AVL Tree

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ALGORITHM

Idea

- The performance (Search, Insertion, Deletion):
  - of binary tree depends on the balance
  - Indeed it is possible to build a nearly balanced tree if all the nodes are available at the beginning.
- AVL tree is a mechanism where a tree can be kept nearly balanced while trees are dynamically added or deleted.
  - The height of an AVL tree will never exceed 1.44 log n.
- AVL: Two Russian Mathematicians G.M. ADELSON_VELSKII and E.M. LANDIS developed this tree in 1962.

AVL Tree

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- An *AVL tree* is a binary search tree in which the heights of the left and right subtrees of the root differ by at most 1 and in which the left and right subtrees are again AVL trees.
- With each node of an AVL tree is associated a ***balance factor*** that is *left higher*, *equal*, or *right higher* according, respectively, as the left subtree has height greater than, equal to, or less than that of the right subtree.
- In each node structure there is an extra field:

  BalanceFactor bf**;**

Examples: Which one are AVL?

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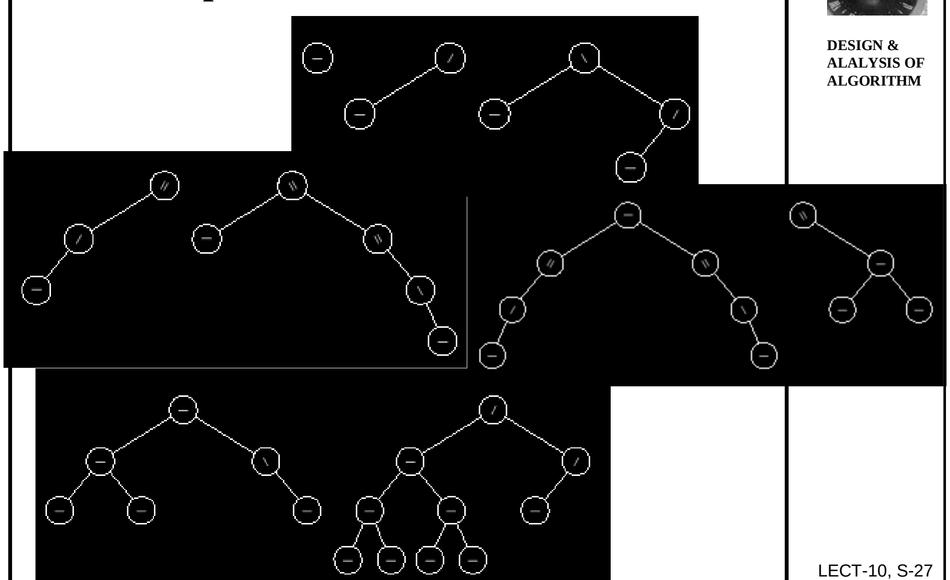

Insertion in AVL

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- Usual Binary tree insertion should work.
  - Check if the new key will go left or right.
  - Insert it recursively in left or right subtree as needed.
- What about the Height?
  - Often it will not result in any increase of the subtree height, do nothing.
  - If it increases the height of the shorter subtree, still do nothing except update the BF of the root.
  - Only if it increases the height of the taller subtree then need to do something special.

Simple Insertion

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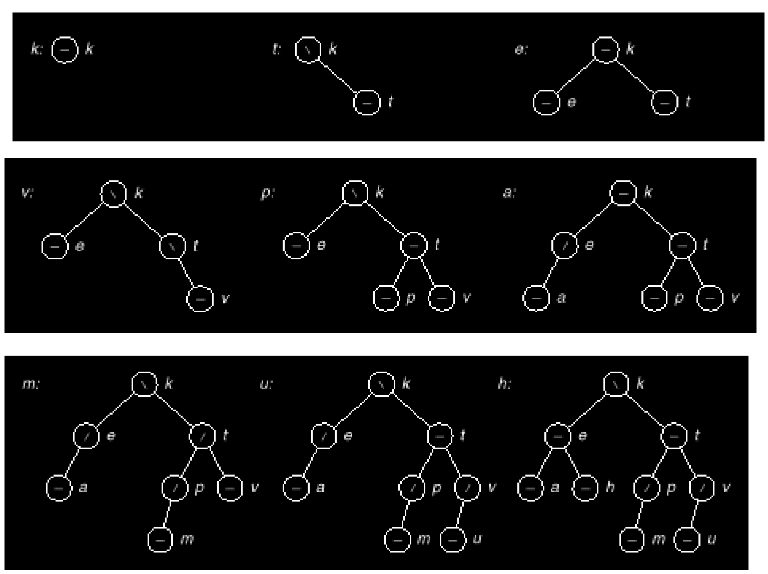

InsertAVL

```
TreeNode *InsertAVL(TreeNode *root, TreeNode *newnode, Boolean *taller)
{
    if (!root) {
        root = newnode;
        root->left = root->right = NULL;
        root->bf = EH;
        *taller = TRUE;

    } else if (EQ(newnode->entry.key, root->entry.key)) {
        Error("Duplicate key is not allowed in AVL tree.");

    } else if (LT(newnode->entry.key, root->entry.key)) {
        root->left = InsertAVL(root->left, newnode, taller);
        if (*taller)                          /* Left subtree is taller.  */
            switch(root->bf) {

            case LH:                          /* Node was left high.      */
                root = LeftBalance(root, taller); break;

            case EH:
                root->bf = LH;  break;        /* Node is now left high.   */

            case RH:
                root->bf = EH;            /* Node now has balanced height.*/
                *taller = FALSE; break;
            }

    } else {

(continued…)
```

First node.

If smaller key

If the height of the left subtree increases

If greater key...

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InsertAVL (continued..)

```
TreeNode *InsertAVL(TreeNode *root, TreeNode *newnode, Boolean *taller)
{

(continued..)

  } else {
    root->right = InsertAVL(root->right, newnode, taller);
    if (*taller)              /* Right subtree is taller. */
      switch(root->bf) {

      case LH:
        root->bf = EH;        /* Node now has balanced height.*/
        *taller = FALSE; break;

      case EH:
        root->bf = RH; break;   /* Node is right high.      */

      case RH:                /* Node was right high.    */
        root = RightBalance(root, taller); break;
      }
  }
  return root;
}
```

If Key goes in right sub-tree

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Balancing unbalanced AVL

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- Problem:
  - let us assume we have used InsertAVL
  - now the right subtree height has grown one and the right subtree was already taller!
  - How to restore the balance?
- Solution:
  - there can be three situations:
  - the right subtree itself is now left heavy
  - the right subtree itself is now right heavy
  - the right subtree now has equal heights in both sides..

Case-1: Right Higher

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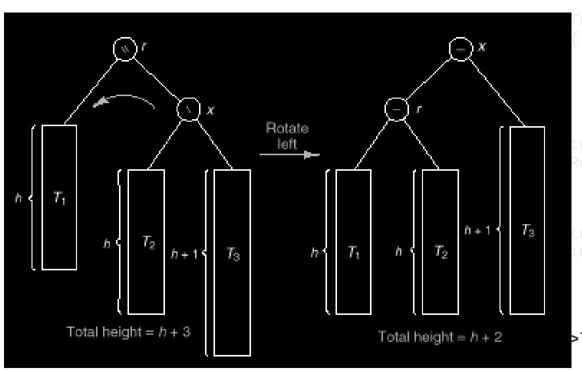
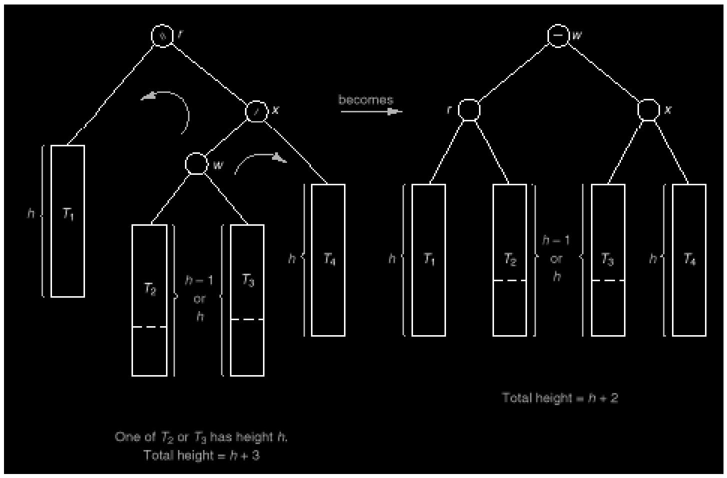
- Left Rotation:




```
TreeNode *RotateLeft(TreeNode *p)
{
    TreeNode *rightchild = p;


    if (!p)
        Error("It is impossible
to rotate an empty tree in
RotateLeft.");

    else if (!p->right)
        Error("It is impossible
to make an empty subtree the root
in RotateLeft.");

    else {
        rightchild = p->right;
        p->right = rightchild-
>left;

        rightchild->left = p;
    }
    return rightchild;
}
```

Case-2: Left Higher

- Double Left Rotation:



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Behavior of Algorithm

- The number of times the function InsertSVL calls itself recursively a new node can be as large as the height of the tree.

- How many times the routine RightBalace or LeftBalance will be called?
  - Both of them makes the BF of the root EQ.
  - Thus it will not further increase the tree height for outer recursive calls.
  - Only once they will be called!
  - Most insertion will induce no rotation.
  - Even when, they usually occur near the leaf.

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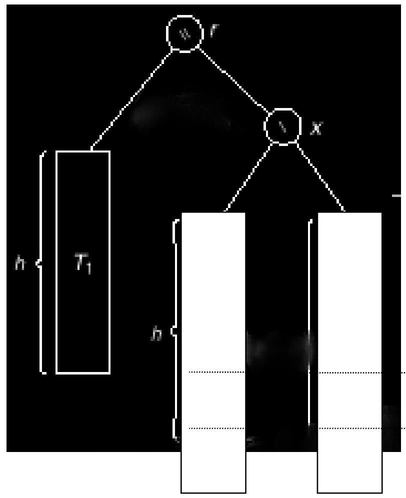
Case-3: Equal Height

- Can it Happen?



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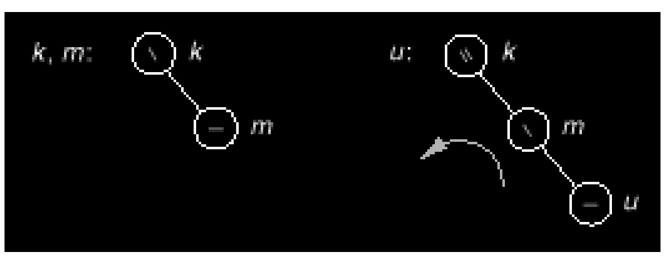
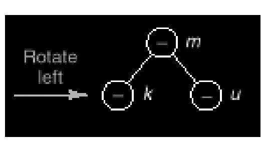
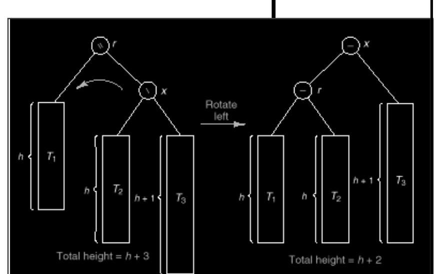
Example-1







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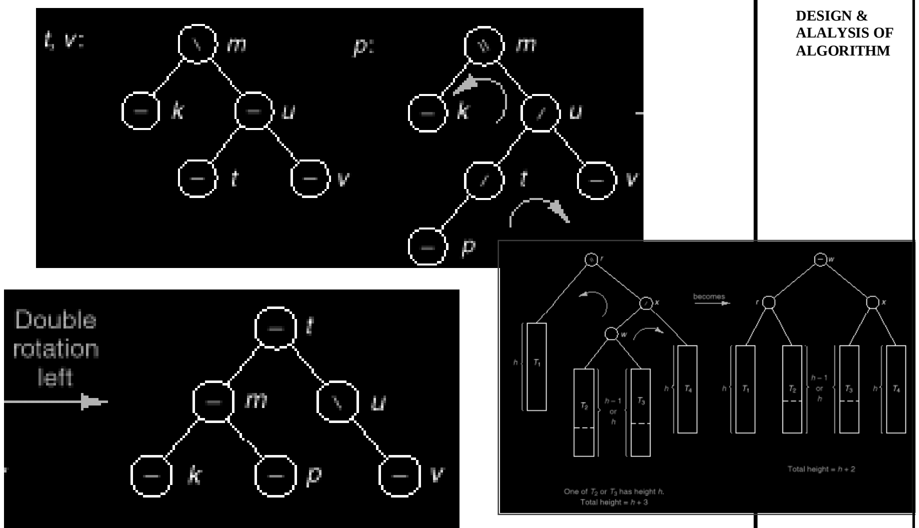
Example-2



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Deletion of a Node

- Reduce the problem to the case when the node *x* to be deleted has at most one child.
- 2. Delete *x*. We use a Boolean variable shorter to show if the height of a subtree has been shortened.
- 3. While shorter is TRUE do the following steps for each node *p* on the path from the parent of *x* to the root of the tree. When shorter becomes FALSE, the algorithm terminates.

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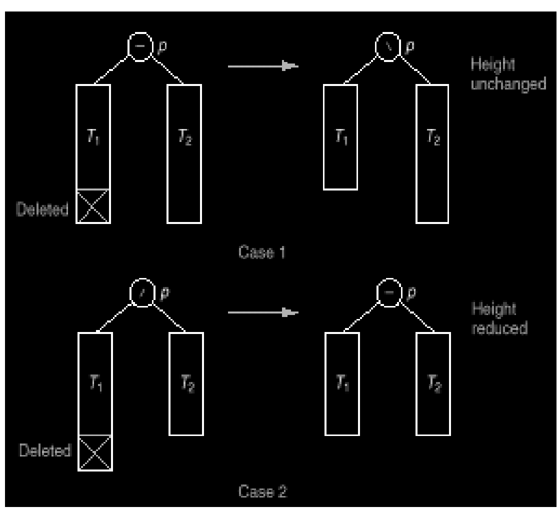
Deletion of a Node

4. *Case 1*: Node *p* has balance factor equal.
5. *Case 2*: The balance factor of *p* is not equal, and the taller subtree was shortened.
6. *Case 3*: The balance factor of *p* is not equal, and the shorter subtree was shortened. Apply a rotation as follows to restore

balance. Let *q* be the root of the taller subtree of *p*.

7. *Case 3a*: The balance factor of *q* is equal.
8. *Case 3b*: The balance factor of *q* is the same as that of *p*.
9. *Case 3c*: The balance factors of *p* and *q* are opposite.

Deletion-1



No operation

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Deletion-2

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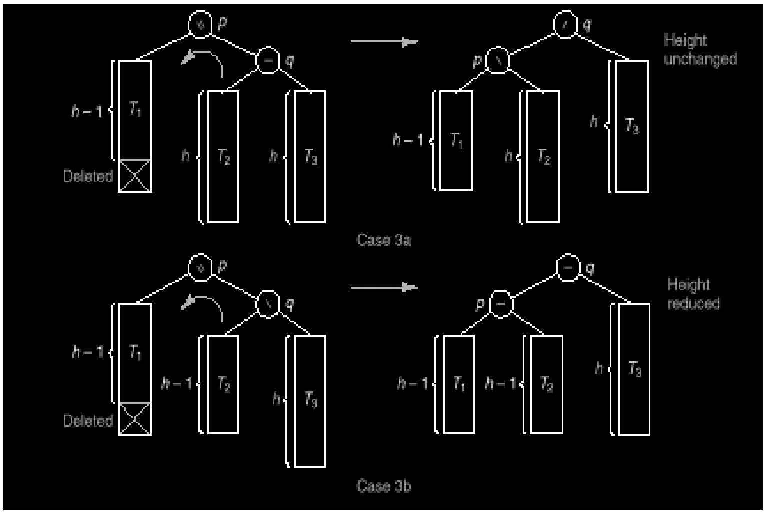

Single Rotation

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Deletion-3

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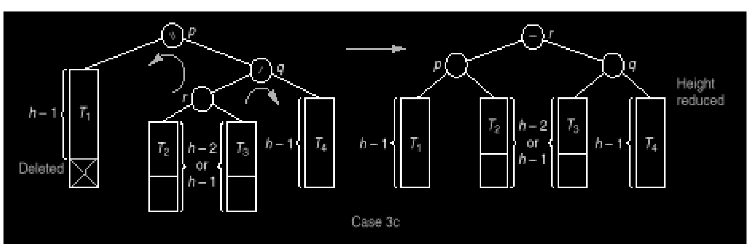

Double Rotation

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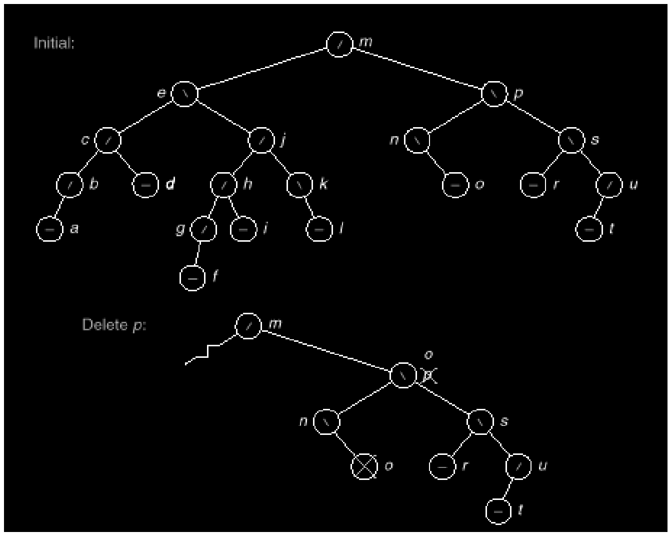
Example



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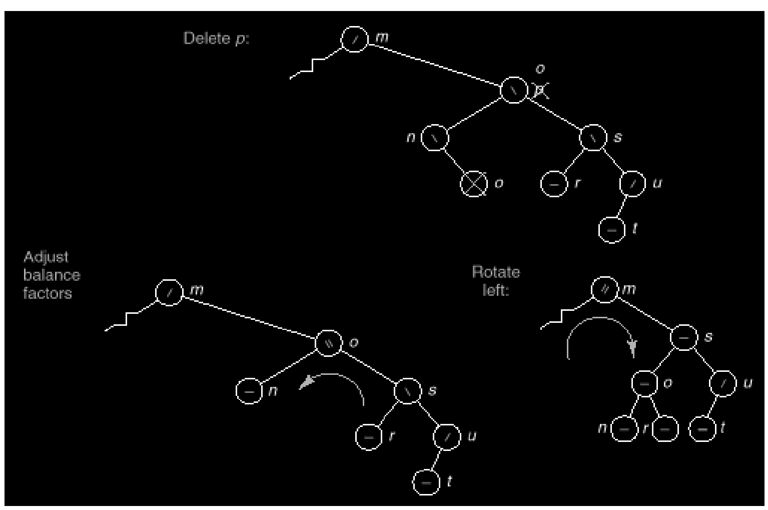
Example (continued..)



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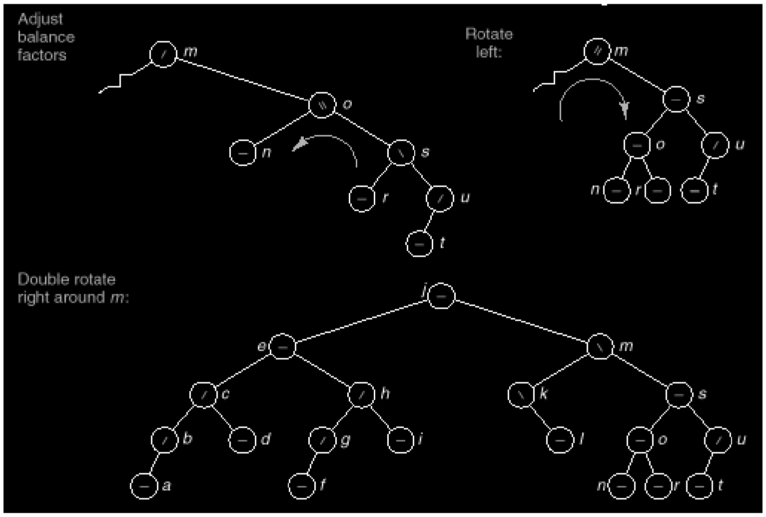
Example (continued..)



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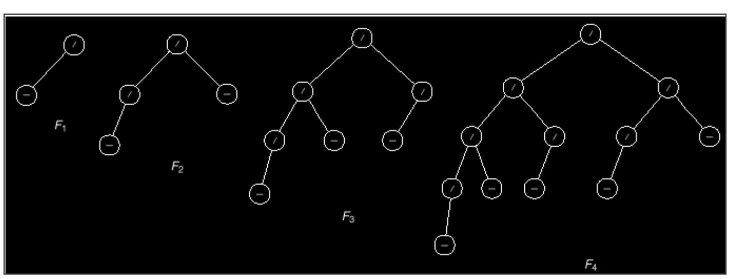
The Height of AVL Tree (WC)

- Let F_h be the minimum number of nodes that a AVL tree of height h can have. Then:

$$|F_h| = |F_{h-1}| + |F_{h-2}| + 1$$



$|F_0| = 1$ $|F_1| = 2$ **Fibonacci Trees**

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The Height of AVL Tree (WC)

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- Fibonacci vs. Our Series (n=h+2)

$$OurSeries: F_{-3}, F_{-2}, F_{-1} = .., F_0 = 1, F_1 = 2, ...F_{h-2}, F_{h-1}, F_h$$
$$Fibonacci: f_0 = 0,\ f_1 = 1,\ f_2 = 1,\ f_3 = 2,\ f_4 = 3,.,\ f_{n+1},\ f_{n+2}$$

- $|F_h|$ +1 satisfies the definition of Fibonacci number.

$$(|F_h| + 1) = (|F_{h-1}| + 1) + (|F_{h-2}| + 1)$$

- By evaluation Fibonacci:

$$(|F_h| + 1) = \frac{1}{\sqrt{5}}\left[\frac{1+\sqrt{5}}{2}\right]^{h+2} = \frac{(GR)^{h+2}}{\sqrt{5}}$$

- By taking log in both sides: $h \approx 1.44 \log|F_h|$
- In the worst case AVL will perform no more than 44% more of the perfect case!