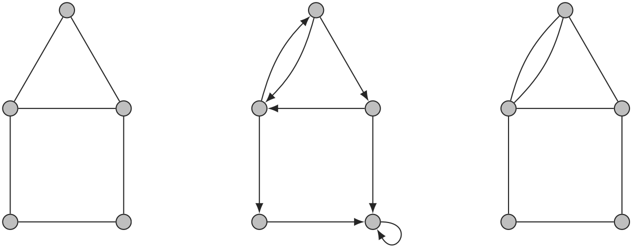
Graphs – Introduction

Graph

Graph

A *graph* $G = (V, E)$ is a set V of vertices connected by an edge set E.



Variations

Multi-Graph: Multiple edges between two vertices.

Directed: Edges have a direction.

Weighted: Vertices and/or edges have weights.

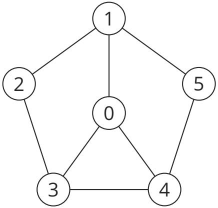
Simple: No multiple edges, no loops.

Simple Undirected Graph

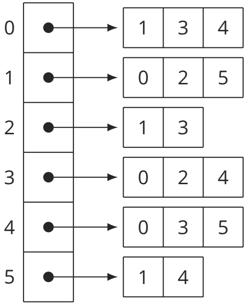
A *simple undirected graph* $G = (V, E)$ is a set V of vertices connected by an edge set $E \subseteq \{ \{u, v\} \mid u, v \in V, u \neq v \}$. An edge $\{u, v\}$ is usually written as uv.

Implementation: Adjacency List

For each vertex, there is an array storing “pointers” to all neighbours.
(Usually, the vertex index is sufficient.)



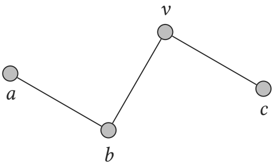
G



Adjacency

Adjacency

Two vertices u and v are *adjacent* if there is an edge connecting them. This is sometimes written as $u \sim v$.

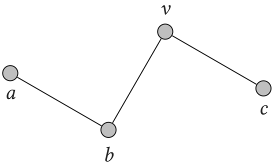


v is adjacent to b and c but not to a.

Neighbourhood

Neighbourhood

The *open neighbourhood* $N(v) = \{ u \in V \mid u \neq v, u \sim v \}$ of a vertex v is the set of vertices adjacent to v (not including v). The *closed neighbourhood* $N[v] = N(v) \cup \{v\}$ includes v.



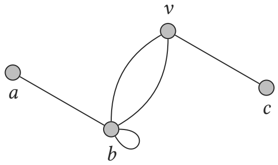
$$N(v) = \{b, c\} \qquad N[v] = \{v, b, c\}$$

Degree

Degree

The *degree* $\deg(v)$ of a vertex v is the number of incident edges.

Note that the degree is not necessarily equal to the cardinality of neighbours.

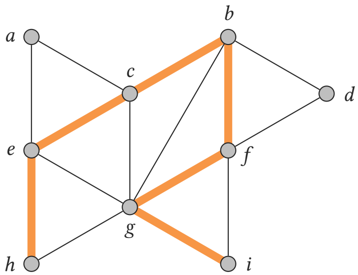


$\deg(v) = 3 \qquad \deg(a) = 1 \qquad \deg(b) = 5 \qquad \deg(c) = 1$

Path

Path

A set $P = \{v_0, v_1, \ldots, v_k\}$ of distinct vertices is called *path* (of *length* k) if v_i is adjacent to v_{i+1} for all i with $0 \leq i < k$.

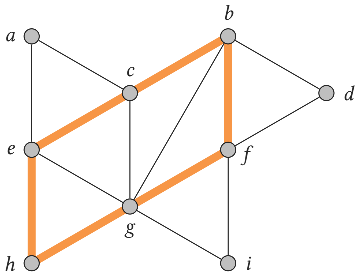


$P = \{h, e, c, b, f, g, i\}$ is a path of length 6.

Cycle

Cycle

A path $P = \{v_0, v_1, \ldots, v_k\}$ is called *cycle* (of *length* $k+1$) if v_0 is adjacent to v_k.

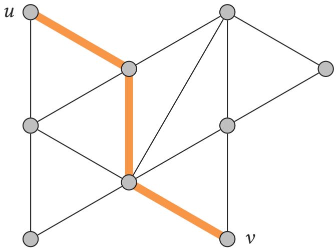


$\{h, e, c, b, f, g\}$ is a cycle of length 6.

Distance

Distance

The *distance* $d(u, v)$ of two vertices u and v is the length of the shortest path from u to v.

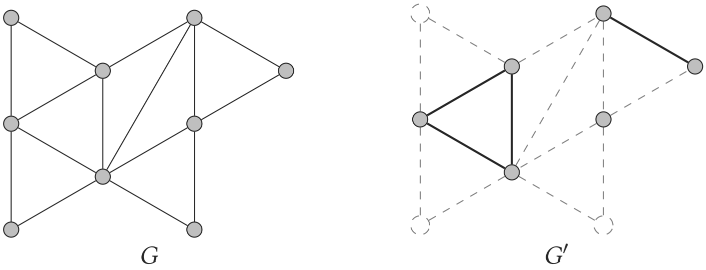


$$d(u, v) = 3$$

Subgraph

Subgraph

A graph $G' = (V', E')$ is a *subgraph* of a graph $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$.

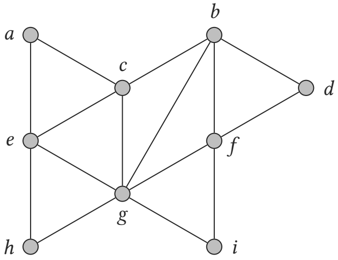


Note that $u, v \in V \cap V'$ and $uv \in E$ does *not* imply $uv \in E'$.

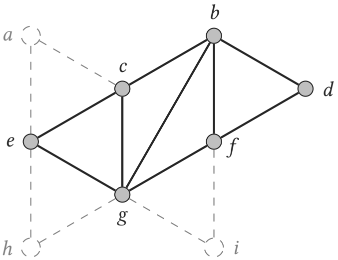
Induced Subgraph

Induced Subgraph

For a graph $G = (V, E)$ and a set $U \subseteq V$, the *induced subgraph* $G[U]$ of G is defined as $G[U] = (U, E')$ with $E' = \{ uv \mid u, v \in U; uv \in E \}$



G

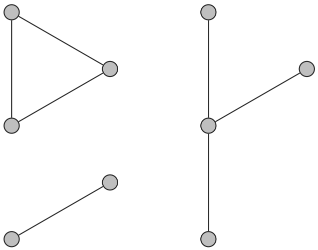


$G\big[\{b, c, d, e, f, g\}\big]$

Connected Component

Connected Component

A *connected component* of an (undirected) graph is a maximal subgraph in which any two vertices can be connected by a path.



A graph with three connected components.