


<b>CS 4/54201 Computer</b>	<b>Kent State University</b> Dept. of Computer Science <a href="http://www.mcs.kent.edu/~javed/class-NET06F/">www.mcs.kent.edu/~javed/class-NET06F/</a>
<b>Communication Network</b>	

	A Course on Networking and Computer Communication

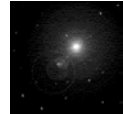
# THEORETICAL BASIS FOR DATA COMMUNICATION

3

## Fourier Decomposition

- Any reasonably behaved periodic function  $g(t)$ , with period  $T$  can be constructed by summing a number of sines and cosines.

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(2\pi nft) + \sum_{n=1}^{\infty} b_n \cos(2\pi nft)$$



COMPUTER  
COMMUNICATION  
NETWORK

$$f = \frac{1}{T}$$

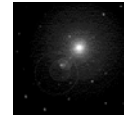
LECT-3, S-4  
NET06F, javed@kent.edu  
Javed I. Khan@1998

## Fourier Transform

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(2\pi nft) + \sum_{n=1}^{\infty} b_n \cos(2\pi nft)$$

- The coefficients can be computed by multiplying both sides with sines and cosines and performing integrals, because:

$$\begin{aligned} & \int_0^T \sin(2\pi kft) \cdot \sin(2\pi nft) dt \\ &= 0 \text{ when } k \neq n \\ &= \frac{T}{2} \text{ for } k = n \end{aligned}$$



COMPUTER  
COMMUNICATION  
NETWORK

LECT-3, S-5  
NET06F, javed@kent.edu  
Javed I. Khan@1998

## Fourier Transform

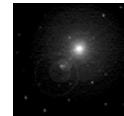
$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(2\pi nft) + \sum_{n=1}^{\infty} b_n \cos(2\pi nft)$$

$$f = \frac{1}{T}$$

$$c_n = \frac{2}{T} \int_0^T g(t) dt$$

$$a_n = \frac{2}{T} \int_0^T g(t) \sin(2\pi nft) dt$$

$$b_n = \frac{2}{T} \int_0^T g(t) \cos(2\pi nft) dt$$

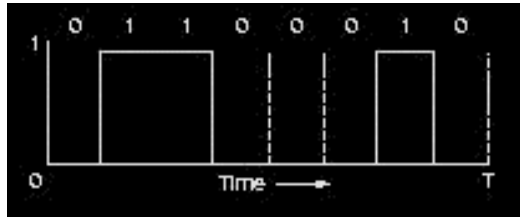


COMPUTER  
COMMUNICATION  
NETWORK

LECT-3, S-6  
NET06F, javed@kent.edu  
Javed I. Khan@1998

# Example

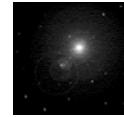
ASCII character b='01100010'



$$a_n = \frac{2}{\pi n} \left[ \cos\left(\frac{\pi n}{4}\right) - \cos\left(\frac{3\pi n}{4}\right) + \cos\left(\frac{6\pi n}{4}\right) - \cos\left(\frac{7\pi n}{4}\right) \right]$$

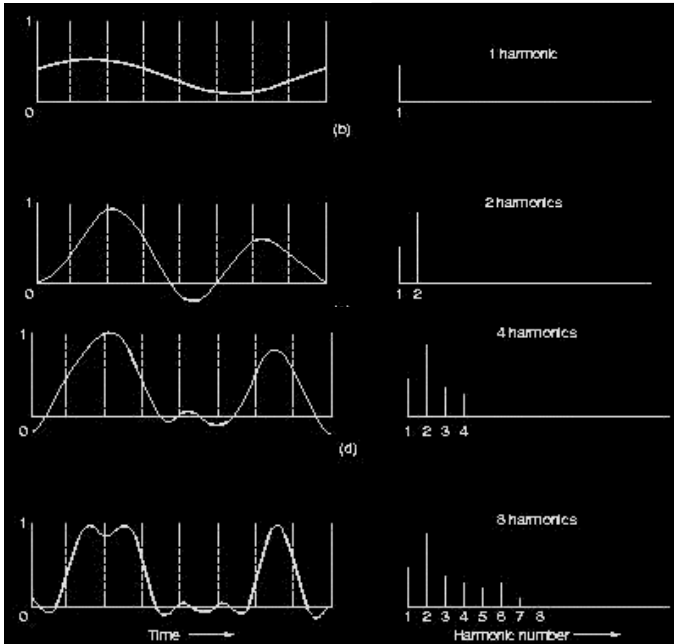
$$b_n = \frac{2}{\pi n} \left[ \sin\left(\frac{3\pi n}{4}\right) - \sin\left(\frac{\pi n}{4}\right) + \sin\left(\frac{7\pi n}{4}\right) - \sin\left(\frac{6\pi n}{4}\right) \right]$$

$$c_n = \frac{3}{4}$$



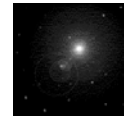
COMPUTER  
COMMUNICATION  
NETWORK

LECT-3, S-7  
NET06F, javed@kent.edu  
Javed I. Khan@1998



Signal Power

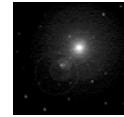
$$\sqrt{a_n^2 + b_n^2}$$



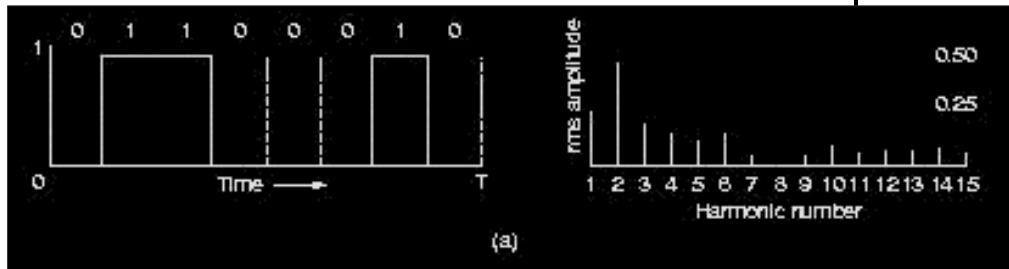
COMPUTER  
COMMUNICATION  
NETWORK

LECT-3, S-8  
NET06F, javed@kent.edu  
Javed I. Khan@1998

# Reconstruction



COMPUTER  
COMMUNICATION  
NETWORK



LECT-3, S-9  
NET06F, javed@kent.edu  
Javed I. Khan@1998

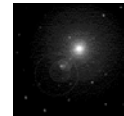
# Data rate and Harmonics

$$T = 8 * 1/\text{bps}$$

$$\text{Hz} = 1/T$$

Via 3000 Hz channel

Bps	T (msec)	First harmonic (Hz)	# Harmonics sent
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

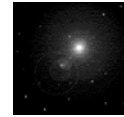
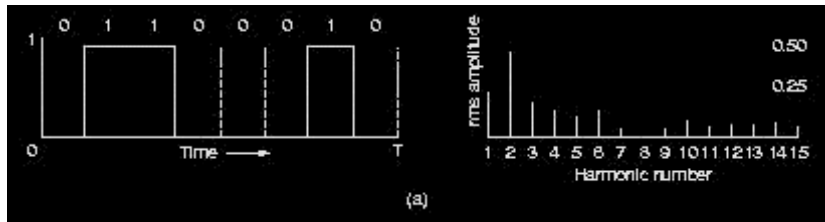


COMPUTER  
COMMUNICATION  
NETWORK

LECT-3, S-10  
NET06F, javed@kent.edu  
Javed I. Khan@1998

## Maximum Data rate of a Channel

- Nyquist Criteria
  - if an arbitrary signal has been run through a low-pass filter of bandwidth  $H$ , the filtered signal can be completely reconstructed by making only  $2H$  (exact) samples per second.
  - Sampling more than  $H$  times per second is pointless because the higher frequency components such sampling could recover have already been filtered out.



COMPUTER  
COMMUNICATION  
NETWORK

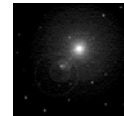
LECT-3, S-11  
NET06F, javed@kent.edu  
Javed I. Khan@1998

## Nyquist Theorem

- For a  $H$  Hz channel (a low pass channel with cutoff at  $H$  hz), if a signal consists of  $V$  discrete levels, then the maximum data rate is:

$$D = 2H \log_2 V \text{ bits/sec}$$

Example: a noiseless 3-kHz channel cannot transmit binary (two level) signals at a rate exceeding 6000 bps



COMPUTER  
COMMUNICATION  
NETWORK

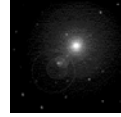
LECT-3, S-12  
NET06F, javed@kent.edu  
Javed I. Khan@1998

## Shannon's Theorem

- Maximum data rate for a noisy channel whose bandwidth is  $H$  hz, and whose signal to noise ratio is  $S/N$ , is given by:

$$M = H \log_2 (1+S/N) \text{ bits/s}$$

$S/N$  is expressed as decibels (DB).  $S/N=10$  is 10 db, a ratio of 100 is 20 db, a ratio of 1000 is 30 db



COMPUTER  
COMMUNICATION  
NETWORK

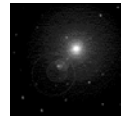
LECT-3, S-13  
NET06F, javed@kent.edu  
Javed I. Khan@1998

## Example Problem

Problem: a noiseless 3-kHz channel has 30db signal to noise ratio. What will be the maximum data rate via this channel according to Shannon's result?

Answer:

- $H=3000$
- 30 db means  $S/N = 1,000$
- $M = 3000 \times \log 1001 = 30,000$  bps



COMPUTER  
COMMUNICATION  
NETWORK

LECT-3, S-14  
NET06F, javed@kent.edu  
Javed I. Khan@1998