $dist(v,u)$ between vertices v and u is the smallest number of edges in a path joining v and u. The *eccentricity* $e(v)$ of a vertex v is the maximum distance from v to any vertex in G. The *radius* $rad(G)$ is the minimum eccentricity of a vertex in G and the *diameter* $diam(G)$ is the maximum eccentricity. Distances in graphs and related graph theoretic parameters such as diameter and radius play an important role

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in the design and analysis of networks in a variety of networking environments like communication networks, electric power grids, and transportation networks.

As yet, no fast algorithms for computing the diameter of an arbitrary graph, avoiding the computation of the whole distance matrix, have been designed. Linear-time algorithms are known only for trees [16], maximal outerplanar graphs [13], interval graphs [21,12], ptolemaic graphs [12], strongly chordal graphs [6], dually chordal graphs [1], distance-hereditary graphs [10,11] and for graphs of benzenoid systems [3].

Even for chordal graphs (a graph is *chordal* if it has no induced cycle of length at least 4) efficient computation of the diameter is an open problem [5], and it seems that the diameter problem on chordal graphs is not easier than $0,1$ matrix multiplication. It can be shown that the diameter problem on chordal graphs is linear time reducible to the diameter problem on split graphs (a subclass of chordal graphs with only three possible values $1,2,3$ for the diameter). Nevertheless, using Lexicographic-Breadth-First-Search (LexBFS) of Rose et al. [22], one can find "almost" diameter of a chordal graph in linear time; namely, the vertex numbered last by LexBFS has eccentricity equal to $diam(G)$ or $diam(G)-1$ (see [12]). Using LexBFS one can find also the diameter of interval graphs, ptolemaic graphs and almost the diameter of distance-hereditary graphs and weak bipolarizable graphs [12].

This result for interval graphs and ptolemaic graphs generalizes the well-known result for trees: the vertex of a tree T visited last by Breadth-First-Search (BFS) has eccentricity $diam(T)$. A linear-time algorithm for computing the diameter of a distance-hereditary graph G, presented in [11], first applies LexBFS to find a vertex with eccentricity at least $diam(G)-1$ and then using only local search either improves this value by 1 or proves that this is the exact diameter of G.

Historically, LexBFS was designed to provide a linear-time recognition algorithm for chordal graphs [22]. Recently in [8] a very simple, optimal recognition algorithm for interval graphs was presented that uses four sweeps of LexBFS. Another linear-time recognition algorithm for interval graphs, developed in [15], is also based on LexBFS. In [7], two sweeps of LexBFS is used to find a dominating pair of AT-free graphs. Note that the interval graphs are exactly the AT-free chordal graphs [19]. A graph is called *AT-free* if it does not have an *asteroidal triple*, i.e. a set of three vertices such that there is a path between any pair of them avoiding the closed neighborhood of the third. A dominating pair of a graph is a pair (x,y) of vertices such that, for any path P connecting x and y, every vertex z either belongs to P or has a neighbor in P. The authors of [7] have shown that the vertex numbered last by LexBFS forms together with some other vertex a dominating pair of an AT-free graph G. It is clear that this vertex again has eccentricity at least $diam(G)-1$.

In this paper we extend the results on diameters of chordal graphs and interval graphs, mentioned above, to three (more general) graph classes: to HH-free graphs, to HHD-free graphs and to HHD-free and AT-free graphs.

A graph is *HH-free* if it does not contain a house or a hole as an induced subgraph. A graph is *HHD-free* if it does not contain a house, a hole or a domino as an induced subgraph (see Fig. 1). HHD-free graphs were introduced and investigated in

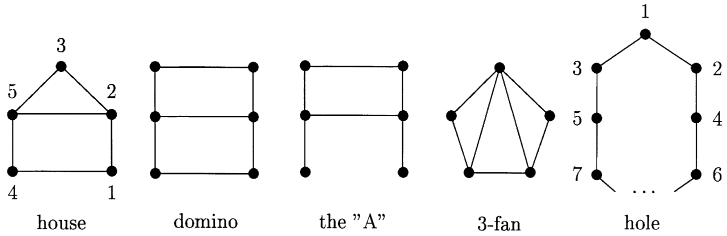

Fig. 1.

[18]. It was shown that G is a HHD-free graph if and only if every ordering of the vertices of G produced by LexBFS is semi-simplicial [18]. Some further properties of HH-free graphs with respect to LexBFS can be found in [4,9]. If a HHD-free graph does not contain the "A" of Fig. 1 as an induced subgraph then this graph is called *weak bipolarizable* [20]. A *distance-hereditary graph* is a HHD-free graph that does not contain 3–fan as an induced subgraph [17]. Chordal distance-hereditary graphs are exactly the ptolemaic graphs [17].

Recall that *LexBFS* orders the vertices of a graph by assigning numbers from $n=|V|$ to 1 in the following way: assign the number k to a vertex v (as yet unnumbered) which has lexically largest vector $(s_n, s_{n-1}, \ldots, s_{k+1})$, where $s_i = 1$ if v is adjacent to the vertex numbered i, and $s_i = 0$ otherwise. An ordering of the vertex set of a graph G generated by LexBFS we will call a *LexBFS-ordering*.

Main results of this paper are the following.

Let v be a vertex of G numbered by "1" in some LexBFS.

- If G is a HH-free graph then $e(v) \in \{diam(G), diam(G) - 1, diam(G) - 2\}$.
- If G is a HHD-free graph then $e(v) \in \{diam(G), diam(G) - 1\}$.
- If G is a HHD-free and AT-free graph then $e(v) = diam(G)$.

To prove these results we use special metric properties of HH-free graphs with respect to LexBFS.

2. LexBFS-orderings in HH-free graphs

Let $\sigma = (v_1, v_2, \ldots, v_n)$ be an ordering of the vertex set of a graph G. We will write $a < b$ whenever in σ vertex a has a smaller number than vertex b. Moreover, $\{a_1, \ldots, a_l\} < \{b_1, \ldots, b_k\}$ is an abbreviation for $a_i < b_j$ $(i = 1, \ldots, l;\ j = 1, \ldots, k)$.

In what follows we will often use the following property (cf.[18]):

(P1) If $a < b < c$ and $ac \in E$ and $bc \notin E$ then there exists a vertex d such that $c < d, db \in E$ and $da \notin E$.

It is well known that any LexBFS-ordering has property (P1) [14]. Moreover, any ordering obeying (P1) can be generated by LexBFS [2,12].

Let G be a HH-free graph and σ be a LexBFS-ordering of G. By P_4 we denote a path on four vertices.

Lemma 1. *G does not have any induced $P_4=(c-a-b-d)$ with $\{a,b\}<\{c,d\}$ in σ.*

Proof. Assume by way of contradiction that there is an induced path $(x_1-x_0-y_0-y_1)$ in G such that $\{x_0,y_0\}<\{y_1,x_1\}$. We may choose such a path with largest sum $\Sigma=\sigma(x_0)+\sigma(y_0)+\sigma(x_1)+\sigma(y_1)$ of the positions of x_0,y_0,x_1,y_1 in σ. Let also $y_1<x_1$.

Consider in G an induced path $P=(x_k-\cdots-x_1-x_0-y_0-y_1-\cdots-y_k)$ $(k\geqslant 1)$ such that $\{x_0,y_0\}<y_1<x_1<\cdots<y_{k-1}<x_{k-1}<y_k<x_k$ and if $k\geqslant 2$ then the following two conditions hold:
1. for each i $(0<i\leqslant k-1)$ and all $y>y_{i+1}$, $yx_{i-1}\in E$ if $yy_i\in E$;
2. for each i $(0<i\leqslant k-1)$ and all $x>x_{i+1}$, $xy_i\in E$ if $xx_i\in E$.

We will extend this path by vertices y_{k+1} and x_{k+1} as follows.

First apply (P1) to $x_{k-1}<y_k<x_k$ and get a vertex $y_{k+1}>x_k$ adjacent to y_k but not to x_{k-1}. We may choose y_{k+1} rightmost in σ, i.e. for every $y>y_{k+1}$, $yx_{k-1}\in E$ if $yy_k\in E$. We claim that y_{k+1} is not adjacent to any x_i $(i=0,\ldots,k)$ and any y_i $(i=0,\ldots,k-1)$. Assume $y_{k+1}x_k\in E$. Since G does not contain holes, the cycle formed by the induced path P and edges $y_{k+1}x_k$, $y_{k+1}y_k$ must have chords. All these chords are incident to the vertex y_{k+1}. Since $y_{k+1}x_{k-1}\notin E$ and the length of this cycle is odd $(=2k+3)$ we cannot avoid an induced house. Hence, $y_{k+1}x_k\notin E$. Let $y_{k+1}x_i\in E$ and i be the largest index with this property. Evidently, $i\leqslant k-2$. Then the path $(y_{k+1}-x_i-x_{i+1}-x_{i+2})$ is induced and $x_i<x_{i+1}<x_{i+2}<y_{k+1}$, contrary to maximality of Σ. Let now $y_{k+1}y_i\in E$ and i be the smallest index with this property. If $i=0$ then we can replace y_1 with y_{k+1} in the path $(x_1-x_0-y_0-y_1)$ and increase the sum Σ. So, $0<i\leqslant k-1$. From $y_{k+1}y_i\in E$, $y_{k+1}x_{i-1}\notin E$ and $y_{k+1}>y_{i+1}$ we get a contradiction with the condition 1.

Now we apply (P1) to $y_k<x_k<y_{k+1}$ and get a vertex $x_{k+1}>y_{k+1}$ adjacent to x_k but not to y_k. Again we may choose x_{k+1} rightmost in σ, i.e. for every $x>x_{k+1}$, $xy_k\in E$ if $xx_k\in E$. We will show that in $P\cup\{y_{k+1},x_{k+1}\}$ vertex x_{k+1} is adjacent to x_k only. Assume $y_{k+1}x_{k+1}\in E$. Since G does not contain holes, the cycle formed by P and edges $y_ky_{k+1},y_{k+1}x_{k+1},x_{k+1}x_k$ must have chords. They are all incident to x_{k+1}. Since $y_kx_{k+1}\notin E$ and G does not contain an induced house, the vertex x_{k+1} is adjacent to x_i, y_j if and only if $i=k,k-2,k-4,\ldots$ and $j=k+1,k-1,k-3,\ldots$. We distinguish between two cases: $x_0x_{k+1}\in E$, i.e. k is even, or $y_0x_{k+1}\in E$, i.e. k is odd. First suppose that $x_0x_{k+1}\in E$. We have x_{k+1} is adjacent to y_1,x_0,x_2 and not to y_0,x_1. Applying (P1) to $x_0<x_1<x_{k+1}$ we will find a vertex $t>x_{k+1}$ adjacent to x_1 but not to x_0. Since $t>x_2$ and $tx_1\in E$ from the condition 2 we have $ty_1\in E$. But then the vertices t,y_1,y_0,x_0,x_1 induce a house or a 5-cycle, that is impossible. Suppose now $y_0x_{k+1}\in E$. Hence, x_{k+1} is adjacent to y_0,x_1,y_2 and not to y_1,x_0. We apply (P1) to $y_0<y_1<x_{k+1}$ to get a vertex $t>x_{k+1}$ adjacent to y_1 and not to y_0. Since $t>y_2$ and $ty_1\in E$ from the condition 1 we have $tx_0\in E$. Furthermore $tx_1\notin E$, otherwise the vertices x_1, x_0, y_0, y_1, t induce a house. Now apply (P1) to $x_0 < x_1 < t$ and get a vertex $s > t$ adjacent to x_1 and not to x_0. Again, $s > x_2$ and the condition 2 give $sy_1 \in E$. Since $sx_0 \notin E$ and the path $(x_1 - x_0 - y_0 - y_1)$ is induced, the vertices s, x_1, x_0, y_0, y_1 induce a house or a hole.

Hence, $y_{k+1}x_{k+1} \notin E$. Let $x_{k+1}y_i \in E$ and i be the largest index with this property. Evidently, $i \leqslant k-1$. Then the path $(x_{k+1} - y_i - y_{i+1} - y_{i+2})$ is induced and $y_i < y_{i+1} < y_{i+2} < x_{k+1}$, contrary to maximality of Σ. Let now $x_{k+1}x_i \in E$ and i be the smallest index with this property. If $i = 0$ then we can replace x_1 with x_{k+1} in the path $(x_1 - x_0 - y_0 - y_1)$ increasing Σ. So, $0 < i \leqslant k-1$. From $x_{k+1}x_i \in E$, $x_{k+1}y_i \notin E$ and $x_{k+1} > x_{i+1}$ we get a contradiction with the condition 2.

Thus, we have extended the path P by vertices y_{k+1} and x_{k+1}. Since G is finite, at a certain step we will arrive at a contradiction. □

In Lemma 1 the condition that G is HH-free is essential. In Fig. 1 a LexBFS-ordering of the house is given for which a forbidden-induced (ordered) P_4 occurs. It is easy to see also that any LexBFS-ordering of a hole produces a forbidden induced $P_4 = (3 - 1 - 2 - 4)$.

Lemma 2. *Let a, b, c be three distinct vertices of G such that $a < \{b, c\}$, $ab, ac \in E$ and $bc \notin E$. Then there is a vertex $d > \{b, c\}$ adjacent to b and c but not to a.*

Proof. Assume without loss of generality, that $b < c$. Applying (P1) to $a < b < c$ gives a vertex $d > c$ adjacent to b but not to a. Since $\{a, b\} < \{c, d\}$, by Lemma 1, the vertices d and c must be adjacent. □

Let $P = (x_0 - x_1 - \cdots - x_{k-1} - x_k)$ be an arbitrary path of G and σ be an ordering of the vertex set of this graph. The path P is *monotonic* (with respect to σ) if $x_0 < x_1 < \cdots < x_{k-1} < x_k$ holds whenever $x_0 < x_k$, and P is *convex* if there is an index i $(1 \leqslant i < k)$ such that $x_0 < x_1 < \cdots < x_{i-1} < x_i > x_{i+1} > \cdots > x_{k-1} > x_k$. Then x_i is called the *switching point* of the convex path P. Let now $P = (x_0 - \cdots - x_k)$ be a shortest path of G connecting x_0 and x_k. We say that P is a *rightmost shortest path* if the sum $\sigma(x_0) + \sigma(x_1) + \cdots + \sigma(x_k)$ of the positions of $x_0, \ldots, x_k$ in σ is largest among all shortest paths connecting x_0 and x_k.

Let G be a HH-free graph and σ be a LexBFS-ordering of G.

Lemma 3. *Every rightmost shortest path of G is either monotonic or convex.*

Proof. Assume that a rightmost shortest path $P = (x_0 - \cdots - x_k)$ has a vertex x_j with $(1 \leqslant j < k)$ such that $x_{j-1} > x_j < x_{j+1}$. Since $x_{j-1}x_{j+1} \notin E$, by Lemma 2, there exists a vertex $y > x_j$ adjacent to both x_{j-1} and x_{j+1}. But this contradicts to the assumption that P is a rightmost shortest path. □

Lemma 3 is implicitly contained in [3], where paths similar to rightmost shortest paths are used. We refine this lemma by the following result.

Lemma 4. *Let $P=(x_0 - \cdots - x_k)$ be a rightmost shortest path in G which is convex and x_i be the switching point of P. Furthermore, let $x_0 < x_k$. Then*
(1) $dist(x_0,x_i) \geqslant dist(x_k,x_i)$ and
(2) if $dist(x_0,x_i)=dist(x_k,x_i)$, i.e. $k=2i$, then $x_0 < x_k < \cdots < x_j < x_{k-j} < \cdots < x_{i-1} < x_{i+1} < x_i$.

Proof. We prove the assertion by induction on k. Note that any subpath of a rightmost shortest path is again a rightmost shortest path.

For $k=2$, evidently $x_0 < x_2 < x_1$ holds. So, let $k \geqslant 3$. Since P is convex we have $x_k < x_{k-1}$ and hence $x_0 < x_{k-1}$. By induction hypothesis, $dist(x_0,x_i) \geqslant dist(x_{k-1},x_i)$. If $dist(x_0,x_i) > dist(x_{k-1},x_i)+1$ then $dist(x_0,x_i) > dist(x_k,x_i)$, and we are done. Now we distinguish between two cases: $dist(x_0,x_i)=dist(x_{k-1},x_i)$ or $dist(x_0,x_i)=dist(x_{k-1},x_i) + 1$. We show that the first case is impossible.

Let $dist(x_0,x_i)=dist(x_{k-1},x_i)$. By induction hypothesis we have $x_0 < x_{k-1} < x_1 < \cdots < x_j < x_{k-j-1} < \cdots < x_{i-1} < x_{i+1} < x_i$. Moreover, from $x_k < x_{k-1}$ we conclude $x_k < x_1$. Applying (P1) to $x_0 < x_k < x_1$ gives a vertex $t > x_1$ adjacent to x_k but not to x_0. Since $x_{k-1} < t$ and P is rightmost $tx_{k-2} \notin E$. From $x_k < x_{k-1} < \{x_{k-2},t\}$ and Lemma 1 the vertices t and x_{k-1} must be adjacent. Since $x_{k-1} < \{x_{k-2},t\}$ and $x_{k-2},t \notin E$, by Lemma 2, there exists a vertex $s > \{t,x_{k-2}\}$ adjacent to x_{k-2},t and not to x_{k-1}. Furthermore, $x_k s \notin E$, otherwise P is not rightmost (note that $s > x_{k-1}$). But then s,t,x_k,x_{k-1},x_{k-2} induce a house, a contradiction.

Assume now that $dist(x_0,x_i)=dist(x_{k-1},x_i)+1$, i.e. $dist(x_0,x_i)=dist(x_k,x_i)$. For the rightmost shortest path $(x_1 - x_2 - \cdots - x_{k-1},x_k)$ with the switching point x_i we have $dist(x_1,x_i) < dist(x_k,x_i)$. Hence, by induction hypothesis, $x_1 > x_k$ must hold. If also $x_{k-1} < x_1$ then, using the same arguments as above, we can construct a house induced by $\{x_k,x_{k-1},x_{k-2}\}$ and two additional vertices t and s. Therefore, $x_{k-1} > x_1$. Since $dist(x_1,x_i) = dist(x_{k-1},x_i)$, by induction, we obtain $x_1 < x_{k-1} < \cdots < x_j < x_{k-j} \cdots < x_{i-1} < x_{i+1} < x_i$. With this and $x_0 < x_k < x_1$ we complete the proof. □

3. Approximation of the diameter of a HH-free graph

A subgraph H of a graph G is *isometric* if the distance between any pair of vertices in H is the same as that in G.

Let v be the first vertex of a LexBFS-ordering of a HH-free graph G.

Lemma 5. *For every two vertices x and y of G such that $dist(x,v)=dist(y,v)=p$, $dist(x,y) \leqslant p+2$ holds. Moreover, if $dist(x,y)=p+2$, then p is even, say $p=2k$, and G contains an induced subgraph isomorphic to the graph H_{k-1} from Fig. 2.*

Proof. Assume that $dist(x,y) \geqslant p+2$. Consider in G rightmost shortest paths P_x and P_y, connecting vertex v with vertices x and y, respectively. Let a be the common vertex of the paths P_x and P_y furthest from v. Since a subpath of a rightmost shortest

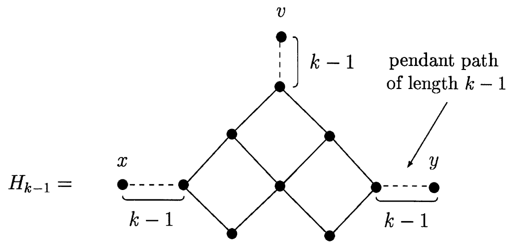

Fig. 2.

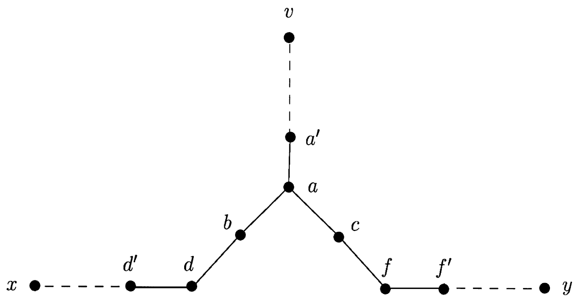

Fig. 3.

path is again a rightmost shortest path, paths P_x and P_y coincide in the part from v to a and do not have any other common vertices. Denote the common subpath of those paths by P_a.

By Lemma 3, P_x and P_y are monotonic or convex. First, we show that these paths cannot have a switching point on the subpath P_a. Assume by way of contradiction that a vertex z of P_a is the switching point of P_x and P_y. Then, by Lemma 4, we obtain $dist(v,z) \geqslant dist(x,z) = dist(y,z) = p - dist(v,z)$. Hence, $p \leqslant 2dist(v,z) \leqslant 2dist(v,a)$, i.e. $p - 2dist(v,a) \leqslant 0$. Thus, $p+2 \leqslant dist(x,y) \leqslant dist(x,a) + dist(a,y) = 2dist(x,a) = 2p - 2dist(v,a)) \leqslant p$, a contradiction.

Let now b and c be the neighbors of a in the paths P_x and P_y, respectively, which do not belong to the path P_a (see Fig. 3). Assume that $b < c$. We claim that b is the switching point of the path P_x. If this is not the case, we will have $a < b < \{c,d\}$, where d is the neighbor of b in the path P_x distinct from a. Since P_x is rightmost and $b < c$, vertices d and c are not adjacent. Applying Lemma 1 to $a < b < \{c,d\}$ we get $bc \in E$. Moreover, from $b < \{d,c\}$, $dc \notin E$ and Lemma 2 we will find a vertex $t > \{c,d\}$ adjacent to c,d and not to b. The vertices t and a are not adjacent, otherwise P_x is not rightmost. Then $\{a,b,c,d,t\}$ induce a house, that is impossible.

So, P_x is a convex path and b is the switching point of P_x. By Lemma 4, $dist(v,b) \geqslant dist(x,b) = p - dist(v,b)$. Hence, $p \leqslant 2dist(v,b) = 2dist(v,a) + 2$, i.e. $p - 2dist(v,a) \leqslant 2$. Thus, $p+2 \leqslant dist(x,y) \leqslant dist(x,a) + dist(a,y) = 2dist(x,a) = 2p - 2dist(v,a) \leqslant$

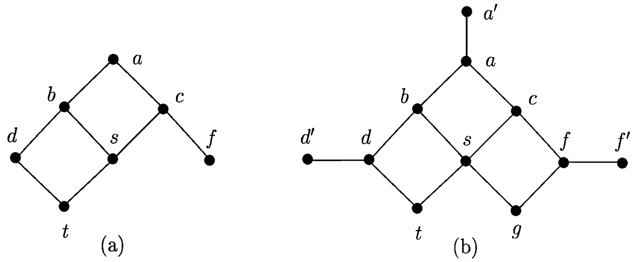

Fig. 4.

$p+2$, i.e. $dist(x,y)=dist(x,a)+dist(a,y)=2dist(x,a)=p+2$, p is even (say $p=2k$), and the graph from Fig. 3 is an isometric subgraph of G.

It remains to construct an induced subgraph of G isomorphic to the graph H_{k-1}. Since P_x is convex, b is the switching point of P_x and $dist(v,b)=dist(x,b)$, by Lemma 4, we have $a<d<b$. Applying (P1) to $a<d<c$ and Lemma 2 to $a<\{b,c\}$, $bc\notin E$ give vertices $s>c$ and $t>c$ such that $as, at\notin E$, s is adjacent to b,c, and t is adjacent to d (see Fig. 4(a)). We choose the vertices s and t rightmost in σ. From distance requirements $sd, sf, tc, tf\notin E$ holds.

Assume that $tb\in E$. Then $ts\notin E$, otherwise we will have an induced house formed by t,b,s,a,c. So, we can apply Lemma 2 to $b<\{t,s\}$ and find a vertex $z>\{t,s\}$ adjacent to t,s and not to b. To avoid a house induced by d,b,s,z,t, the vertices d and z must be adjacent. From the choice of t we conclude $za\in E$. Hence, the path P_x is not rightmost – a contradiction. So, $tb\notin E$ and since $\{d,b\}<\{t,s\}$, by Lemma 1, t and s must be adjacent.

Claim 1. *If there exists a vertex g adjacent to s, f and not to a, then G has an induced subgraph isomorphic to H_{k-1}.*

Proof. Since the graph from Fig. 3 is an isometric subgraph of G we have $gb, gd, gt\notin E$. Furthermore, $gc\notin E$, otherwise g,s,c,a,b induce a house. To see now that the vertices of the graph from Fig. 3 together with s,t and g induce H_{k-1} (see Fig. 4(b)), it is enough from distance requirements to show that $sa'\notin E$. But this is immediate, because $s>a$ and P_x is rightmost. $\square$

So, we may assume that $ga\in E$ for every vertex g adjacent to both f and s. Moreover, since P_y is rightmost, for every such vertex g, $g<c$ must hold. From this we infer also that $c>f$, otherwise Lemma 2, applied to $c<\{s,f\}$ and $sf\notin E$, gives a vertex $g>c$ adjacent to both s and f. Hence, the path P_y is convex too, and c is the switching point of P_y. By Lemma 4, $a<f<c$ holds.

Claim 2. *For every vertex g adjacent to f, $g\leqslant c$ holds.*

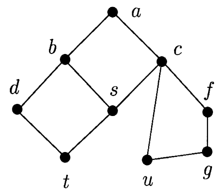

Fig. 5.

Proof. If $g > c$ then from the previous discussion g and s cannot be adjacent. Hence, by Lemma 1, g is adjacent to c (note that $\{c, f\} < \{s, g\}$). Now we have $c < \{s, g\}$ and $gs \notin E$. By Lemma 2 there exists a vertex $z > \{s, g\}$ adjacent to s, g and not to c. To avoid a house induced by z, s, g, c, f we must have $zf \in E$. But then the vertex z with $z > c$ adjacent to both s and f, a contradiction. □

From this claim we deduce that $f < b$, otherwise (P1) applied to $b < f < s$ will give a vertex $g > s > c$ adjacent to f. Now we apply (P1) to $a < f < b$ and find a vertex $g > b$ adjacent to f and not to a. Hence $sg \notin E$ and $g < c$. Then we can apply (P1) to $b < g < s$ and get a vertex $u > s$ adjacent to g and not to b. Since $u > c$, by Claim 2, $uf \notin E$. If $ua \in E$ then (P1) applied to $a < b < u$ will give a vertex $p > u$ adjacent to b and not to a. From $\{a, b\} < \{c, p\}$ and Lemma 1 the vertices p and c must be adjacent. But since $p > s$ this contradicts to the choice of s. So, u and a cannot be adjacent. We have $a < \{d, f\} < b < g < c < \{s, t\}$ and $s < u$.

Assume $gc \in E$. Then $\{c, g\} < \{s, u\}$ and Lemma 1 yield $uc \in E$ or $us \in E$. If $us \in E$ we obtain an induced house formed by s, u, g, c, f when $uc \notin E$ or by u, s, c, b, a otherwise. Hence, $su \notin E$ and $uc \in E$. Applying now Lemma 2 to $c < \{s, u\}$, $su \notin E$ we find a vertex $q > \{s, u\}$ adjacent to u, s and not to c. To avoid an induced house, q must be adjacent to g. By Claim 2, the vertex q with $q > c$ cannot be adjacent to f. Hence, q, s, g, c, f induce a house.

Thus, $gc \notin E$. From $\{f, g\} < \{c, u\}$ and Lemma 1 we infer $uc \in E$. Furthermore, $us \notin E$, otherwise we will have an induced house. From $dist(f, d) = 4$ we deduce $gb, gd, gt, ud \notin E$. If $ut \in E$ then we obtain an induced 6-cycle, that is impossible. Hence, $ut \notin E$ too. So, we have constructed an induced subgraph of G isomorphic to the graph from Fig. 5. Then $t < s$ must hold, since otherwise $\{s, c\} < \{t, u\}$ and a contradiction to the Lemma 1 arises.

Claim 3. *For every vertex z adjacent to d, $z \leqslant t$ holds.*

Proof. If $z > t$ then from the choice of t vertices a and z must be adjacent. But then the path P_x is not rightmost since $z > t > b$. □

Now we apply Lemma 2 to $c < \{s, u\}$ and (P1) to $c < t < u$ and get vertices q and p such that $u < \{q, p\}$, $qc, pc \notin E$ and $qs, qu, pt \in E$. Moreover, Claim 3 gives

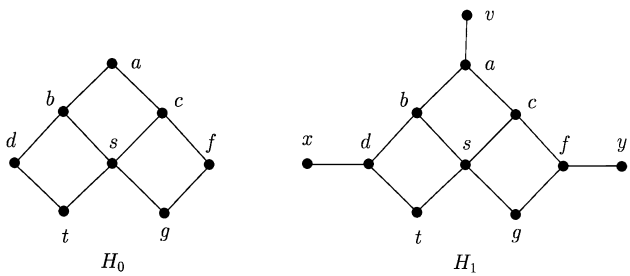

Fig. 6.

$pd, qd \notin E$. If $q \neq p$ then, by Lemma 1, from $\{t,s\} < \{q,p\}$ we obtain $ps \in E$ or $tq \in E$ or $pq \in E$. If $p = q$ or $tq \in E$ or $ps \in E$ then, to avoid an induced house formed by z,t,s,d,b where $z \in \{p,q\}$, we must have $zb \in E$. For the path $(z-b-a-c)$ with $\{b,a\} < \{z,c\}$ and $bc, zc \notin E$, by Lemma 1, we have $za \in E$. But then vertices z,t,d,b,a induce a house. Finally, we have $pq \in E$ and $tq, ps \notin E$. Since G is an HH-free graph cycle $(p-t-d-b-a-c-u-q-p)$ of G must have chords. All these chords are incident either to p or to q. From $dp, dq, tq \notin E$ we deduce that p and b must be adjacent. Now we can proceed as before and get that the vertices a and p must be adjacent, obtaining in this way an induced house formed by p,t,d,b,a. □

Corollary 6. *For every two vertices x and y of G, $dist(x,y) \leqslant \max\{dist(x,v), dist(y,v)\} + 2$ holds. Moreover, if G does not contain the graph H_0 as an isometric subgraph, then $dist(x,y) \leqslant \max\{dist(x,v), dist(y,v)\} + 1$.*

Proof. Assume that $dist(x,v) \geqslant dist(y,v)$, and let z be a vertex from a shortest path connecting vertices x and v, and such that $dist(y,v) = dist(z,v)$. By Lemma 5, $dist(y,z) \leqslant dist(z,v) + 2$. Hence, $dist(x,y) \leqslant dist(x,z) + dist(z,y) \leqslant dist(x,z) + dist(z,v) + 2 = dist(x,v) + 2$. Furthermore, if $dist(x,y) = dist(x,v) + 2$ then $dist(y,z) = dist(z,v) + 2$, and hence G contains the graph H_0 as an induced subgraph. Let H_0 be induced by vertices a,b,c,d,s,f,t,g as shown in Fig. 6. From the proof of Lemma 5 we have $dist(a,d) = dist(a,f) = 2$ and $dist(f,d) = 4$. To see that H_0 is an isometric subgraph of G, we need only to show that $dist(t,a) = dist(g,a) = 3$. Assume $dist(t,a) = 2$, and let w be a common neighbor of a and t. To avoid an induced cycle C_5 or an induced house, vertex w must be adjacent to all vertices of H_0. Hence, a contradiction to $dist(f,d) = 4$ arises. Thus, if G does not contain the graph H_0 as an isometric subgraph, then $dist(x,y) \leqslant dist(x,v) + 1$. □

Corollary 7. *If $e(v) = 2$ then $diam(G) \leqslant 3$.*

Proof. Assume $diam(G) = 4 = dist(x,y)$. Then we have $dist(x,v) = dist(y,v) = 2$. By Lemma 5, vertices v,x,y together with some vertices b,c,s,t,g induce a subgraph isomorphic to the graph H_0. As we have shown in the proof of Corollary 6, H_0 is an isometric subgraph of G. But then, $dist(v,t) = 3$ contradicts $e(v) = 2$. □

Theorem 8. *Let v be the first vertex of a LexBFS-ordering of a HH-free graph G. Then $e(v) \in \{diam(G), diam(G)-1, diam(G)-2\}$.*

Moreover, if G does not contain the graph H_0 as an isometric subgraph then $e(v) \in \{diam(G), diam(G)-1\}$.

Proof. Let x and y be vertices of G such that $dist(x,y) = diam(G)$. By Corollary 6, $diam(G) = dist(x,y) \leqslant \max\{dist(x,v), dist(y,v)\} + 2 \leqslant e(v) + 2 \leqslant diam(G) + 2$, i.e. $diam(G) - 2 \leqslant e(v) \leqslant diam(G)$. Analogously, if G does not contain the graph H_0 as an isometric subgraph, then $diam(G) - 1 \leqslant e(v) \leqslant diam(G)$. $\square$

Notice that the result of Theorem 8 is sharp. The graph H_1 from Fig. 6 is HH-free, but the first vertex of LexBFS-ordering $\sigma = (v, y, x, a, f, d, b, c, g, t, s)$ has the eccentricity $diam(G) - 2$; namely, $e(v) = dist(v,x) = 4 = dist(x,y) - 2 = diam(G) - 2$.

4. Approximation of the diameter of a HHD-free graph

Let σ be a LexBFS-ordering of a HHD-free graph G and v be the first vertex in σ. For HHD-free graphs, the following stronger version of Lemma 1 holds (see [18]).

Lemma 9 (Jamison and Olariu [18]). *G does not have any induced $P_4 = (c - a - b - d)$ with $a < \{b, c, d\}$.*

Lemma 10. *For every two vertices x and y of G with $dist(x,v) = dist(y,v) = p$, $dist(x,y) \leqslant p + 1$ holds. Moreover, if $dist(x,y) = p + 1 \geqslant 3$, then G contains one of the graphs from Fig. 7 as an isometric subgraph.*

Proof. That $dist(x,y) \leqslant p + 1$ follows from Lemma 5. Assume now that $dist(x,y) = p + 1 \geqslant 3$ and consider, as in the proof of Lemma 5, rightmost shortest paths P_x and P_y, connecting v with x and y, respectively. Let again a be the common vertex of the paths P_x and P_y furthest from v, and b, c with $b < c$ be the neighbors of a in the paths P_x and P_y, respectively (see Fig. 3). Since G is HH-free (even HHD-free) again one can show that P_x is a convex path of G and b is the switching point of P_x. By Lemma 4, we have $dist(v,b) \geqslant dist(x,b) = p - dist(v,b)$.

Claim 4. *If $dist(v,b) > dist(x,b)$, then G contains the graph* (a) *of Fig. 7 as an isometric subgraph.*

Proof. If $dist(v,b) > dist(x,b)$ then $p < 2dist(v,b) = 2dist(v,a) + 2$, i.e. $p - 2dist(v,a) < 2$. Hence, $p + 1 = dist(x,y) \leqslant dist(x,a) + dist(a,y) = 2dist(x,a) = 2p - 2dist(v,a) < p + 2$, i.e. $dist(x,y) = dist(x,a) + dist(a,y) = 2dist(x,a) = p + 1$, p is odd, and the graph from Fig. 3 is an isometric subgraph of G. Since $a < \{b, c\}$ and $bc \notin E$, by Lemma 2, there exists a vertex $s > \{b, c\}$ adjacent to b, c and not to a. From distance requirements we infer $sd, sf \notin E$ (recall that $p \geqslant 3$ and hence the vertices d, f, a' exist).

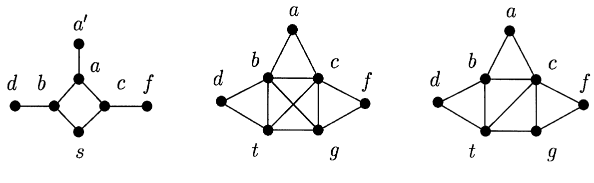

Fig. 7.

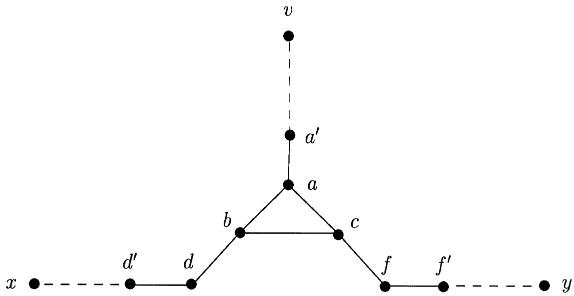

Fig. 8.

Also $sa' \notin E$ holds, otherwise P_x is not rightmost. To see now that a', a, b, c, d, f and s (see Fig. 3) induce the graph (a) of Fig. 7 as an isometric subgraph, we need only to show that $dist(s, a') = 3$. Assume $dist(s, a') = 2$, and let z be a common neighbor of a' and s rightmost in σ. To avoid an induced 5-cycle or an induced house, vertex z must be adjacent to a, b, c. Since P_x is rightmost and $zb, za' \in E$ we must have $z < a$. Applying (P1) to $z < a < s$ we get a vertex $u > s$ adjacent to a and not to z. From $z < \{a, s, u\}$, $zu, as \notin E$ and Lemma 9 vertices s and u must be adjacent. We have also $a'u \notin E$, otherwise a contradiction to the choice of z will arise. But then we obtain an induced house formed by a', a, z, u, s. □

So, we may assume $dist(v, b) = dist(x, b) = p - dist(v, b)$, i.e. $p = 2dist(v, b)$. Since P_x is convex and b is the switching point of P_x, by Lemma 4, we have $a < d < b$. From $b < c$ and P_x is rightmost we infer $dc \notin E$. Furthermore, Lemma 9 applied to $a < \{d, b, c\}, da, dc \notin E$ gives $bc \in E$. Hence, $p + 1 = dist(x, y) \leqslant dist(x, b) + 1 + dist(c, y) = 2dist(x, b) + 1 = 2dist(v, b) + 1 = p + 1$, i.e. $dist(x, y) = dist(x, b) + 1 + dist(c, y)$ holds and the graph from Fig. 8 is an isometric subgraph of G.

Applying (P1) to $a < d < c$ gives a vertex $t > c$ adjacent to d and not to a. We choose t rightmost in σ.

Claim 5. *For every vertex z adjacent to d, $z \leqslant t$ holds.*

Proof. If $z > t$ then from the choice of t the vertices a and z must be adjacent. But then the path P_x is not rightmost since $z > t > b$. $\square$

From distance requirement $tf \notin E$ holds. Since $d < \{b,c,t\}$, by Lemma 9, we have $tb \in E$ or $tc \in E$. But if $tc \in E$ then $tb \in E$ too, otherwise d,t,b,c,a will induce a house. So, in any case $tb \in E$. If now $tc \notin E$ then from $b < \{c,t\}$ and Lemma 2 we will have a vertex $s > \{t,c\}$ adjacent to t,c and not to b. To avoid an induced house, vertex s must be adjacent to d. Since $t < s$ a contradiction with Claim 5 arises. Thus, $tc \in E$ as well.

If $c < f$ then Lemma 2 applied to $c < \{t,f\}$ gives a vertex $s > \{t,f\}$ adjacent to t,f and not to c. Again to avoid an induced house, we must have $bs \in E$ and hence $as \in E$. Since $s > c$ and P_y is rightmost a contradiction arises.

So, $f < c$ and hence P_y is a convex path with the switching point c. By Lemma 4 $a < f < c$ holds. We distinguish between two cases.

Case 1. $b < f$. Applying (P1) to $b < f < t$ gives a vertex $s > t$ adjacent to f and not to b. Since P_y is rightmost and $s > c$ we infer $sa \notin E$. From $f < \{s,c,t\}$ and Lemma 9 we have $st \in E$ or $sc \in E$. If $st \in E$ then $sc \in E$ too, otherwise s,t,b,c,f induce a house. Hence, we have constructed the graph (c) of Fig. 7. Since $dist(f,d) = 3$ it is an isometric subgraph of G. Let now $st \notin E$ but $sc \in E$. Then Lemma 2 applied to $c < \{t,s\}$ gives a vertex $u > \{t,s\}$ adjacent to t,s and not to c. Since G does not contain any induced house we must have $bu, fu \in E$ and hence $au \in E$. Thus, a contradiction arises (recall that P_y is rightmost but $u > c$).

Case 2. $f < b$. We apply (P1) to $a < f < b$ and get a vertex $s > b$ adjacent to f and not to a. We choose s rightmost in σ. If $ts \in E$ then, to avoid an induced house, we must have $sc \in E$. But then vertices a,b,c,d,f,t,s induce either graph (b) or graph (c) of Fig. 7, depending on whether b and s are adjacent. Again since $dist(f,d) = 3$ these graphs are isometric subgraphs of G.

Let now $st \notin E$. Then Lemma 9 applied to $f < \{s,c,t\}$ gives $sc \in E$. If $c < s$ then, by Lemma 2, there exists a vertex $u > \{t,s\} > c$ adjacent to both t and s but not to c. Furthermore, $uf, ub, ua \in E$, otherwise we will have an induced house. But then again a contradiction to P_y is rightmost arises. Hence, $c > s$ and the following claim holds.

Claim 6. *For every vertex z adjacent to f, $z \leqslant c$ holds.*

If $bs \in E$ then Lemma 2 applied to $b < \{s,t\}, st \notin E$ gives a vertex $z > \{s,t\}$ adjacent to s,t and not to b. To avoid an induced house, vertices d and z must be adjacent, contradicting to Claim 5. So, $bs \notin E$.

Now apply (P1) to $b < s < t$ to get a vertex $u > t$ adjacent to s and not to b. First assume $tu \notin E$. Then from $s < \{c,t,u\}$ and Lemma 9 we infer $uc \in E$. Again Lemma 2 applied to $c < \{u,t\}$, $ut \notin E$ will give a vertex $z > \{u,t\}$ adjacent to u,t and not to c. Since G does not contain any induced house, vertex z must be adjacent to b,s and hence to f. But then a contradiction with Claim 6 arises.

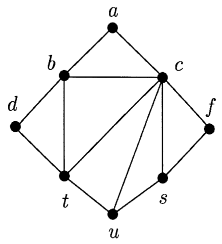

Fig. 9.

So, $tu \in E$. Hence, $uc \in E$ too, otherwise vertices t, u, s, c, b induce a house. Moreover, from $u > t > c$, Claims 5 and 6 we conclude $ud, uf \notin E$. Then $ua \notin E$ too, since otherwise u, a, b, t, d will induce a house. Thus, we have constructed an induced subgraph of G isomorphic to the graph from Fig. 9.

If $dist(s, d) = 3$ then vertices a, b, c, d, s, t, u form a subgraph isometric to the graph (c) of Fig. 7. So, assume that $dist(s, d) = 2$ and let z be a common neighbor of s and d. Since G is HH-free in cycle formed by z, d, b, c, s we must have chords zb and zc. If $za \notin E$ then vertices a, b, c, d, f, s, z form again a subgraph of G isometric to the graph (c) of Fig. 7. So, let $za \in E$. Since P_x is rightmost we conclude $z < b$. Applying (P1) to $z < b < s$ gives a vertex $w > s$ adjacent to b and not to z. Since $z < \{b, s, w\}$, by Lemma 9, $ws \in E$. To avoid an induced house, w must be adjacent to both d and a. Then a contradiction arises since P_x was rightmost but $w > b$. □

Corollary 11. *For every two vertices x and y of G, $dist(x, y) \leqslant \max\{dist(x, v), dist(y, v)\} + 1$ holds. Moreover, if G does not contain any graph of Fig. 7 as an isometric subgraph, then $dist(x, y) \leqslant \max\{dist(x, v), dist(y, v), 2\}$.*

Proof. Assume that $dist(x, v) \geqslant dist(y, v)$, and let z be a vertex from a shortest path connecting vertices x and v and such that $dist(y, v) = dist(z, v)$. By Lemma 10, $dist(y, z) \leqslant dist(z, v) + 1$. Hence, $dist(x, y) \leqslant dist(x, z) + dist(z, y) \leqslant dist(x, z) + dist(z, v) + 1 = dist(x, v) + 1$. Let $dist(x, y) = dist(x, v) + 1$. Then $dist(y, z) = dist(z, v) + 1$. If $dist(y, v) \geqslant 2$ then, by Lemma 10, G contains one of the graphs of Fig. 7 as an isometric subgraph. So, assume that $dist(z, v) = dist(y, v) = 1$. If $z = x$ then $dist(x, y) = 2$ and we are done. Now let $z \neq x$ and u be the neighbor of z on a shortest path connecting vertices x and z. Vertices u and y cannot be adjacent, otherwise $dist(x, y) \leqslant dist(x, z) = dist(x, v) - 1$. Furthermore, $zy, vu \notin E$. Thus, in induced path $(u - z - v - y)$ we have $v < \{u, z, y\}$, contradicting Lemma 9. □

Corollary 12. *If $e(v) = 1$ then $diam(G) = 1$.*

Proof. Assume that $diam(G) = dist(x, y) = 2$. Then we have $dist(x, v) = dist(y, v) = 1$ and $xy \notin E$. Hence, Lemma 2 applied to $v < \{x, y\}$ gives a vertex s such that $vs \notin E$. But this contradicts with $e(v) = 1$. □

Theorem 13. *Let v be the first vertex of a LexBFS-ordering of a HHD-free graph G. Then* $e(v) \in \{diam(G), diam(G) - 1\}$.

Moreover, if G does not contain any graph of Fig. 7 as an isometric subgraph, then $e(v) = diam(G)$.

Proof. Let x and y be vertices of G such that $dist(x, y) = diam(G)$. By Corollary 11, $diam(G) = dist(x, y) \leqslant \max\{dist(x, v), dist(y, v)\} + 1 \leqslant e(v) + 1 \leqslant diam(G) + 1$, i.e. $diam(G) - 1 \leqslant e(v) \leqslant diam(G)$. Moreover, if $e(v) = 1$ then $diam(G) = 1 = e(v)$. Let now $e(v) > 1$ and G does not contain any graph of Fig. 7 as an isometric subgraph. Then $diam(G) = dist(x, y) \leqslant \max\{dist(x, v), dist(y, v), 2\} \leqslant e(v) \leqslant diam(G)$, i.e. $e(v) = diam(G)$. □

Again the result of Theorem 13 is sharp. Each of the graphs from Fig. 7 has a LexBFS-ordering σ such that the eccentricity of the first vertex in σ equals $diam(G) - 1$ (check ordering (a', d, f, a, b, c, s) of the graph (a) and ordering (a, f, d, b, c, g, t) of graphs (b) and (c)).

Corollary 14. *Let v be the first vertex of a LexBFS-ordering of a HHD-free and AT-free graph G. Then* $e(v) = diam(G)$.

Proof. It is easy to see that vertices a', d, f of the graph (a) as well as vertices a, d, f of graphs (b) and (c) (see Fig. 7) form an asteroidal triple. Hence, each HHD-free graph, which does not have an asteroidal triple, does not contain any graph of Fig. 7 as an isometric subgraph. □

Corollary 15 (Dragan et al. [12]). *Let v be the first vertex of a LexBFS-ordering of a chordal, or a distance-hereditary, or a weak bipolarizable graph G. Then* $e(v) \in \{diam(G), diam(G) - 1\}$.

Corollary 16 (Dragan et al. [12]). *Let v be the first vertex of a LexBFS-ordering of an interval or a ptolemaic graph G. Then* $e(v) = diam(G)$.

5. Conclusion

In this paper we have proven that the vertex visited last by LexBFS has eccentricity at least $diam(G) - 2$ for house-hole-free graphs, at least $diam(G) - 1$ for house-hole-domino-free graphs, and equal to $diam(G)$ for house-hole-domino-free and AT-free graphs. This generalizes results known from [12] on diameters of chordal, distance-hereditary, weak bipolarizable, interval and ptolemaic graphs. An open question remains, for which other classes of graphs, the diameter can be computed via LexBFS? In [6] we continue investigations in this direction by proving that the diameter of a directed path graph and a chordal comparability graph can be computed in linear-time using two sweeps of LexBFS. As the next result shows, for general graphs,

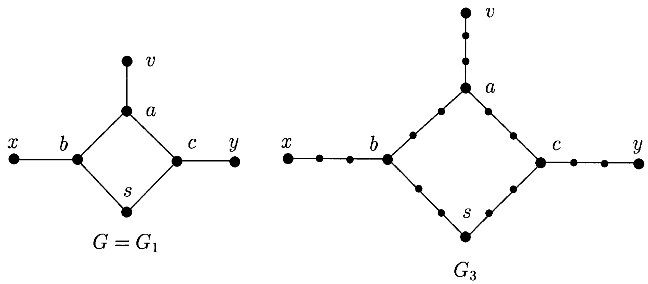

Fig. 10.

there is no constant k such that the eccentricity of the last visited by LexBFS vertex is at least $diam(G) - k$. So, we have to restrict ourselves to some well-structured classes of graphs or/and to considering a few sweeps of LexBFS.

Proposition 17. *For any constant k, there exists a graph G_k and a LexBFS-ordering σ of it such that the first vertex of σ will have the eccentricity equal to $diam(G_k) - k$.*

Proof. Let G be the graph (a) from Fig. 7 and G_k be the graph obtained from G by replacing each edge of G with a path of length k (see Fig. 10 for $k = 3$). It is easy to see that in LexBFS-ordering σ of G_k started from s, i.e. $v_n = s$, the vertex v will be numbered by 1, but $e(v) = dist(v, x) = 3k = 4k - k = dist(x, y) - k = diam(G_k) - k$. □

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