OpenGL Transformations

Objectives

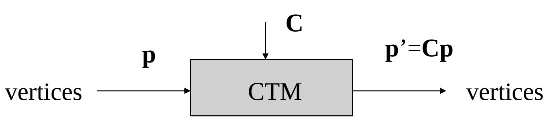
- Learn how to carry out transformations in OpenGL
  - Rotation
  - Translation
  - Scaling
- Introduce OpenGL matrix modes
  - Model-view
  - Projection

OpenGL Matrices

- In OpenGL matrices are part of the state
- Multiple types
  - Model-View (**`GL_MODELVIEW`**)
  - Projection (**`GL_PROJECTION`**)
  - Texture (**`GL_TEXTURE`**) (ignore for now)
  - Color(**`GL_COLOR`**) (ignore for now)
- Single set of functions for manipulation
- Select which to manipulated by
  - **`glMatrixMode(GL_MODELVIEW);`**
  - **`glMatrixMode(GL_PROJECTION);`**

Current Transformation Matrix (CTM)

- Conceptually there is a 4 x 4 homogeneous coordinate matrix, the *current transformation matrix* (CTM) that is part of the state and is applied to all vertices that pass down the pipeline
- The CTM is defined in the user program and loaded into a transformation unit



CTM operations

- The CTM can be altered either by loading a new CTM or by post-mutiplication

Load an identity matrix: $\mathbf{C} \leftarrow \mathbf{I}$
Load an arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{M}$

Load a translation matrix: $\mathbf{C} \leftarrow \mathbf{T}$
Load a rotation matrix: $\mathbf{C} \leftarrow \mathbf{R}$
Load a scaling matrix: $\mathbf{C} \leftarrow \mathbf{S}$

Postmultiply by an arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{CM}$
Postmultiply by a translation matrix: $\mathbf{C} \leftarrow \mathbf{CT}$
Postmultiply by a rotation matrix: $\mathbf{C} \leftarrow \mathbf{C\,R}$
Postmultiply by a scaling matrix: $\mathbf{C} \leftarrow \mathbf{C\,S}$

Rotation about a Fixed Point

Start with identity matrix: $\mathbf{C} \leftarrow \mathbf{I}$
Move fixed point to origin: $\mathbf{C} \leftarrow \mathbf{CT}$
Rotate: $\mathbf{C} \leftarrow \mathbf{CR}$
Move fixed point back: $\mathbf{C} \leftarrow \mathbf{CT}^{-1}$

Result: $\mathbf{C} = \mathbf{TR}\,\mathbf{T}^{-1}$ which is **backwards**.

This result is a consequence of doing postmultiplications.
Let's try again.

Reversing the Order

We want $\mathbf{C} = \mathbf{T}^{-1}\,\mathbf{R}\,\mathbf{T}$
so we must do the operations in the following order

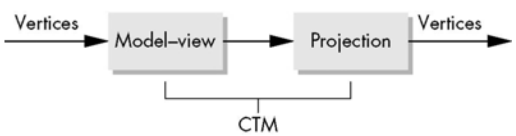
$\mathbf{C} \leftarrow \mathbf{I}$
$\mathbf{C} \leftarrow \mathbf{CT}^{-1}$
$\mathbf{C} \leftarrow \mathbf{CR}$
$\mathbf{C} \leftarrow \mathbf{CT}$

Each operation corresponds to one function call in the program.

Note that the last operation specified is the first executed in the program

CTM in OpenGL

- OpenGL has a model-view and a projection matrix in the pipeline which are concatenated together to form the CTM
- Can manipulate each by first setting the correct matrix mode



Rotation, Translation, Scaling

Load an identity matrix:

```
glLoadIdentity()
```

Multiply on right:

```
glRotatef(theta, vx, vy, vz)
```

`theta` in degrees, (`vx, vy, vz`) define axis of rotation

```
glTranslatef(dx, dy, dz)
glScalef( sx, sy, sz)
```

Each has a float (f) and double (d) format (`glScaled`)

Example

- Rotation about z axis by 30 degrees with a fixed point of (1.0, 2.0, 3.0)

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
glTranslatef(1.0, 2.0, 3.0);
glRotatef(30.0, 0.0, 0.0, 1.0);
glTranslatef(-1.0, -2.0, -3.0);
```

- Remember that last matrix specified in the program is the first applied

Arbitrary Matrices

- Can load and multiply by matrices defined in the application program

```
glLoadMatrixf(m)
glMultMatrixf(m)
```

- The matrix **m** is a one dimension array of 16 elements which are the components of the desired 4 x 4 matrix stored by <u>columns</u>
- In **glMultMatrixf**, **m** multiplies the existing matrix on the right

Matrix Stacks

- In many situations we want to save transformation matrices for use later
  - Traversing hierarchical data structures (Chapter 10)
  - Avoiding state changes when executing display lists
- OpenGL maintains stacks for each type of matrix
  - Push/Pop present type (as set by **glMatrixMode)** by

```
glPushMatrix()
glPopMatrix()
```


Reading Back Matrices

- Can also access matrices (and other parts of the state) by *query* functions

```
glGetIntegerv
glGetFloatv
glGetBooleanv
glGetDoublev
glIsEnabled
```

- For matrices, we use as

```
double m[16];
glGetFloatv(GL_MODELVIEW, m);
```

Using Transformations

- Example: use idle function to rotate a cube and mouse function to change direction of rotation
- Start with a program that draws a cube (`colorcube.c`) in a standard way
  - Centered at origin
  - Sides aligned with axes
  - Will discuss modeling in next lecture

main.c

```
void main(int argc, char **argv)
{
    glutInit(&argc, argv);
    glutInitDisplayMode(GLUT_DOUBLE | GLUT_RGB |
       GLUT_DEPTH);
    glutInitWindowSize(500, 500);
    glutCreateWindow("colorcube");
    glutReshapeFunc(myReshape);
    glutDisplayFunc(display);
    glutIdleFunc(spinCube);
    glutMouseFunc(mouse);
    glEnable(GL_DEPTH_TEST);
    glutMainLoop();
}
```


Idle and Mouse callbacks

```
void spinCube()
{
 theta[axis] += 2.0;
 if( theta[axis] > 360.0 ) theta[axis] -= 360.0;
 glutPostRedisplay();
}
void mouse(int btn, int state, int x, int y)
{
   if(btn==GLUT_LEFT_BUTTON && state == GLUT_DOWN)
           axis = 0;
   if(btn==GLUT_MIDDLE_BUTTON && state == GLUT_DOWN)
           axis = 1;
   if(btn==GLUT_RIGHT_BUTTON && state == GLUT_DOWN)
           axis = 2;
}
```


Display callback

```
void display()
{
   glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT);
   glLoadIdentity();
   glRotatef(theta[0], 1.0, 0.0, 0.0);
   glRotatef(theta[1], 0.0, 1.0, 0.0);
   glRotatef(theta[2], 0.0, 0.0, 1.0);
   colorcube();
   glutSwapBuffers();
}
```

Note that because of fixed from of callbacks, variables such as `theta` and `axis` must be defined as globals

Camera information is in standard reshape callback

Using the Model-view Matrix

- In OpenGL the model-view matrix is used to
  - Position the camera
    - Can be done by rotations and translations but is often easier to use `gluLookAt`
  - Build models of objects
- The projection matrix is used to define the view volume and to select a camera lens

Model-view and Projection Matrices

- Although both are manipulated by the same functions, we have to be careful because incremental changes are always made by postmultiplication
  - For example, rotating model-view and projection matrices by the same matrix are not equivalent operations.
  - Postmultiplication of the model-view matrix is equivalent to premultiplication of the projection matrix

Smooth Rotation

- From a practical standpoint, we often want to use transformations to move and reorient an object smoothly
  - Problem: find a sequence of model-view matrices $\mathbf{M}_0,\mathbf{M}_1,\ldots,\mathbf{M}_n$ so that when they are applied successively to one or more objects we see a smooth transition
- For orientating an object, we can use the fact that every rotation corresponds to part of a great circle on a sphere
  - Find the axis of rotation and angle
  - Virtual trackball (see text)

Incremental Rotation

- Consider the two approaches
  - For a sequence of rotation matrices $\mathbf{R}_0,\mathbf{R}_1,\ldots,\mathbf{R}_n$, find the Euler angles for each and use $\mathbf{R}_i = \mathbf{R}_{iz}\,\mathbf{R}_{iy}\,\mathbf{R}_{ix}$
    - Not very efficient
  - Use the final positions to determine the axis and angle of rotation, then increment only the angle
- Quaternions can be more efficient than either

Quaternions

- Extension of imaginary numbers from two to three dimensions
- Requires one real and three imaginary components **i**, **j**, **k**

$$q=q_0+q_1\mathbf{i}+q_2\mathbf{j}+q_3\mathbf{k}$$

- Quaternions can express rotations on sphere smoothly and efficiently. Process:
  - Model-view matrix → quaternion
  - Carry out operations with quaternions
  - Quaternion → Model-view matrix

Interfaces

- One of the major problems in interactive computer graphics is how to use two-dimensional devices such as a mouse to interface with three dimensional obejcts
- Example: how to form an instance matrix?
- Some alternatives
  - Virtual trackball
  - 3D input devices such as the spaceball
  - Use areas of the screen
    - Distance from center controls angle, position, scale depending on mouse button depressed