Following Derek's footsteps

Feodor Dragan

May, 2014

Derek's Primary Universe

- The talk is not about this Derek
- These footsteps are hard to follow

Derek's Parallel Universe

decomposition

domination

minors

Interval

LBFS

LDFS

Tree-width

AT-free

Co-comp

Search

Clique-width

Partial k-tree

Path cover

NP-hard

cographs

Linear

Cliques

Dom. path

Dom. pair

Tree spanner

Perfect graphs

Efficient

Tree power

- ***Not a complete picture***
- ***I didn't want to block the Sun***

Talk outline

- **collaborating with Derek**
  - fast estimation of diameters
  - representing approximately graph distances with few tree distances
- **following Derek's footsteps**
  - tree- and path-decompositions and new graph parameters
  - Approximating tree t-spanner problem using tree-breadth
  - Approximating bandwidth using path-length
  - Approximating line-distortion using path-length

Talk outline

- collaborating with Derek
  - fast estimation of diameters

2003		
4	EE	Derek G. Corneil, Feodor F. Dragan, Ekkehard Köhler: On the power of BFS to determine a graph's diameter. Networks 42(4):209-222 (2003)
2002		
3	EE	Derek G. Corneil, Feodor F. Dragan, Ekkehard Köhler: On the Power of BFS to Determine a Graphs Diameter. LATIN 2002:209-223
2001		
2	EE	Derek G. Corneil, Feodor F. Dragan, Michel Habib, Christophe Paul: Diameter determination on restricted graph families. Discrete Applied Mathematics (DAM) 113(2-3):143-166 (2001)
1998		
1	EE	Derek G. Corneil, Feodor F. Dragan, Michel Habib, Christophe Paul: Diameter Determination on Restricted Graph Faminlies. WG 1998:192-202

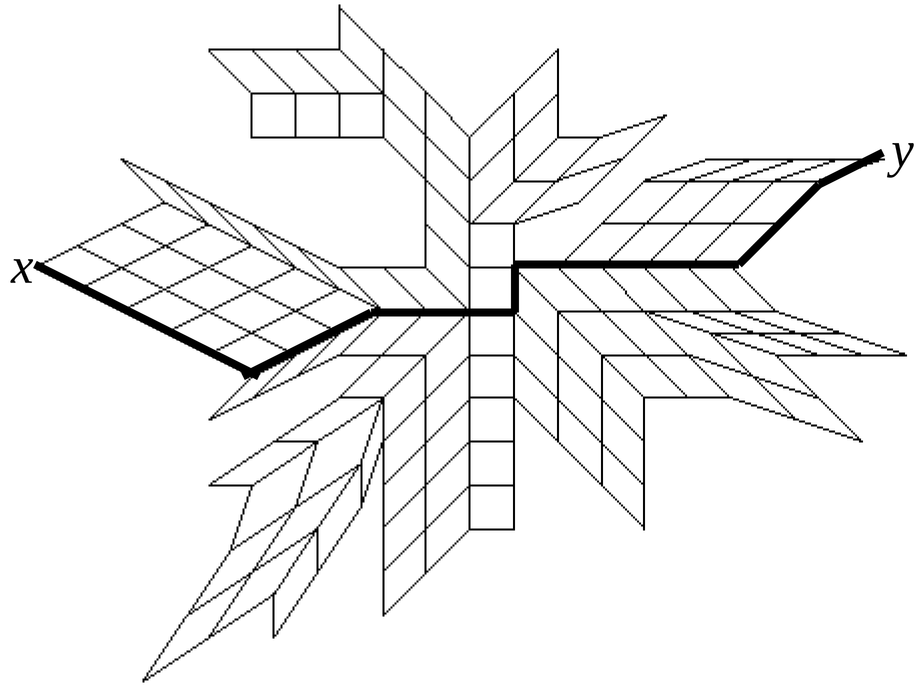
The Diameter Problem

- The ***eccentricity*** $\mathrm{ecc}(v) = diam(G)$ of a vertex v is the maximum distance from v to a vertex in G

- The ***diameter*** $diam(G)$ is the maximum eccentricity of a vertex of G

- **The diameter problem** (find a longest shortest path in a graph)**:**

  find $diam(G)$ and x, y
  such that $d(x, y) = diam(G)$
  (in other words, find a vertex of maximum eccentricity)



$$diam(G) = d(x, y) = 20$$

Our Approach

4 | EE | Derek G. Corneil, Feodor F. Dragan, Ekkehard Köhler: On the power of BFS to determine a graph's diameter. Networks 42(4):209-222 (2003)

- Examine the naïve algorithm of
  - choosing a vertex
  - performing some version of BFS from this vertex and then
  - showing a nontrivial bound on the eccentricity of the last vertex visited in this search.
- This approach has already received considerable attention
  - (classical result [Handler'73]) for trees this method produces a vertex of maximum eccentricity
  - [Dragan et al' 97] if LexBFS is used for chordal graphs, then $\mathrm{ecc}(v) \geq diam(G) - 1$ whereas for interval graphs and Ptolemaic graphs $\mathrm{ecc}(v) = diam(G)$
  - [Corneil et al'99] if LexBFS is used on AT-free graphs, then $\mathrm{ecc}(v) \geq diam(G) - 1$
  - [Dragan'99] if LexBFS is used, then $\mathrm{ecc}(v) \geq diam(G) - 2$ for HH-free graphs, $\mathrm{ecc}(v) \geq diam(G) - 1$ for HHD-free graphs and $\mathrm{ecc}(v) = diam(G)$ for HHD-free and AT-free graphs
  - [Corneil et al'01] considered multi sweep LexBFSs …

Variants of BFS used

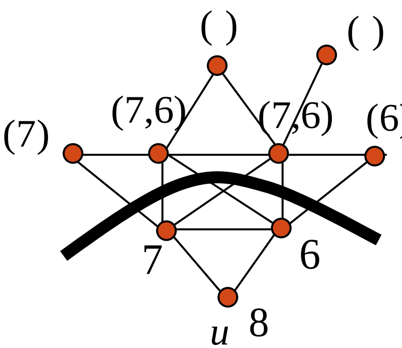
Algorithm BFS: Breadth First Search
Input: graph $G(V, E)$ and vertex u
Output: vertex v, the last vertex visited by a BFS starting at u

Initialize queue Q to be $\{u\}$ and mark u as "visited".
while $Q \neq \emptyset$ **do**
  Let v be the first vertex of Q and remove it from Q.
  Each unvisited neighbour of v is added to the end of Q and marked as "visited".

Can be implemented to run in linear time

Algorithm LBFS: Lexicographic Breadth First Search
Input: graph $G(V, E)$ and vertex u
Output: vertex v, the last vertex visited by an LBFS starting at u

Assign label $\emptyset$ to each vertex in V.
for $i = n$ **downto** 1 **do**
  Pick an unmarked vertex v with the largest (with respect to lexicographic order) label.
  Mark v "visited".
  For each unmarked neighbour y of v, add i to the label of y.



Algorithm LL: Last Layer
Input: graph $G(V, E)$ and vertex u
Output: vertex v, a vertex in the last layer of u

Run BFS to get the layering of V with respect to u.
Choose v to be an arbitrary vertex in the last layer.

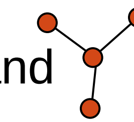
Algorithm LL+: Last Layer, Minimum Degree
Input: graph $G(V, E)$ and vertex u
Output: vertex v, a vertex in the last layer of u, that has minimum degree with respect to the vertices in the previous layer

Our Results on Restricted Families of Graphs

GRAPH CLASS	LL	LL+	BFS	LBFS
chordal graphs	$\geq D-2$ [2] Fig. 4	$\geq D-2$ [2] Fig. 5	$\geq D-1$ [*] Fig. 2	$\geq D-1$ [6] Fig.6
AT-free graphs	$\geq D-2$ [*] Fig. 3	$\geq D-1$ [*] Fig. 7	$\geq D-2$ [*] Fig. 3	$\geq D-1$ [3] Fig. 7
{AT,claw}-free graphs	$\geq D-1$ [*] Fig. 2	$= D$ [*]	$\geq D-1$ [*] Fig. 2	$= D$ [*]
interval graphs	$\geq D-1$ [*] Fig. 2	$= D$ [*]	$\geq D-1$ [*] Fig. 2	$= D$ [6]
hole-free graphs	$\geq D-2$ [*] Fig. 8	$\geq D-2$ [*] Fig. 8	$\geq D-2$ [*] Fig. 8	$\geq D-2$ [*] Fig. 8

No induced cycles of length >3

No asteroidal triples

No asteroidal triples and 

The intersection graph of intervals of a line

No induced cycles of length >4

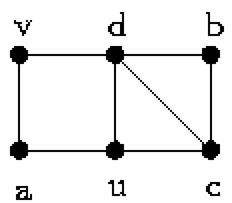


Figure 7: LBFS: u|cda|bv

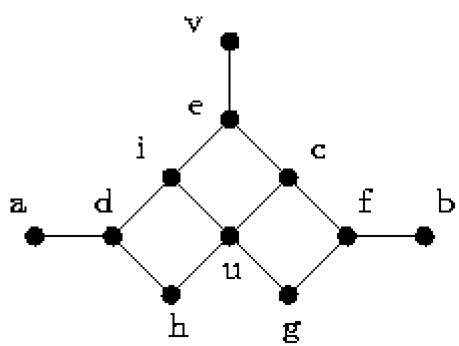
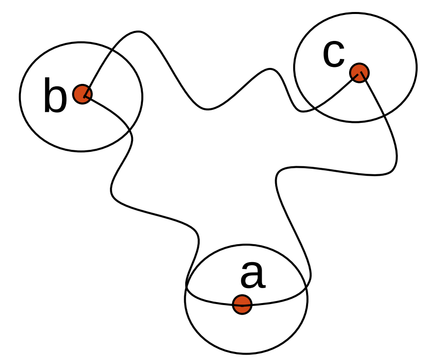


Figure 8: LBFS: u|ghic|dfe|abv



asteroidal triple a,b,c

Arbitrary k-Chordal graphs

- a graph is k-*chordal* if it has no induced cycles of length greater than k.

4 EE Derek G. Corneil, Feodor F. Dragan, Ekkehard Köhler: On the power of BFS to determine a graph's diameter. Networks 42(4):209-222 (2003)

  - if LL is used for k-chordal graphs ($k > 3$), then $\mathrm{ecc(v)} \geq diam(G) - \lfloor k/2 \rfloor$

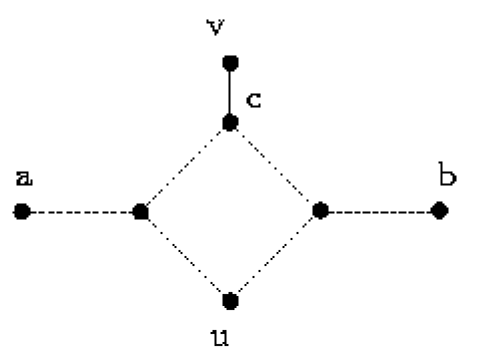
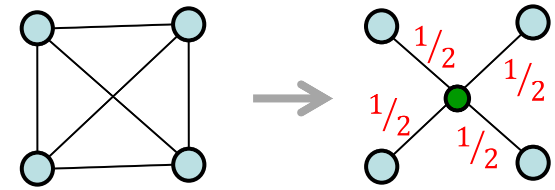


Figure 14: LBFS: u|. . . |acb|v

  - $k = 4l$
  - $diam(G) = 4l = k = d(a, b)$
  - $ecc(v) = 2l + 1 = 4l - 2l + 1$
    $= diam(G) - k/2 + 1$

- **Conclusion**:
  - Full power of LBFS is not needed
  - Good bounds hold for other graph families

Hyperbolic graphs

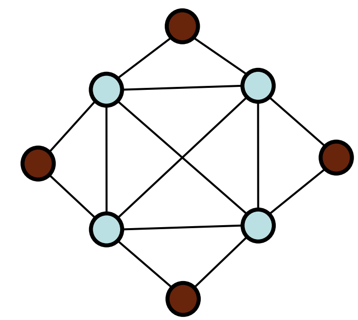


$hb(K_n) = 0$ (is a tree metrically)

δ-Hyperbolicity (M. Gromov, 1987)

for any four points u, v, w, x of a metric space (X, d), the two larger of the distance sums $d(u,v) + d(w,x)$, $d(u,w) + d(v,x)$, $d(u,x) + d(v,w)$ differ by at most 2δ.

δ-Hyperbolicity measures the local deviation of a metric from a tree metric: *a metric is a tree metric iff it is 0-hyperbolic.*



$hb(S_4) = 1$

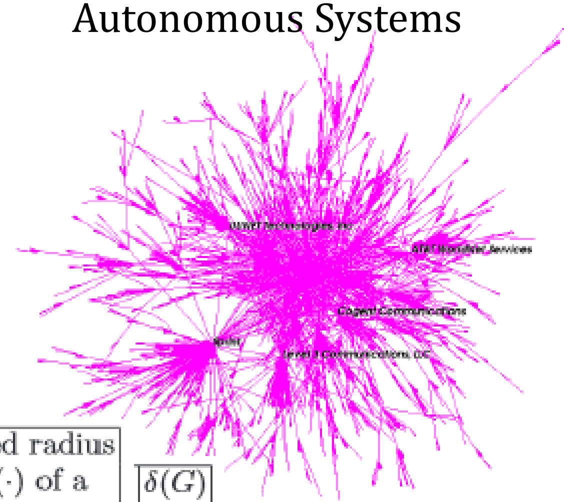
- $hb(G) = 0$ iff G is a block graph (metrically a tree)
- Chordal graphs: $hb(G) \leq 1$ [Brinkmann, Koolen, Moulton: (2001)]
- k-Chordal graphs (k>3): $hb(G) \leq {}^{k}/_{4}$ [Wu, Zhang: (2011)]

74	EE	Victor Chepoi, Feodor F. Dragan, Bertrand Estellon, Michel Habib, Yann Vaxès: Diameters, centers, and approximating trees of delta-hyperbolicgeodesic spaces and graphs. SoCG 2008:59-68

  - if LL is used for δ-hyperbolic graphs, then $\mathrm{ecc}(v) \geq diam(G) - 2\delta$

Real-Life datasets

Autonomous Systems



Graph $G=(V,E)$	n= $\lvert V\rvert$	m= $\lvert E\rvert$	diameter $diam(G)$	radius $rad(G)$	# of BFS scans needed to get $diam(G)$	estimated radius or $ecc(\cdot)$ of a middle vertex	$\delta(G)$
PPI [46]	1458	1948	19	11	3	12	3.5
Yeast [14]	2224	6609	11	6	3	6	2.5
DutchElite [29]	3621	4311	22	12	4	13	4
EPA [1]	4253	8953	10	6	2	7	2.5
EVA [57]	4475	4664	18	10	2	10	1
California [49]	5925	15770	13	7	2	8	3
Erdös [10]	6927	11850	4	2	2	3	2
Routeview [2]	10515	21455	10	5	2	5	2.5
Homo release 3.2.99 [63]	16711	115406	10	5	2	6	2
AS_Caida_20071105 [18]	26475	53381	17	9	2	9	2.5
Dimes 3/2010 [61]	26424	90267	8	4	2	5	2
Aqualab 12/2007- 09/2008 [19]	31845	143383	9	5	2	5	2
AS_Caida_20120601 [16]	41203	121309	10	5	2	5	2
itdk0304 [17]	190914	607610	26	14	2	15	
DBLB-coauth [67]	317080	1049866	23	12	2	14	
Amazon [67]	334863	925872	47	24	2	26	

104 EE Muad Abu-Ata, Feodor F. Dragan: Metric tree-like structures in real-life networks: an empirical study. CoRR abs/1402.3364 (2014)

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8	EE	Feodor F. Dragan, Derek G. Corneil, Ekkehard Köhler, Yang Xiang: Additive Spanners for Circle Graphs and Polygonal Graphs. WG 2008:110-121
2006		
7	EE	Feodor F. Dragan, Chenyu Yan, Derek G. Corneil: Collective Tree Spanners and Routing in AT-free Related Graphs. J. Graph Algorithms Appl. (JGAA) 10(2):97-122 (2006)
2005		
6	EE	Derek G. Corneil, Feodor F. Dragan, Ekkehard Köhler, Chenyu Yan: Collective Tree 1-Spanners for Interval Graphs. WG 2005:151-162
2004		
5	EE	Feodor F. Dragan, Chenyu Yan, Derek G. Corneil: Collective Tree Spanners and Routing in AT-free Related Graphs. WG 2004:68-80

Tree *t* -Spanner Problem

Defined this object

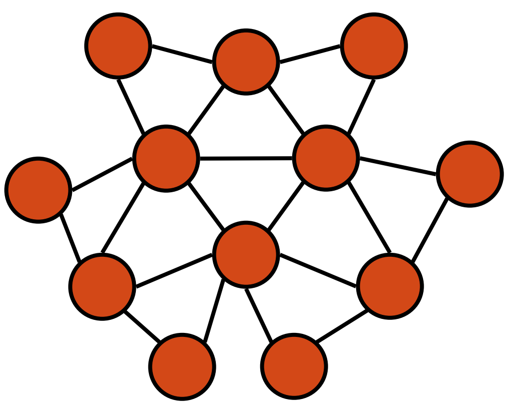
35 EE Leizhen Cai, Derek G. Corneil: Tree Spanners. SIAM J. Discrete Math. (SIAMDM) 8(3):359-387 (1995)

- Given unweighted undirected graph $G=(V,E)$ and integers t, s.
- Does G admit a spanning tree $T=(V,E')$ such that

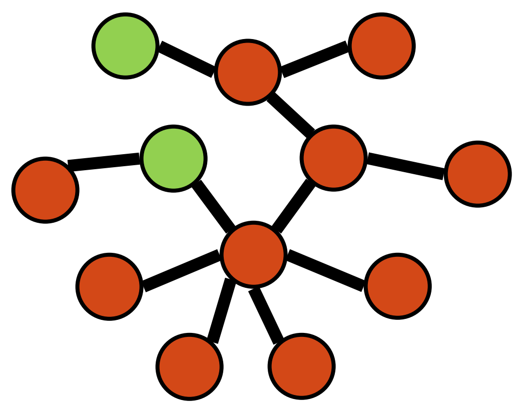
$$\forall u,v \in V, \quad dist_T(v,u) \le t \times dist_G(v,u)$$ (a *multiplicative* tree *t-spanner* of G)

or

$$\forall u,v \in V, \quad dist_T(u,v) - dist_G(u,v) \le s$$ (an *additive* tree *s-spanner* of G)?



G



multiplicative tree 4- and

additive tree 3- spanner of G

Some known results for the tree spanner problem (mostly multiplicative case)

- general graphs [CC'95]
  - $t \geq 4$ is NP-complete. ($t=3$ is still open, $t \leq 2$ is P)
- approximation algorithm for general graphs [EP'04]
  - $O(\log n)$ approximation algorithm
- chordal graphs [BDLL'02]
  - $t \geq 4$ is NP-complete. ($t=3$ is still open.)
- planar graphs [FK'01]
  - $t \geq 4$ is NP-complete. ($t=3$ is polynomial time solvable.)
- AT-free graphs and their subclasses
  - additive tree 3-spanner [Pr'99, PKLMW'03]
  - a permutation graph admits a multiplicative tree 3-spanner [MVP'96]
  - an interval graph admits an additive tree 2-spanner

Collective Additive Tree *r* -Spanners Problem

Feodor F. Dragan, Chenyu Yan, Irina Lomonosov: Collective Tree Spanners of Graphs. SWAT 2004: 64-76

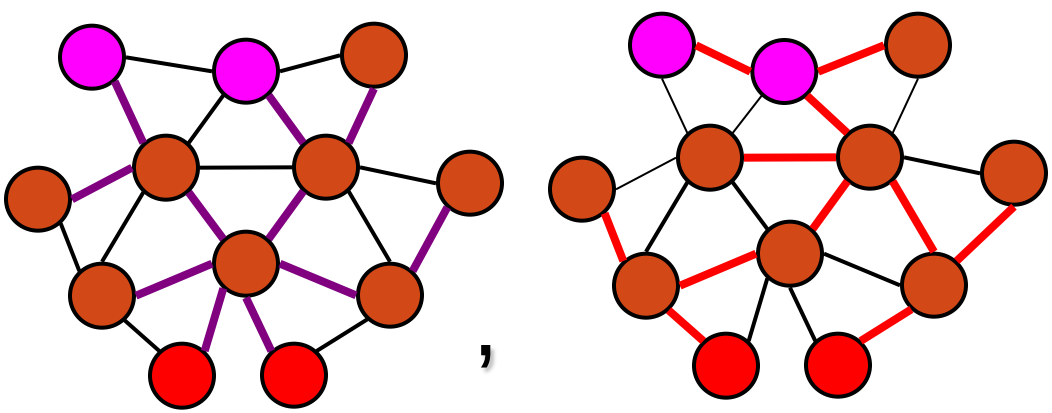
Feodor F. Dragan, Chenyu Yan, Derek G. Corneil: Collective Tree Spanners and Routing in AT-free Related Graphs. WG 2004: 68-80

- Given unweighted undirected graph $G=(V,E)$ and integers μ, r.
- Does G admit a system of μ collective additive tree *r-spanners* $\{T_1, T_2 \ldots, T\mu\}$ such that

$$\forall u,v \in V \; and \; \exists 0 \le i \le \mu, \; dist_{T_i}(v,u) - dist_G(v,u) \le r$$

surplus

(a system of μ collective *additive tree r-spanners* of G)?



2 collective additive tree 2-spanners

collective multiplicative *tree t-spanners* *can be defined similarly*

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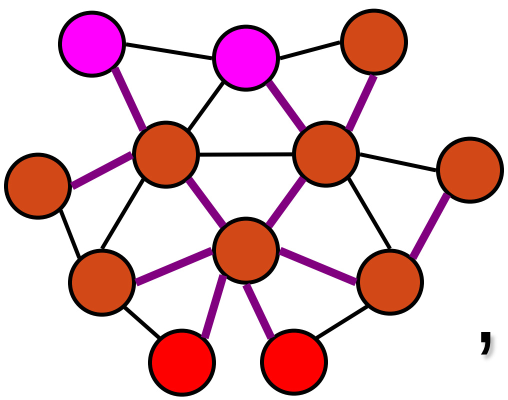
(a system of μ collective *additive tree r-spanners* of G)?



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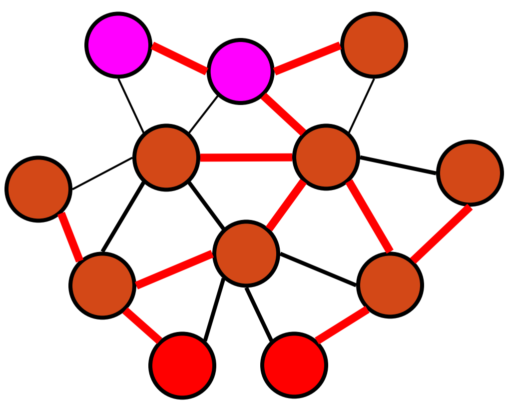

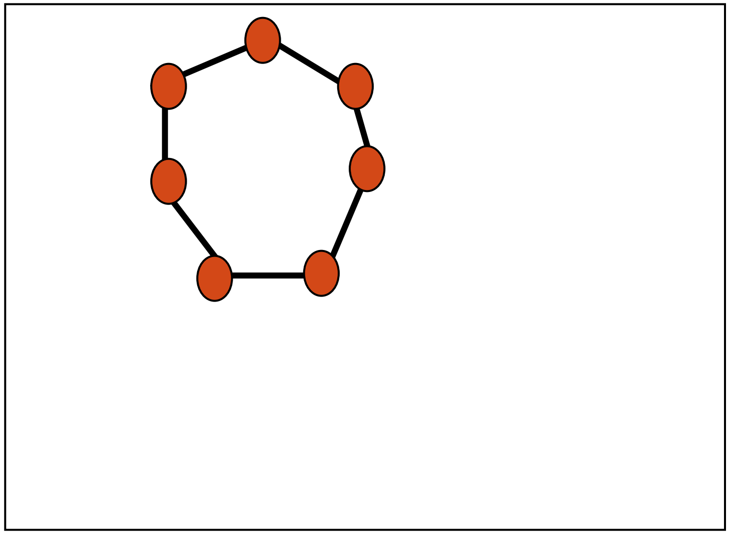

2 **collective additive tree *2*-spanners**

Collective Additive Tree *r* -Spanners Problem

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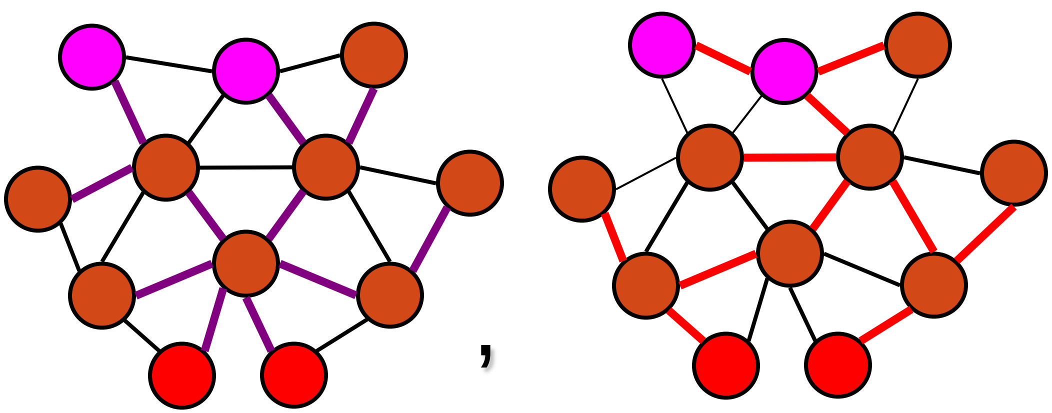
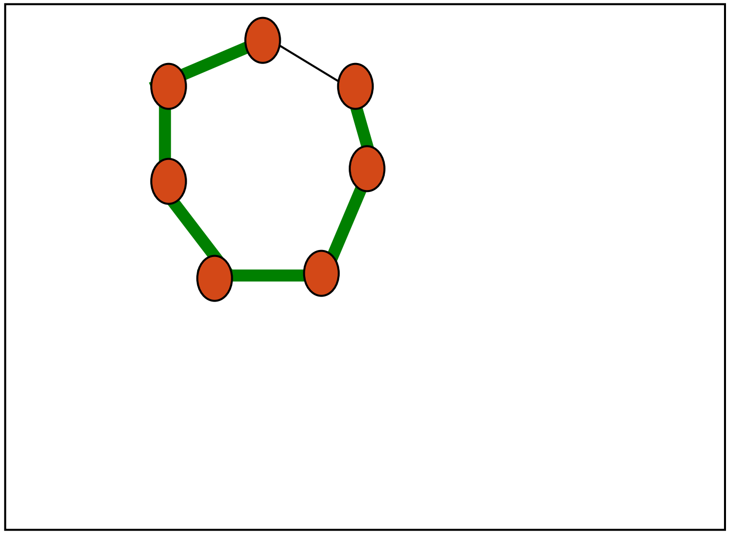
(a system of μ collective *additive tree r-spanners* of G)?





2 collective additive tree 2-spanners

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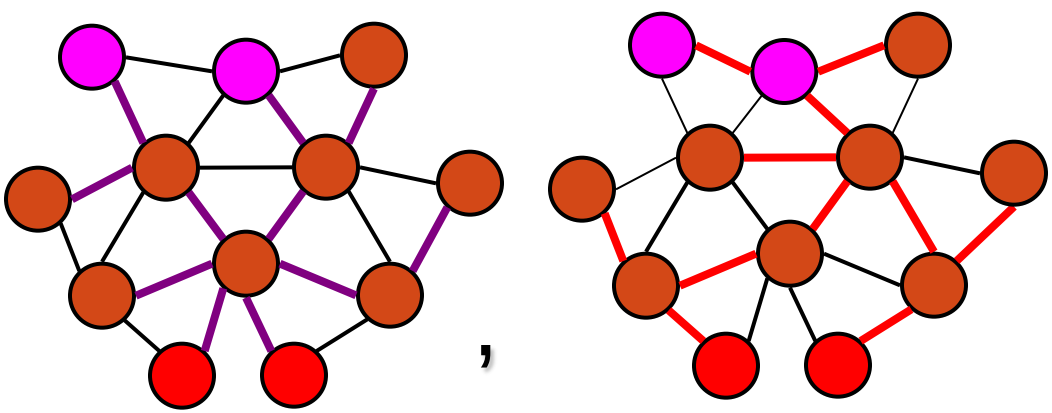
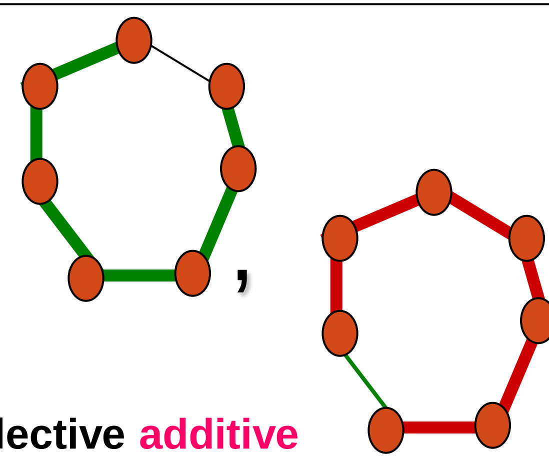
(a system of μ collective *additive tree r-spanners* of G)?



2 collective additive tree 2-spanners



2 collective additive tree 0-spanners

Applications of Collective Tree Spanners

- **message routing in networks**

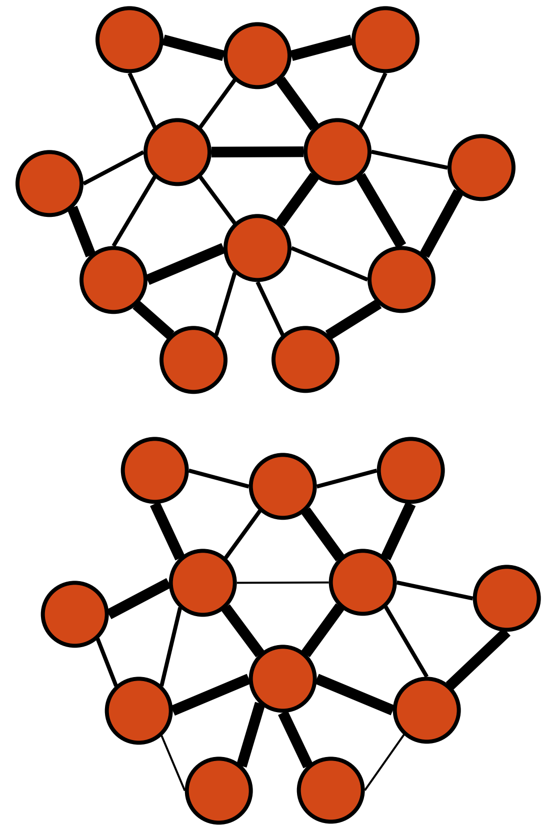
  Efficient routing schemes are known for trees

  but not for general graphs. For any two nodes, we can route the message between them in one of the trees which approximates the distance between them.

  - *($\mu \log^2 n$)-bit labels,*
  - *$O(\mu)$ initiation, $O(1)$ decision*

- **solution for sparse t-spanner problem**

  If a graph admits a system of μ collective additive tree r-spanners, then the graph admits a sparse additive r-spanner with at most $\mu(n-1)$ edges, where n is the number of nodes.



2 collective tree 2-spanners for G

Some results on collective tree spanners

Feodor F. Dragan, Chenyu Yan, Irina Lomonosov: Collective Tree Spanners of Graphs. SWAT 2004: 64-76
Feodor F. Dragan, Chenyu Yan, Derek G. Corneil: Collective Tree Spanners and Routing in AT-free Related Graphs. WG 2004: 68-80

- chordal graphs, chordal bipartite graphs
  - $\log n$ collective additive tree 2-spanners in polynomial time
  - $\Omega(n^{1/2})$ or $\Omega(n)$ trees necessary to get $+1$
  - no constant number of trees guaranties $+2$ $(+3)$
- circular-arc graphs
  - 2 collective additive tree 2-spanners in polynomial time
- k-chordal graphs
  - $\log n$ collective additive tree $2\lfloor k/2 \rfloor$-spanners in polynomial time
- interval graphs
  - $\log n$ collective additive tree 1-spanners in polynomial time
  - no constant number of trees guaranties $+1$

Results for AT-free graphs

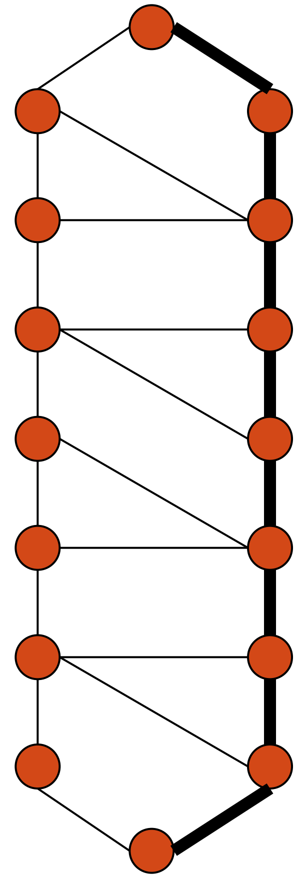
Feodor F. Dragan, Chenyu Yan, Derek G. Corneil: Collective Tree Spanners and Routing in AT-free Related Graphs. J. Graph Algorithms Appl. 10(2): 97-122 (2006)

- AT-free graphs
  - include: interval, permutation, trapezoid, co-comparability
  - 2 collective additive tree *2*-spanners in linear time
  - an additive tree *3*-spanner in linear time (before)
- graphs with a dominating shortest path
  - an additive tree *4*-spanner in polynomial time (before)
  - 2 collective additive tree *3*-spanners in polynomial time
  - 5 collective additive tree *2*-spanners in polynomial time
- graphs with asteroidal number an(G)=k
  - k(k-1)/2 collective additive tree 4-spanners in polynomial time
  - k(k-1) collective additive tree 3-spanners in polynomial time

Results for AT-free graphs

Feodor F. Dragan, Chenyu Yan, Derek G. Corneil: Collective Tree Spanners and Routing in AT-free Related Graphs. J. Graph Algorithms Appl. 10(2): 97-122 (2006)

- Any AT-free graph G admits an additive tree 3-spanner [PKLMW'03]
- **Thm:** Any AT-free graph G admits a system of 2 collective additive tree 2-spanners which can be constructed in linear time.
- To get $+2$, one needs at least 2 spanning trees
- To get $+1$, one needs at least $\Omega(n)$ spanning trees



an AT-free graph with its backbone

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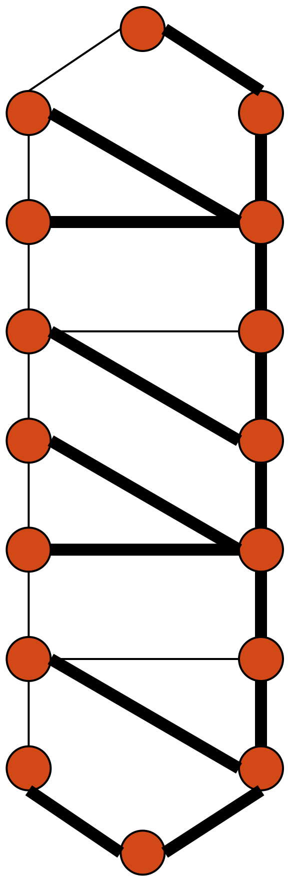
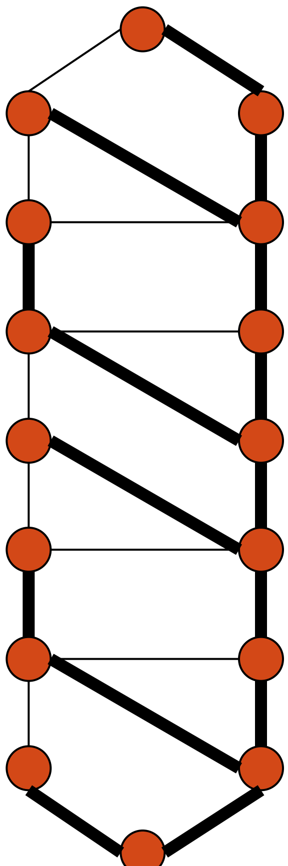
- 2 collective additive tree 2-spanners of G



caterpillar-tree



cactus-tree

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  - fast estimation of diameters
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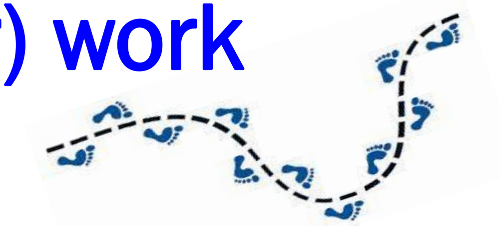
9	EE	Feodor F. Dragan, Derek G. Corneil, Ekkehard Köhler, Yang Xiang: Collective additive tree spanners for circle graphs and polygonal graphs. Discrete Applied Mathematics (DAM) 160(12):1717-1729 (2012)
		2008
8	EE	Feodor F. Dragan, Derek G. Corneil, Ekkehard Köhler, Yang Xiang: Additive Spanners for Circle Graphs and Polygonal Graphs. WG 2008:110-121
		2006
7	EE	Feodor F. Dragan, Chenyu Yan, Derek G. Corneil: Collective Tree Spanners and Routing in AT-free Related Graphs. J. Graph Algorithms Appl. (JGAA) 10(2):97-122 (2006)
		2005
6	EE	Derek G. Corneil, Feodor F. Dragan, Ekkehard Köhler, Chenyu Yan: Collective Tree 1-Spanners for Interval Graphs. WG 2005:151-162
		2004
5	EE	Feodor F. Dragan, Chenyu Yan, Derek G. Corneil: Collective Tree Spanners and Routing in AT-free Related Graphs. WG 2004:68-80
		2003
4	EE	Derek G. Corneil, Feodor F. Dragan, Ekkehard Köhler: On the power of BFS to determine a graph's diameter. Networks 42(4):209-222 (2003)
		2002
3	EE	Derek G. Corneil, Feodor F. Dragan, Ekkehard Köhler: On the Power of BFS to Determine a Graphs Diameter. LATIN 2002:209-223
		2001
2	EE	Derek G. Corneil, Feodor F. Dragan, Michel Habib, Christophe Paul: Diameter determination on restricted graph families. Discrete Applied Mathematics (DAM) 113(2-3):143-166 (2001)
		1998
1	EE	Derek G. Corneil, Feodor F. Dragan, Michel Habib, Christophe Paul: Diameter Determination on Restricted Graph Faminlies. WG 1998:192-202

Papers that influenced my (later) work



(among many others)

❑ Graph searches and their algorithmic use

93	EE	Derek G. Corneil, Barnaby Dalton, Michel Habib: LDFS-Based Certifying Algorithm for the Minimum Path Cover Problem on Cocomparability Graphs. SIAM J. Comput. (SIAMCOMP) 42(3):792-807 (2013)
85	EE	Derek G. Corneil, Ekkehard Köhler, Jean-Marc Lanlignel: On end-vertices of Lexicographic Breadth First Searches. Discrete Applied Mathematics (DAM) 158(5):434-443 (2010)
84	EE	Derek G. Corneil, Stephan Olariu, Lorna Stewart: The LBFS Structure and Recognition of Interval Graphs. SIAM J. Discrete Math. (SIAMDM) 23(4):1905-1953 (2009)
82	EE	Derek G. Corneil, Richard Krueger: A Unified View of Graph Searching. SIAM J. Discrete Math. (SIAMDM) 22(4):1259-1276 (2008)
68	EE	Derek G. Corneil: A simple 3-sweep LBFS algorithm for the recognition of unit interval graphs. Discrete Applied Mathematics (DAM) 138(3):371-379 (2004)
53	EE	Derek G. Corneil, Stephan Olariu, Lorna Stewart: LBFS Orderings and Cocomparability Graphs. SODA 1999:883-884

❑ AT-free graphs

54	EE	Derek G. Corneil, Stephan Olariu, Lorna Stewart: Linear Time Algorithms for Dominating Pairs in Asteroidal Triple-free Graphs. SIAM J. Comput. (SIAMCOMP) 28(4):1284-1297 (1999)
45	EE	Derek G. Corneil, Stephan Olariu, Lorna Stewart: Asteroidal Triple-Free Graphs. SIAM J. Discrete Math. (SIAMDM) 10(3):399-430 (1997)
36	EE	Derek G. Corneil, Stephan Olariu, Lorna Stewart: A Linear Time Algorithm to Compute a Dominating Path in an AT-Free Graph. Inf. Process. Lett. (IPL) 54(5):253-257 (1995)

Papers that influenced my (later) work

(among many others)

- Tree spanners, tree powers

49	EE	Paul E. Kearney, Derek G. Corneil: Tree Powers. J. Algorithms (JAL) 29(1):111-131 (1998)
35	EE	Leizhen Cai, Derek G. Corneil: Tree Spanners. SIAM J. Discrete Math. (SIAMDM) 8(3):359-387 (1995)

- Graph decompositions and their parameters

92	EE	Derek G. Corneil, Michel Habib, Jean-Marc Lanlignel, Bruce A. Reed, Udi Rotics: Polynomial-time recognition of clique-width ≤3 graphs. Discrete Applied Mathematics (DAM) 160(6):834-865 (2012)
72	EE	Derek G. Corneil, Udi Rotics: On the Relationship Between Clique-Width and Treewidth. SIAM J. Comput. (SIAMCOMP) 34(4):825-847 (2005)
23	EE	Stefan Arnborg, Andrzej Proskurowski, Derek G. Corneil: Forbidden minors characterization of partial 3-trees. Discrete Mathematics (DM) 80(1):1-19 (1990)

- first paper that I got from Derek (long time ago)

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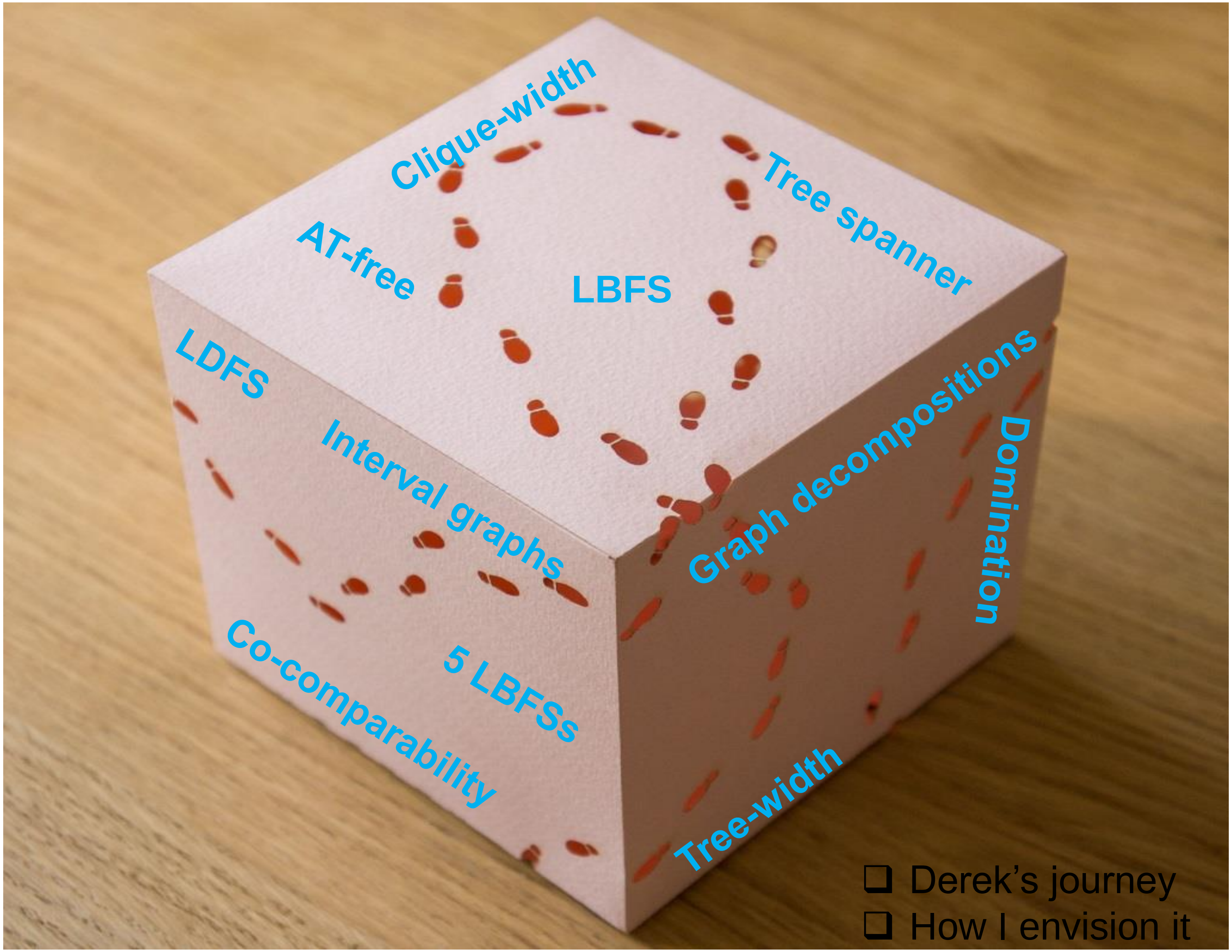
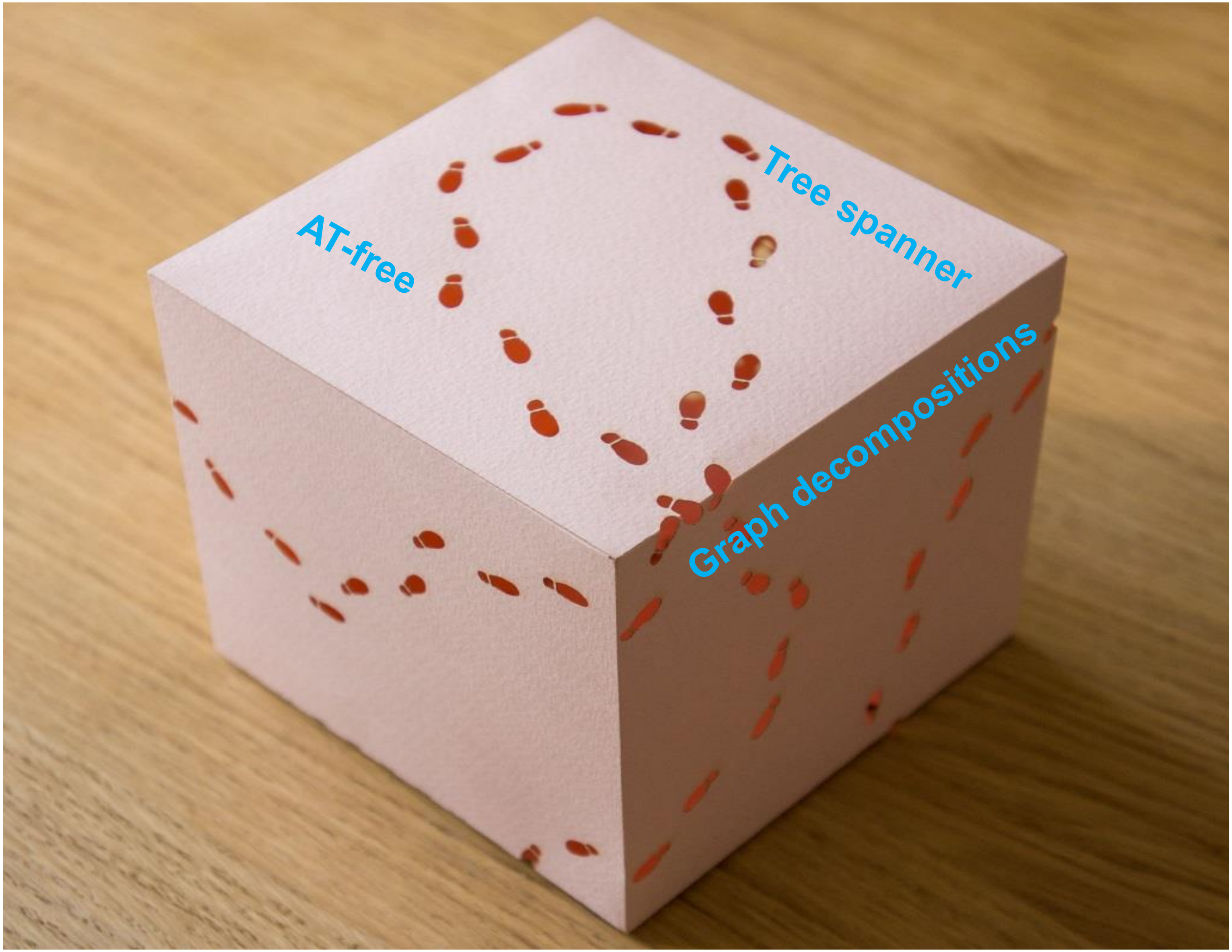
Following Derek's footsteps



- ❑ Derek's journey
- ❑ How I envision it

Following Derek's footsteps



Talk outline

- **collaborating with Derek**
  - fast estimation of diameters
  - representing approximately graph distances with few tree distances
- **following Derek's footsteps**
  - tree- and path-decompositions and new graph parameters
  - Approximating tree t-spanner problem using tree-breadth
  - Approximating bandwidth using path-length
  - Approximating line-distortion using path-length

Talk outline

- **following Derek's footsteps**
  - tree- and path-decompositions and new graph parameters
  - Approximating tree t-spanner problem using tree-breadth

Graph decompositions and their parameters + Tree spanners =

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=

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Tree-Decomposition

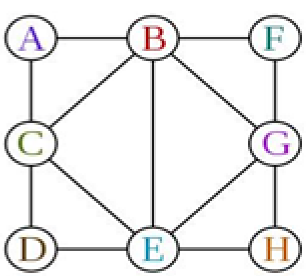
[Robertson, Seymour]

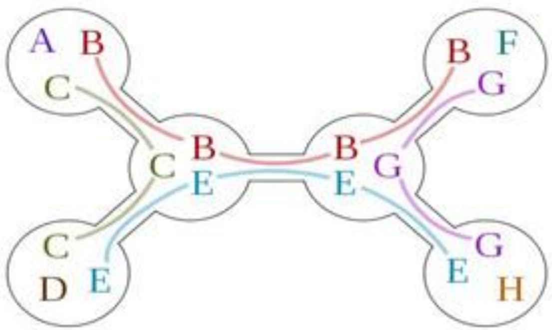
- Tree-decomposition $T(G)$ of a graph $G = (V, E)$ is a pair $(\{X_i : i \in I\}, T = (I, F))$ where $\{X_i : i \in I\}$ is a collection of subset of V (bags) and T is a tree whose nodes are the bags satisfying:

1) $\bigcup_{i \in I} X_i = V$

2) $\forall\, uv \in E, \exists\, i \in I \; s.t. \; u, v \in X_i$

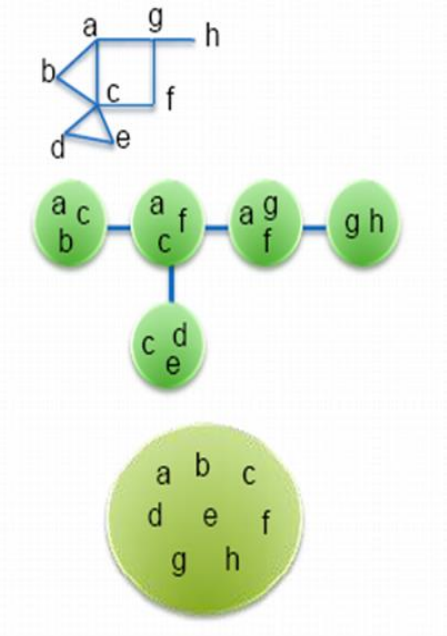
3) $\forall\, v \in V, \mathit{the\ set\ of\ bags}\ \{i \in I, v \in X_i\}\ \mathit{form\ a\ subtree}\ T_v\ \mathit{of}\ T$





Tree-Decomposition and Graph Parameters

- **Tree-width $tw(G)$:**
  - Width of $T(G)$ is $\max_{i \in I} |X_i| - 1$
  - $tw(G)$: minimum width over all tree-decompositions
- **Tree-length $tl(G)$:**
  - Length of $T(G)$ is $\max_{i \in I} \max_{u,v \in X_i} d_G(u,v)$
  - $tl(G)$: minimum length over all tree-decompositions
- **Tree-breadth $tb(G)$:**
  - Breadth is minimum r such that $\forall i \in I, \exists v_i$ with $X_i \subseteq D_r(v_i, G)$
  - $tb(G)$: minimum breadth over all tree-decompositions



Tree-length was introduced in [Dourisboure, Gavoille: *DM* (2007)] and [Dragan,Lomonosov: *DAM* (2007)]

Tree-breadth was introduced in [Dragan,Lomonosov: *DAM* (2007)] and [Dragan, Köhler: *APPROX* (2011)]

(R,D)-acyclic clustering

Tree-Decomposition and Graph Parameters

- Tree-width $\boldsymbol{tw(G)}$:
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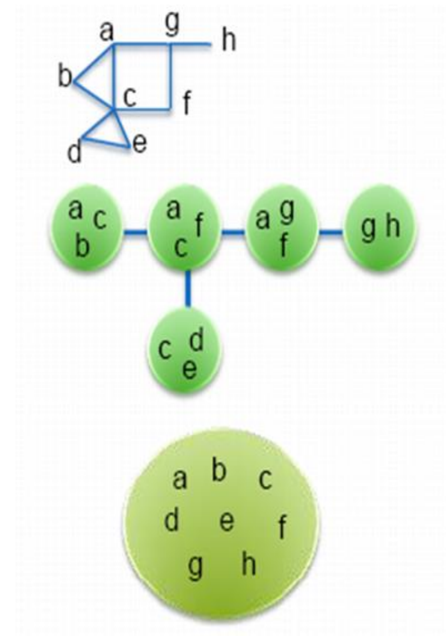


- $\forall\, G,\ tb(G) \le tl(G) \le 2tb(G)$ $\quad$ *as* $\forall\, S \subseteq V(G),\ rad_G(S) \le diam_G(S) \le 2rad_G(S)$
- $tw(G)$ and $tl(G)$ are not comparable (check cycles and cliques)

$$tw(C_{3k}) = 2, \qquad tl(C_{3k}) = k$$
$$tw(K_n) = n - 1, \qquad tl(K_n) = 1$$

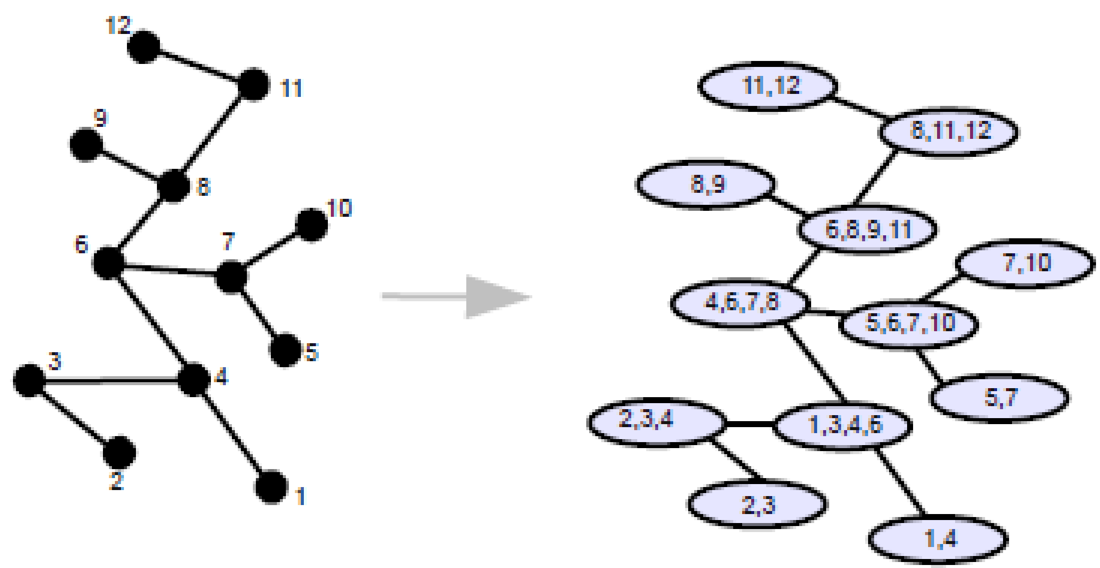
Tree-stretch vs tree-breadth

Tree t-spanner problem:

- Given unweighted undirected graph $G=(V,E)$ and integer t.
- Does G admit a spanning tree $T=(V,E')$ such that

$$\forall u,v \in V, \quad dist_T(v,u) \le t \times dist_G(v,u)$$

- If a graph G admits a tree t-spanner then $\mathrm{tb}(G) \le \lceil t/2 \rceil$.



Tree spanners in bounded tree-breadth graphs

Lm1) Each graph G has balanced disk separator $D_r(v,G)$, where $r \leq \mathrm{tb}(G)$. It can be found in $O(nm)$.

Lm2) $\mathrm{tb}(G_i^+) \leq \mathrm{tb}(G)$.

Lm3) T_is are α-spanners $\Rightarrow T$ is $(\alpha + 2r)$-spanner, where $r \leq \mathrm{tb}(G)$.

Tm2) Any connected graph G admits a tree $(2\mathrm{tb}(G)\lfloor \log_2 n \rfloor)$-spanner constructible in $O(nm\log^2 n)$ time.

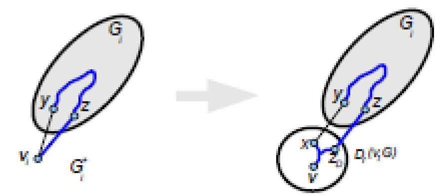
Tree_Spanner(G)

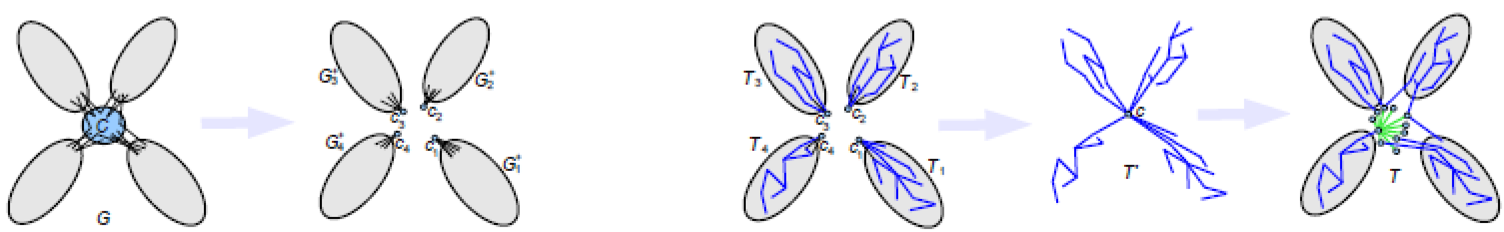
If G has at most 9 vertices

Find a tree t-spanner T of G with minimum t directly;
Output T.

Else

Find a balanced disk-separator $D_r(v,G)$ of G with minimum r;
Find connected components $G_1,\dots,G_k$ of graph $G[V \setminus D_r(v,G)]$;
Build graphs $G_1^+,\dots,G_k^+$;
Set $T_i :=$Tree_Spanner(G_i^+), for each $i = 1,\dots,k$;
Construct a shortest path tree SPT_D of $G[D_r(v,G)]$ rooted at vertex v;
Construct a spanning tree T of G from trees $T_1,\dots,T_k$ and SPT_D;
Output T.



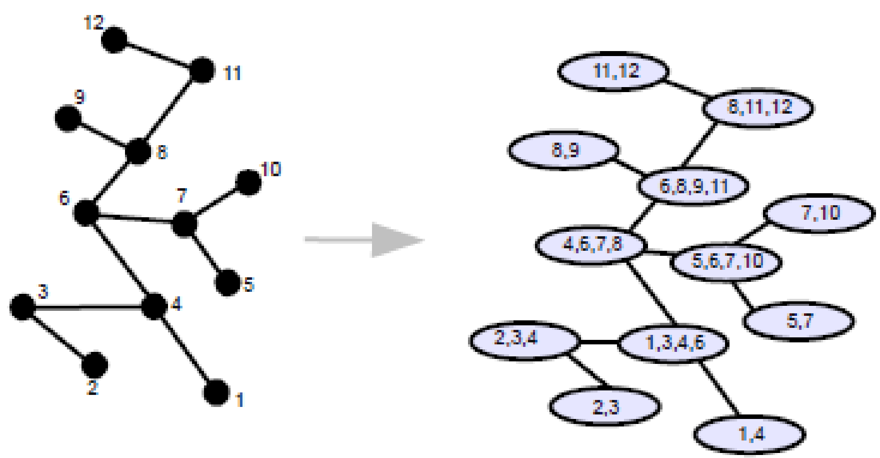


- leaves have tree $4\mathrm{tb}(G)$-spanners
- depth is at most $\log_2 n - 2$
- total number of edges per level of recursion is $O(m)$; total number of vertices is $O(n\log n)$

Approximating tree t-spanner problem in general unweighted graphs

Tm2) Any connected graph G admits a tree $(2\mathrm{tb}(G)\lfloor \log_2 n \rfloor)$-spanner constructible in $O(nm\log^2 n)$ time.

- If a graph G admits a tree t-spanner then $\mathrm{tb}(G) \leq \lceil t/2 \rceil$.

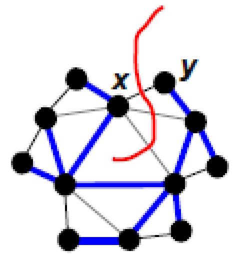


Tm3) Any connected graph G admits a tree $(2\lceil t/2 \rceil \lfloor \log_2 n \rfloor)$-spanner constructible in $O(nm\log^2 n)$ time.

95	EE	Feodor F. Dragan, Ekkehard Köhler: An Approximation Algorithm for the Tree t-Spanner Problem on Unweighted Graphs via Generalized Chordal Graphs. APPROX-RANDOM 2011:171-183

Our results vs known results

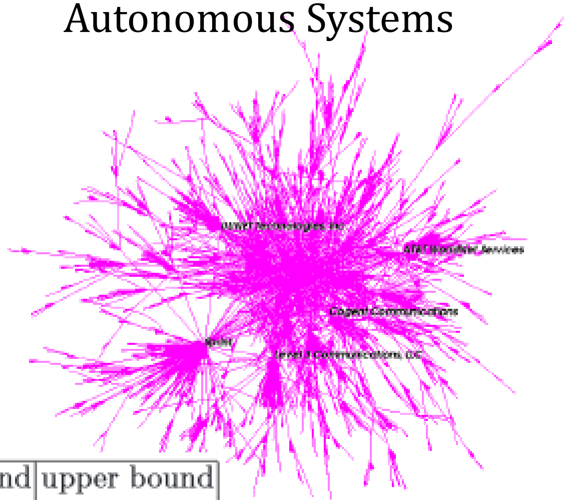
- G chordal $\Rightarrow$
  - $\exists$ a tree $(2\lfloor \log_2 n \rfloor)$-spanner in $O(m \log n)$ time
  - no t-spanner with $t < \log_2 \frac{n}{3} + 2$
    - NP-complete for every $t \geq 4$ (BDLL '04)
- $\mathrm{tb}(G) = \rho \Rightarrow$
  - a tree $(2\rho \lfloor \log_2 n \rfloor)$-spanner in $O(mn \log^2 n)$ time or
  - a tree $(12\rho \lfloor \log_2 n \rfloor)$-spanner in $O(m \log n)$ time
    - no previous result known
- if G admits a tree t-spanner we construct
  - a tree $(2\lceil t/2 \rceil \lfloor \log_2 n \rfloor)$-spanner in $O(mn \log^2 n)$ time or
  - a tree $(6t \lfloor \log_2 n \rfloor)$-spanner in $O(m \log n)$ time
    - if G admits a tree t-spanner, Emek & Peleg (2008) construct a tree $(6t \lceil \log_2 n \rceil)$-spanner in $O(mn \log^2 n)$ time.



- k-snowflake has no tree t-spanner with $t < k+1 = \log_2 \frac{n}{3} + 2$

Real-Life datasets

Autonomous Systems



Graph $G=(V,E)$	n= $\|V\|$	m= $\|E\|$	diameter $diam(G)$	radius $rad(G)$	lower bound on $tb(G)$	upper bound on $tb(G)$
PPI [46]	1458	1948	19	11	2	5
Yeast [14]	2224	6609	11	6	2	4
DutchElite [29]	3621	4311	22	12	2	6
EPA [1]	4253	8953	10	6	2	4
EVA [57]	4475	4664	18	10	2	5
California [49]	5925	15770	13	7	2	4
Erdös [10]	6927	11850	4	2	1	2
Routeview [2]	10515	21455	10	5	1	4
Homo release 3.2.99 [63]	16711	115406	10	5	1	3
AS_Caida_20071105 [18]	26475	53381	17	9	1	3
Dimes 3/2010 [61]	26424	90267	8	4	1	2
Aqualab 12/2007- 09/2008 [19]	31845	143383	9	5	1	3
AS_Caida_20120601 [16]	41203	121309	10	5	1	3
itdk0304 [17]	190914	607610	26	14	2	6
DBLB-coauth [67]	317080	1049866	23	12	3	7
Amazon [67]	334863	925872	47	24	4	12

104 EE Muad Abu-Ata, Feodor F. Dragan: Metric tree-like structures in real-life networks: an empirical study. CoRR abs/1402.3364 (2014)

Talk outline

- following Derek's footsteps
  - Approximating bandwidth using path-length
  - Approximating line-distortion using path-length

- Graph decompositions and their parameters +
- AT-free graphs =

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- =

[F. Dragan, E. Köhler, A. Leitert: Line-distortion, Bandwidth and Path-length of a graph, SWAT 2014]

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[Robertson, Seymour]

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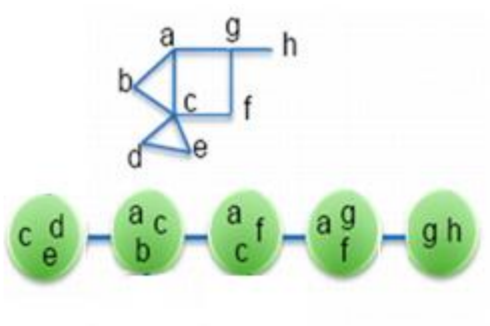
3) $\forall\, v \in V, \mathit{the\ set\ of\ bags}\ \{i \in I, v \in X_i\}\ \mathit{form\ a\ subpath\ of}\ P$



Path-Decomposition and new Graph Parameters

- path-width $pw(G)$:
  - Width of $P(G)$ is $\max_{i \in I} |X_i| - 1$
  - $pw(G)$: minimum width over all path-decompositions
- path-length $pl(G)$:
  - Length of $P(G)$ is $\max_{i \in I} \max_{u,v \in X_i} d_G(u, v)$
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- path-breadth $pb(G)$:
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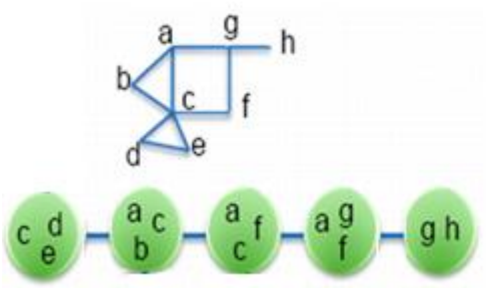
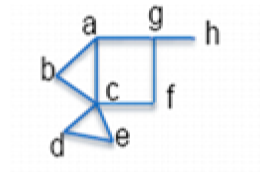
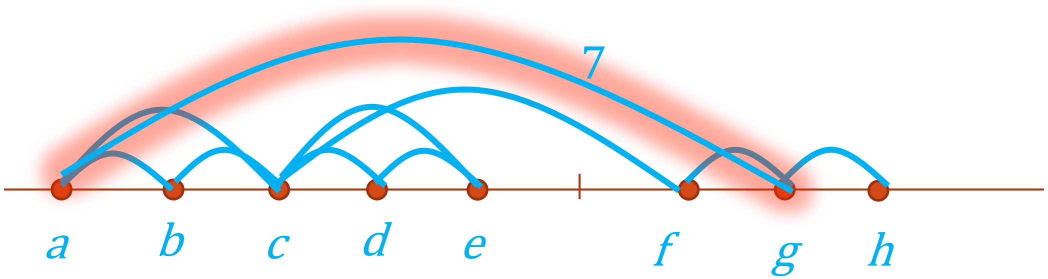
  - $pb(G)$: minimum breadth over all path-decompositions



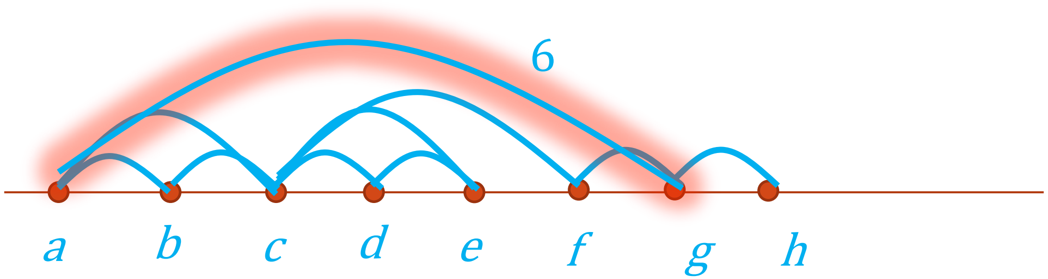
Line distortion and bandwidth



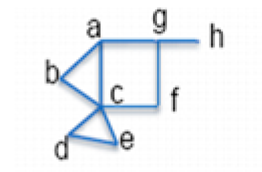
- Line-distortion $\boldsymbol{ld(G)}$: $\boldsymbol{f}$: $\boldsymbol{V \to l}$ with minimum k such that $\forall x, y \in V$
  - Non-contractiveness: $d_G(x,y) \leq |f(x) - f(y)|$
  - minimum distortion k: $|f(x) - f(y)| \leq k\, d_G\,(x,y)$



- Bandwidth $\boldsymbol{bw(G)}$: $\boldsymbol{b}$: $\boldsymbol{V \to N}$ with minimum k such that $\forall xy \in E$
  - minimum bandwidth k: $|b(x) - b(y)| \leq k$



Line distortion and bandwidth



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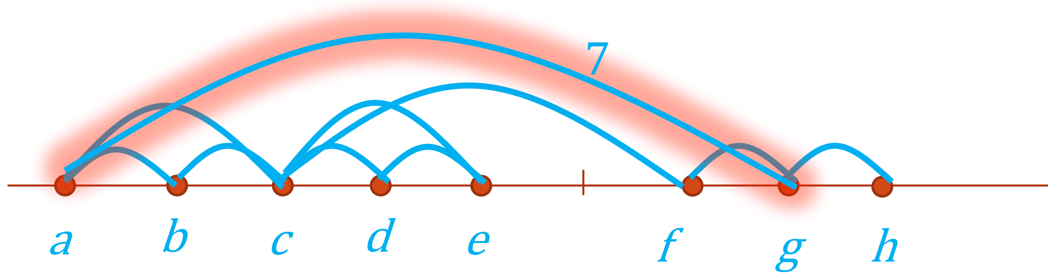
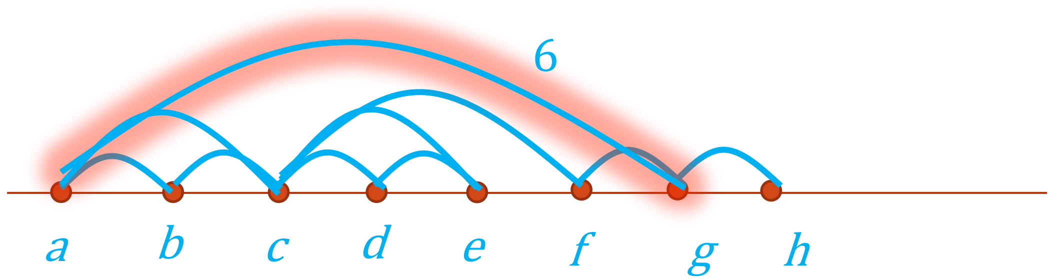
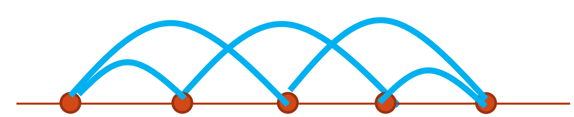


- Bandwidth $\boldsymbol{bw(G)}$: $\boldsymbol{b}: \boldsymbol{V} \rightarrow \boldsymbol{N}$ with minimum k such that $\forall xy \in E$
  - minimum bandwidth k: $|b(x) - b(y)| \leq k$

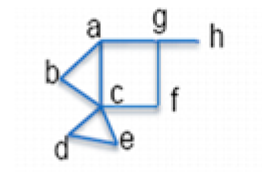
$bw(G) \leq ld(G)$
$bw(C_k) = 2$
$ld(C_k) = k - 1$



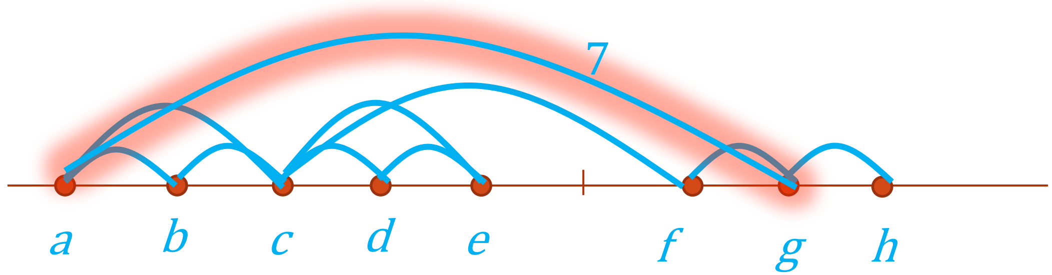


k=5

Line distortion and bandwidth

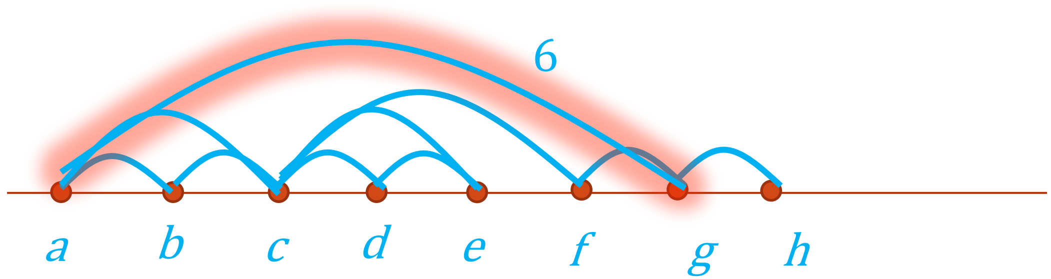


- Line-distortion $\boldsymbol{ld(G)}$: $\boldsymbol{f}$: $\boldsymbol{V} \rightarrow \boldsymbol{l}$ with minimum k such that $\forall x, y \in V$
  - Non-contractiveness: $d_G(x,y) \leq |f(x) - f(y)|$
  - minimum distortion k: $|f(x) - f(y)| \leq k\, d_G\,(x,y)$



Hard to approximate within a constant factor

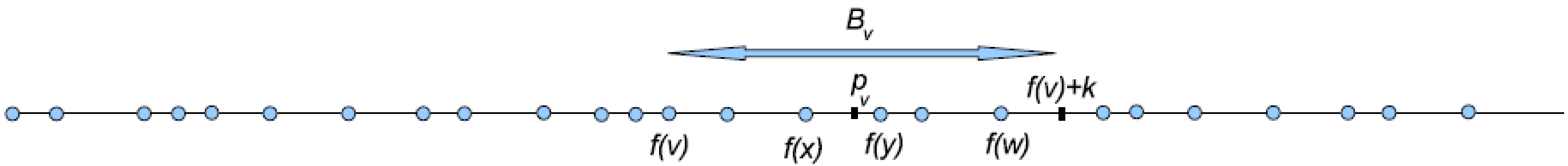
- Bandwidth $\boldsymbol{bw(G)}$: $\boldsymbol{b}$: $\boldsymbol{V} \rightarrow \boldsymbol{N}$ with minimum k such that $\forall xy \in E$
  - minimum bandwidth k: $|b(x) - b(y)| \leq k$



Hard to approximate within a constant factor

Line-distortion vs path-length

- *For an arbitrary graph G,* $\mathsf{pl}(G) \leq \mathsf{ld}(G)$, $\mathsf{pw}(G) \leq \mathsf{ld}(G)$ *and* $\mathsf{pb}(G) \leq \lceil \mathsf{ld}(G)/2 \rceil$.



- Line-distortion is hard to approximate within a constant factor
- **Theorem:** *a factor 2 approximation of the path-length of an arbitrary n-vertex graph can be computed in $\mathcal{O}(n^3)$ total time.*

Line-distortion vs path-length

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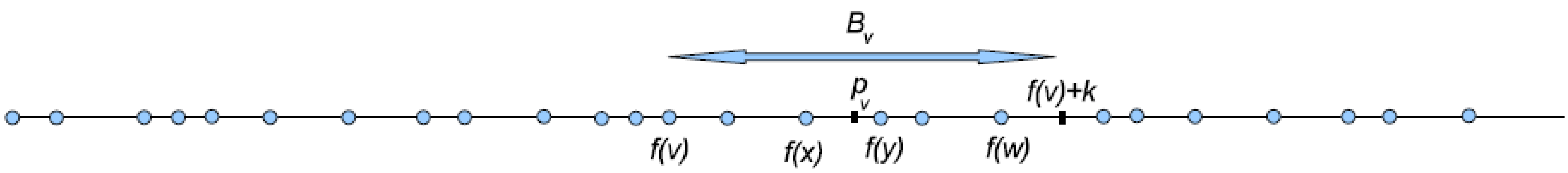


- Line-distortion is hard to approximate within a constant factor
- **Theorem:** *a factor 2 approximation of the path-length of an arbitrary n-vertex graph can be computed in $\mathcal{O}(n^3)$ total time.*

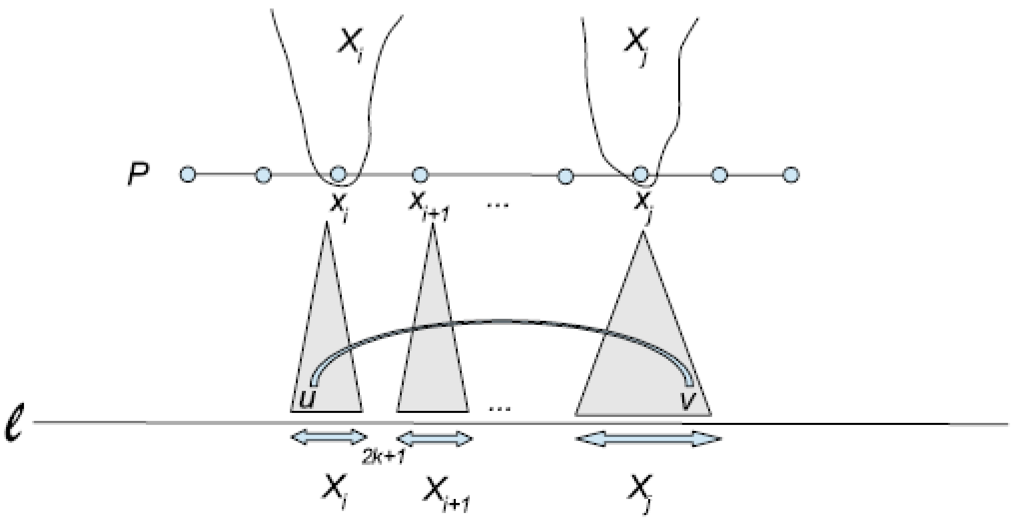
Path-length and AT-free graphs

- *For a graph G with* $\mathsf{pl}(G) \leq \lambda$, $G^{2\lambda}$ *is an AT-free graph.*
- *Every graph G with* $\mathsf{pl}(G) \leq \lambda$ *has a λ-dominating pair.*

Approximating line-distortion

hard to approximate within a constant factor in general graphs

- **Proposition:** *Every graph G with a k-dominating shortest path admits an embedding f of G into the line with distortion at most $(8k+4)\mathrm{ld}(G)+(2k)^2+2k+1$. If a k-dominating shortest path of G is given in advance, then such an embedding f can be found in linear time.*



$k \leq pl(G) \leq ld(G)$

- **Corollary:** *For every n-vertex m-edge graph G, an embedding into the line with distortion at most $(12\mathrm{pl}(G)+7)\mathrm{ld}(G)$ can be found in $\mathcal{O}(n^2m)$ time.*

- **Theorem:** *For every class of graphs with path-length bounded by a constant, there is an efficient constant-factor approximation algorithm for the minimum line-distortion problem.*

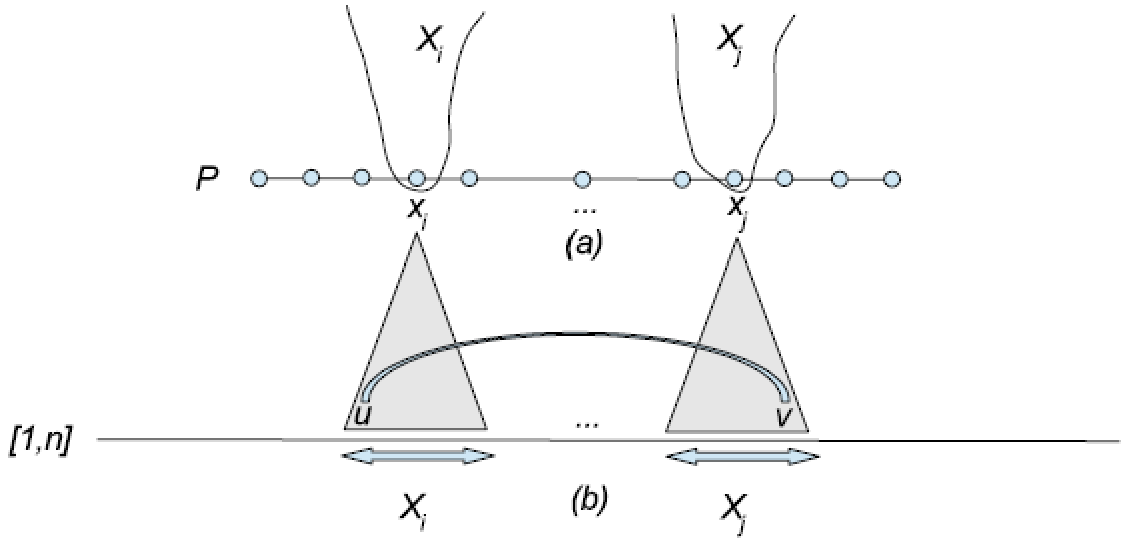
- **Corollary:**[4] *For every graph G with $\mathrm{ld}(G)=c$, an embedding into the line with distortion at most $\mathcal{O}(c^2)$ can be found in polynomial time.*

([BDGRRRS: SODA'05])

Bandwidth approximation

hard to approximate within a constant factor in general graphs

- **Proposition:** *Every graph G with a k-dominating shortest path has a layout f with bandwidth at most $(4k+2)\mathsf{bw}(G)$. If a k-dominating shortest path of G is given in advance, then such a layout f can be found in linear time.*

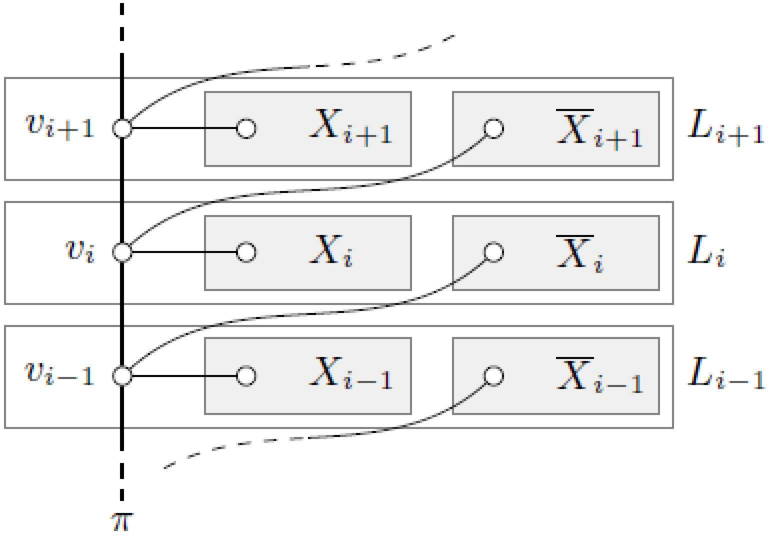


- **Corollary:** *For every n-vertex m-edge graph G, a layout with bandwidth at most $(4\mathsf{pl}(G)+2)\mathsf{bw}(G)$ can be found in $\mathcal{O}(n^2m)$ time.*

- **Theorem:** *For every class of graphs with path-length bounded by a constant, there is an efficient constant-factor approximation algorithm for the minimum bandwidth problem.*

AT-free graphs

- *If G is an AT-free graph, then* $\mathsf{pb}(G) \leq \mathsf{pl}(G) \leq 2$.
- *There is a linear time algorithm to compute an 8-approximation of the line-distortion of an AT-free graph.*

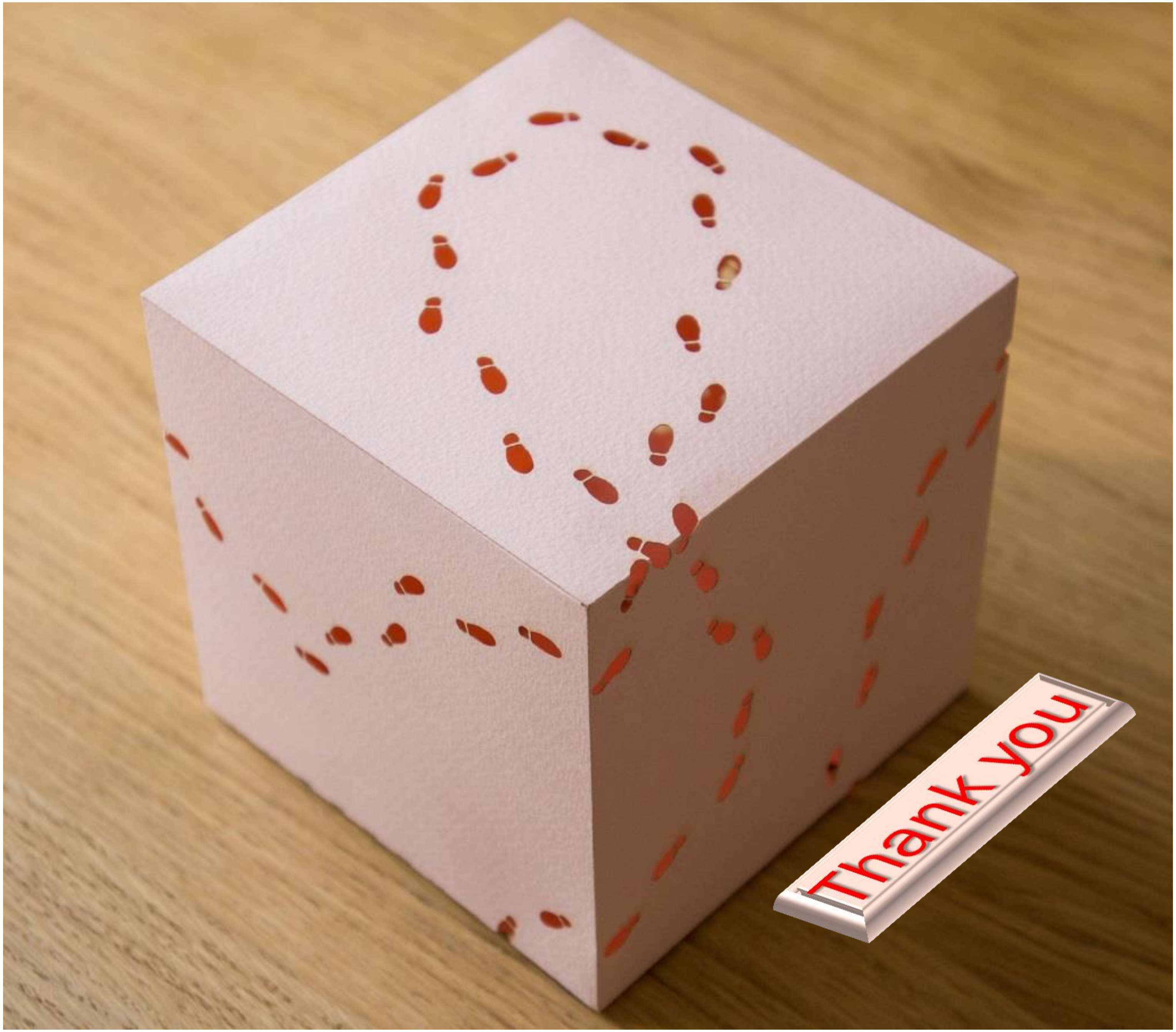


- *There is a linear time algorithm to compute a 4-approximation of the minimum bandwidth of an AT-free graph.*

Thank you

Thank you

Thank you


Thank you