Simple Topologies

Applications of Computational Geometry in Wireless Networks

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Simple Toplolgies - Outline

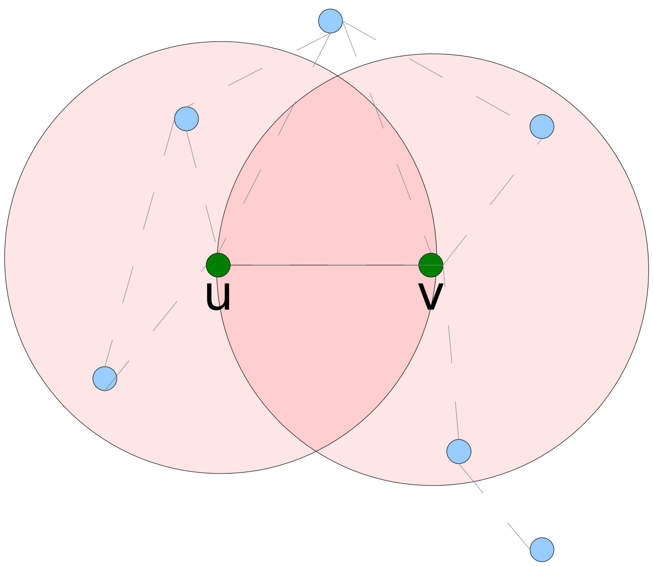
- Unit Disk Graph (UDG)
- Relative Neighborhood Graph (RNG)
- Gabriel Graph (GG)
- Yao-Graph
- The Power Stretch Factor
  - RNG, GG, Yao
- Bounded Degree Topologies
  - Sink, YaoYao, Symmetric Yao

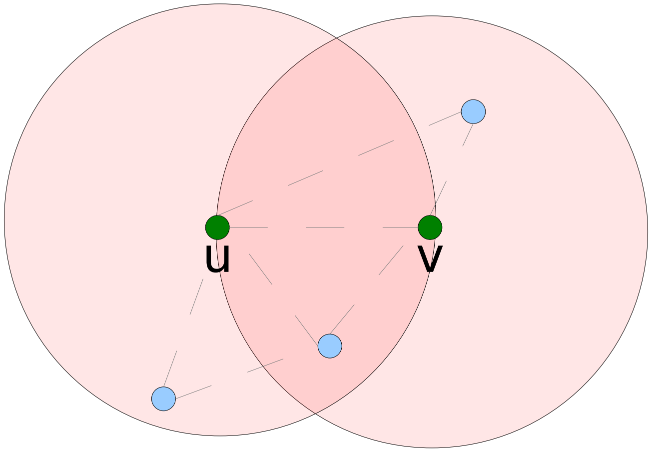
Unit Disk Graph

- Let V be a set of n wireless nodes distributed in a 2D plane
- These nodes define a unit disk graph UDG(V)
- $\|uv\|$ means the norm (distance) between nodes u and v
- There is an edge between nodes u and v iff $\|uv\| <= 1$

Relative Neighborhood Graph (RNG)

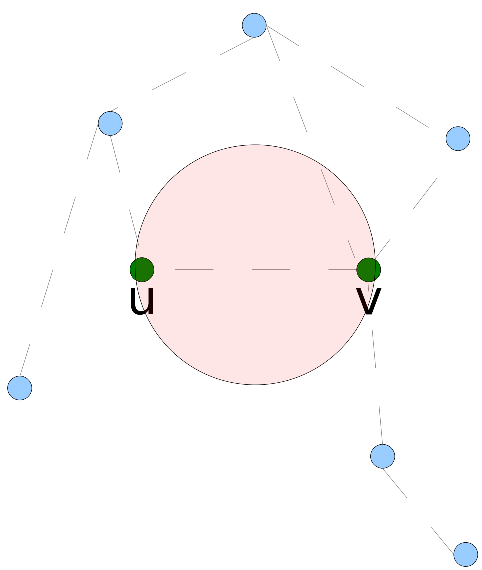
- All edges *uv*∈*E* such that there is no point w∈*V* with edges uw and wv in E satisfying ||uw|| < ||uv|| and ||wv|| < ||uv||

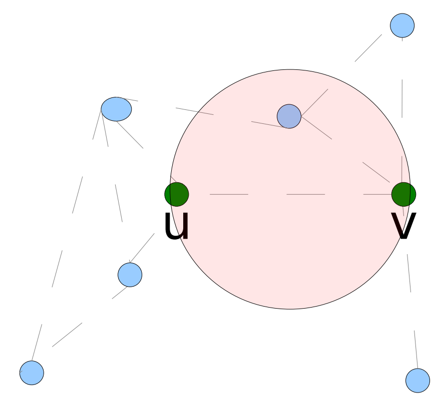




Gabriel Graph

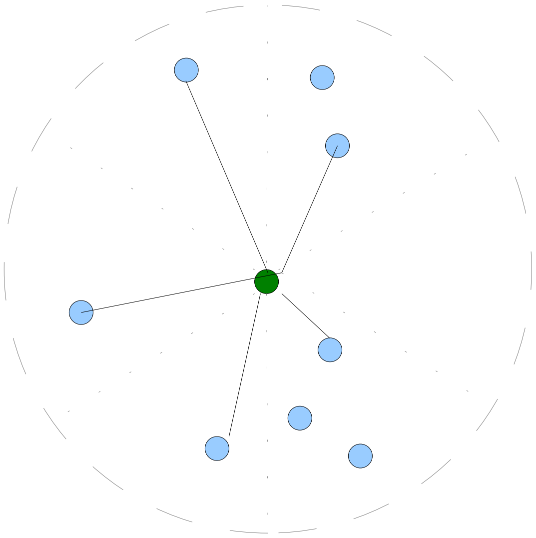
- Contains an edge from G iff disk(u, v) contains no other vertex $w \in V$ such that there exist edges uw and wv from G satisfying $\|uw\| < \|uv\|$ and $\|wv\| < \|uv\|$





Yao Graph

- Graph with an integer parameter k >= 6. At each node u, any k equally separated rays originated at u define k cones. The shortest (if any) edge from u is added to the graph



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The Power Stretch Factor

- Total transmission power consumed by path Π is:

$$p(\Pi) = \sum_{i=1}^{h} \|v_{i-1}v_i\|^{\beta}$$

- The power stretch factor is:

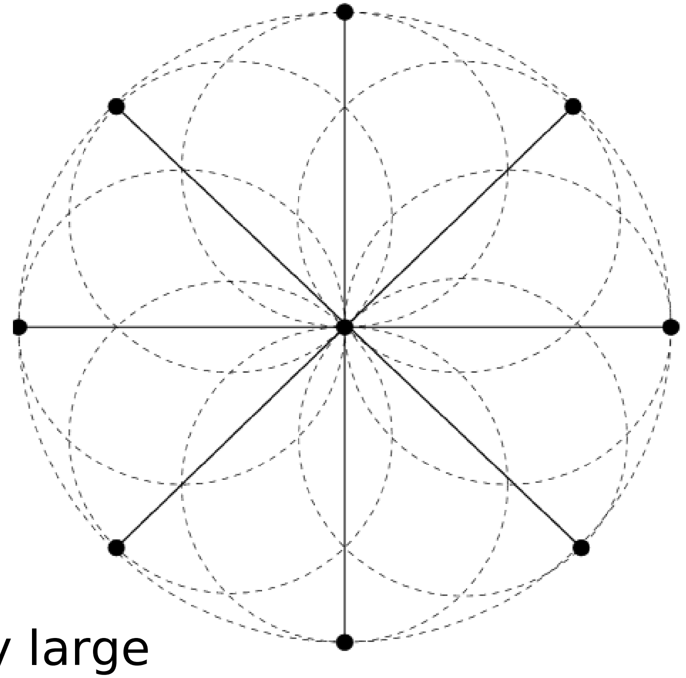
$$\rho_H(G) = \max_{u,v \in V} \frac{p_H(u,v)}{p_G(u,v)}$$

Power Stretch Factor of RNG

- At most n – 1
- Path between u and v in EMST(V) has at most n – 1 edges and each edge has length at most $\|uv\|$
- EMST(V) $\subset$ RNG(V) if UDG(V) is connected

Power Stretch Factor of GG

- Always = 1; Gabriel Graph is an optimal power spanner
- Proof involves showing that we cannot add an edge to the GG which will reduce the energy



Problem though:
The degree may be very large

Power Stretch Factor of YG

$$\frac{1}{1-(2\sin\frac{\pi}{k})^{\beta}}$$

- Important thing: bounded by a constant

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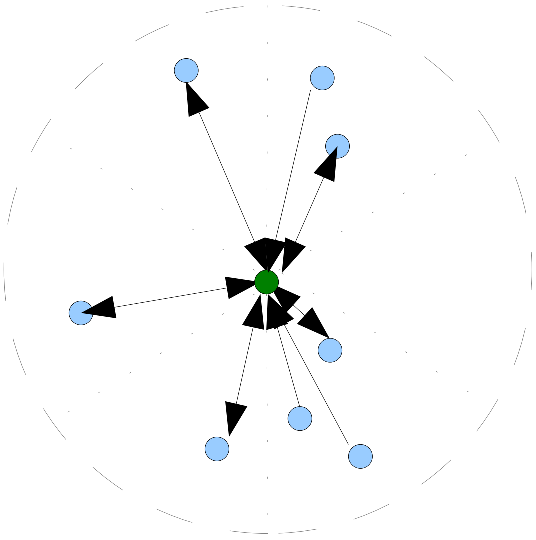
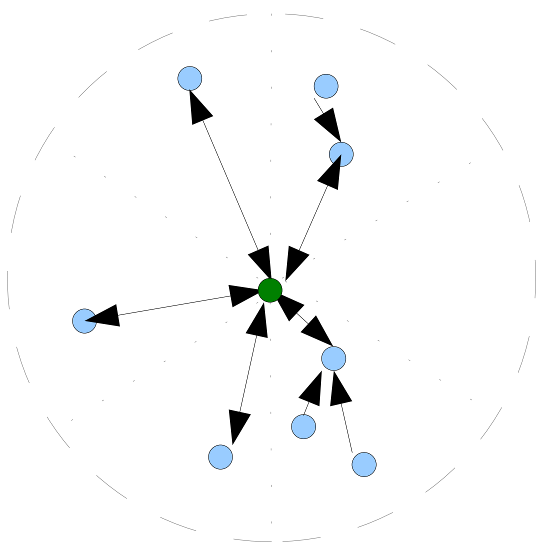
Bounded Degree Spanners

- Yao graph and its variants have a bounded power stretch factor and bounded out-degree k
- No bound on in-degree, however, which results in too much overhead at u



Sink Structure

- Idea: Use a directed tree to point further nodes toward (not directly to) central node u



YaoYao Structure

- Each node v_i of V has a unique identification number $ID(v_i) = i$
- ID of edge uv is defined $ID(uv) = (\|uv\|, ID(u), ID(v))$
- Order ranks by their identities and build the tree based on the minimum in each cone

Symmetric Yao Graph

- An edge $\overrightarrow{uv}$ is included if both directed edges $\overrightarrow{uv}$ and $\overrightarrow{vu}$ are in the Yao graph $\overrightarrow{Yg_k}(V)$
- Maximum node degree is k

References

- Xiang-Yang Li, “Applications of Computational Geometry in Wireless Networks,” in Ad Hoc Wireless Networking, 2003
- Xiang-Yang Li, Peng-Jun Wan, Yu Want, and Ophir Frieder, “Sparse Power Efficient Topology for Wireless Networks,” in Proceedings of HICSS-35, 2002

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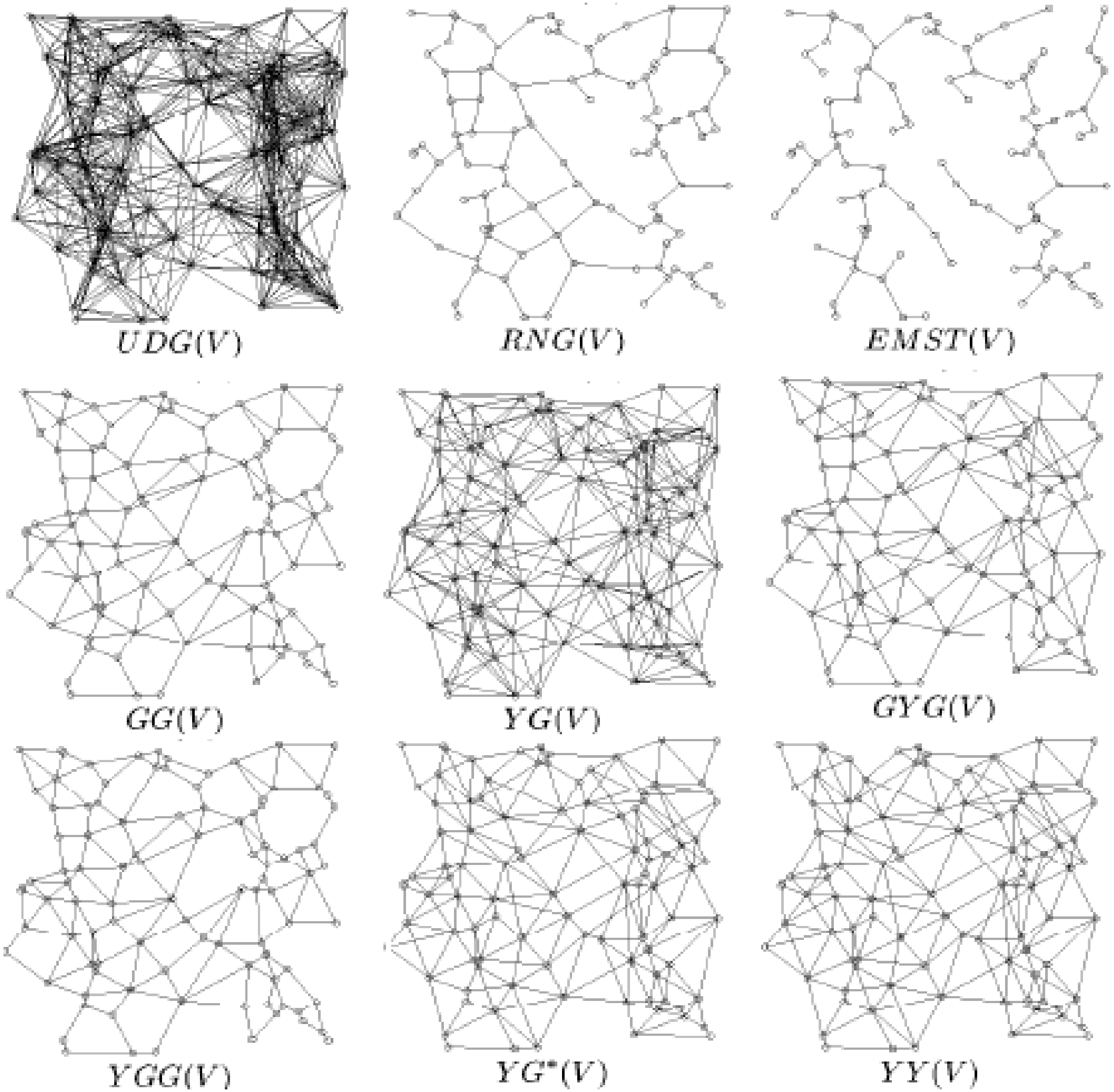

Figure 4: Different topologies generated from the same *unit disk graph* $UDG(V)$.

Source: Xiang-Yang Li, Peng-Jun Wan, Yu Want, and Ophir Frieder, “Sparse Power Efficient Topology for Wireless Networks”