## Computational Geometry: Homework 1

## Problem 1.

a) If a line $L$ does not intersect a diagonal of a convex polygon $\boldsymbol{P}$ then $\boldsymbol{L}$ can intersect only one of the two subpolygons defined by that diagonal. Proof this.
b) Suggest an $\mathbf{O}(1)$ time method for recognizing which of the two subpolygons may intersect $L$. Your method should detect also a trivial case when the line $L$ cannot intersect any of the subpolygons. Hint: Consider the distances between $L$ and three vertices of the polygon: closest to $L$ end-vertex of the diagonal and two neighbors of this vertex on $\boldsymbol{P}$. The distance between a point $\left(\mathbf{x}^{\prime}, \boldsymbol{y}^{\prime}\right)$ and line $\boldsymbol{A x + B y}+\boldsymbol{C}=\mathbf{O}$ is proportional to $\left|A x^{\prime}+B y^{\prime}+C\right|$.
c) Design an algorithm which finds the intersection of a line $L$ with a convex polygon $P$ in $O(l o g n)$ time. Hint: Use a), b) and binary search.

## Problem 2.

Design an O(logn) time algorithm which finds the leftmost and rightmost vertices of a convex polygon. Hint: Use binary search.

## Problem 3.

Problem 5 in the textbook Preparata \& Shamos (p.94).
Apply the locus approach to solve the following problem (fixed-radius circular range search): given $\boldsymbol{N}$ points in the plane and a constant $\mathbf{d > O}$, report (possibly, with logarithmictime overhead) the points that are at most at distance d from a given query point $q$.

