Discrete Structures for Computer Science

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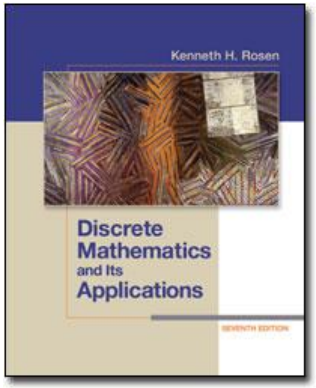
Spring 2012

Textbook

Discrete Mathematics and Its Applications
By Kenneth H. Rosen, McGraw Hill (7th ed.)



Use lecture notes as study guide.

Course Requirements

- Attendance 5%
- Quizzes 20%
- Homework 20%
- Midterm Exam 25%
- Extra Credit Problem 2-5%
- Final Exam 30%

Why Discrete Math?

Design efficient computer systems.

- How did Google manage to build a fast search engine?
- What is the foundation of internet security?

algorithms, data structures, database,
parallel computing, distributed systems,
cryptography, computer networks…

Logic, sets/functions, counting, graph theory…

What is discrete mathematics?

Logic: artificial intelligence (AI), database, circuit design

Counting: probability, analysis of algorithm

Graph theory: computer network, data structures

Number theory: cryptography, coding theory

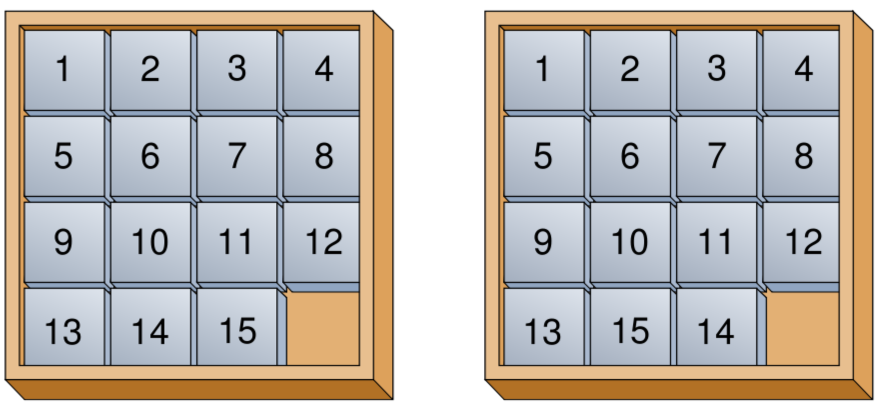
logic, sets, functions, relations, etc

Topic 1: Logic and Proofs

How do computers think?

Logic: propositional logic, first order logic

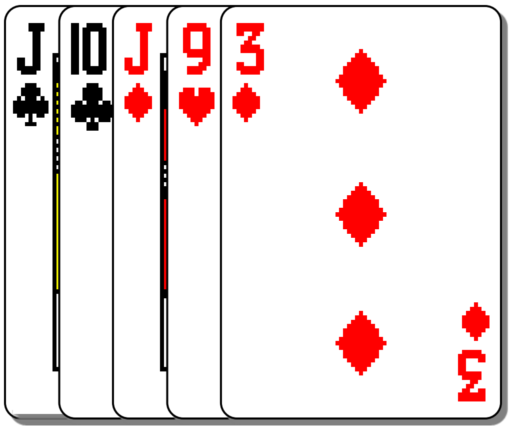
Proof: induction, contradiction



Artificial intelligence, database, circuit, algorithms

Topic 2: Counting

- Sets
- Combinations, Permutations, Binomial theorem
- Functions
- Counting by mapping, pigeonhole principle
- Recursions, generating functions



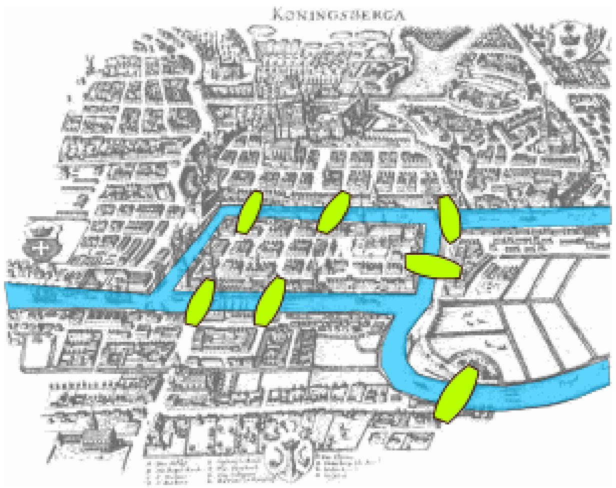
Probability, algorithms, data structures

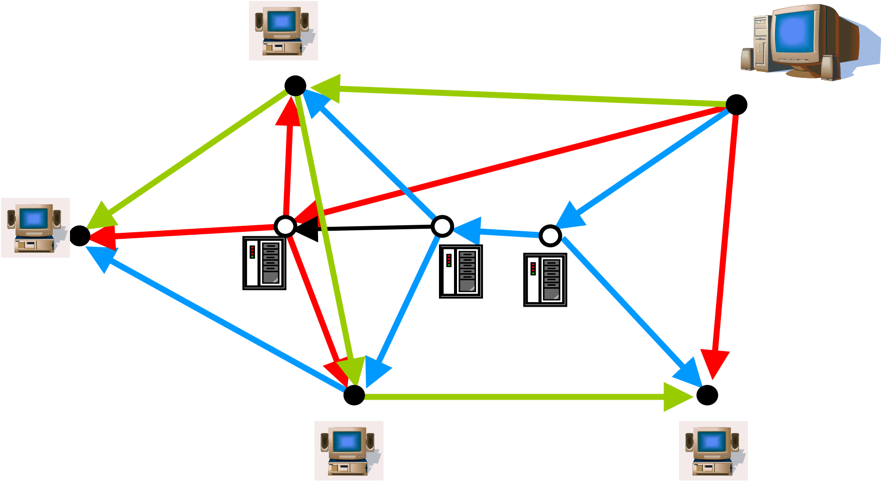
Topic 2: Counting

How many steps are needed to sort n numbers?

Topic 3: Graph Theory

- Relations, graphs
- Degree sequence, isomorphism, Eulerian graphs
- Trees

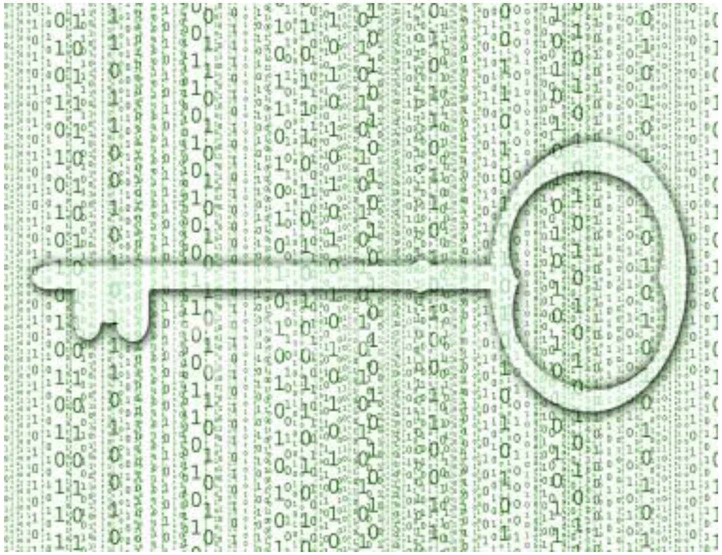




Computer networks, circuit design, data structures

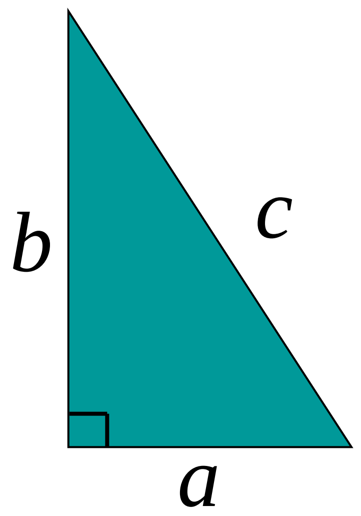
Topic 4: Number Theory

- Number sequence
- Euclidean algorithm
- Prime number
- Modular arithmetic



Cryptography, coding theory, data structures

Pythagorean theorem

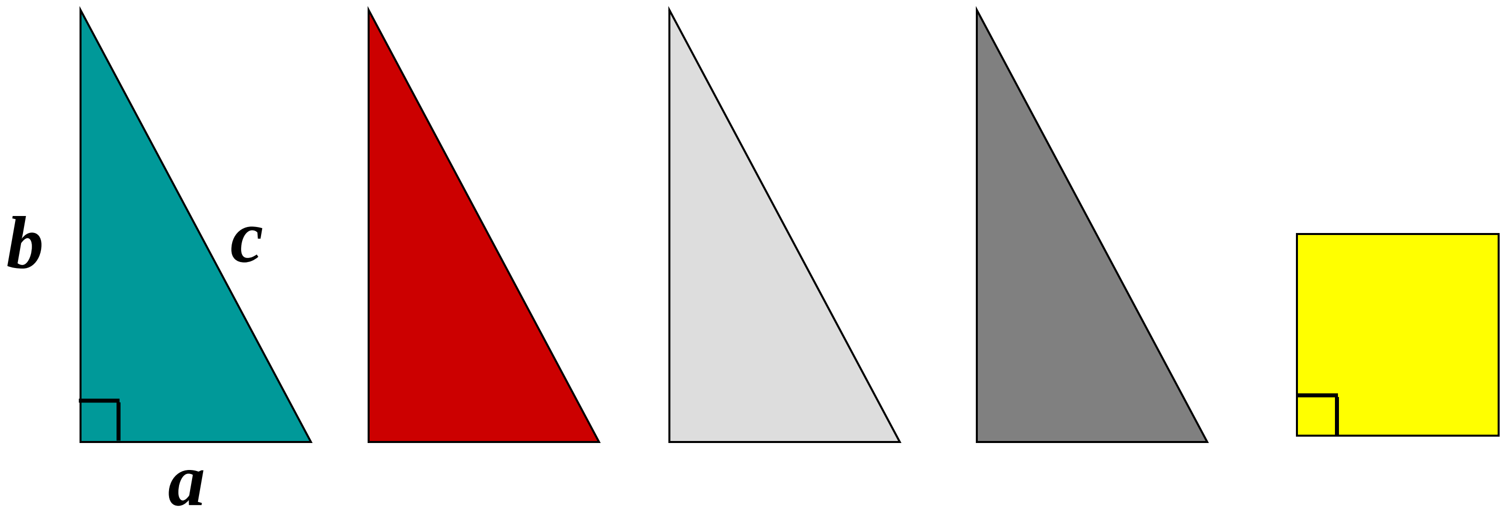


$$a^2 + b^2 = c^2$$

Familiar?

Obvious?

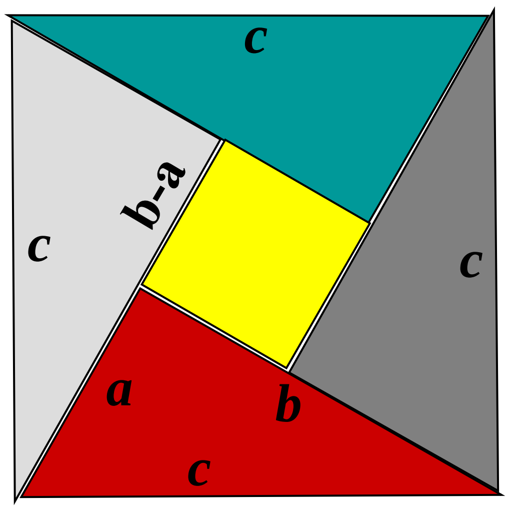
Good Proof



Rearrange into: (i) a c×c square, and then

(ii) an a×a & a b×b square

Good Proof



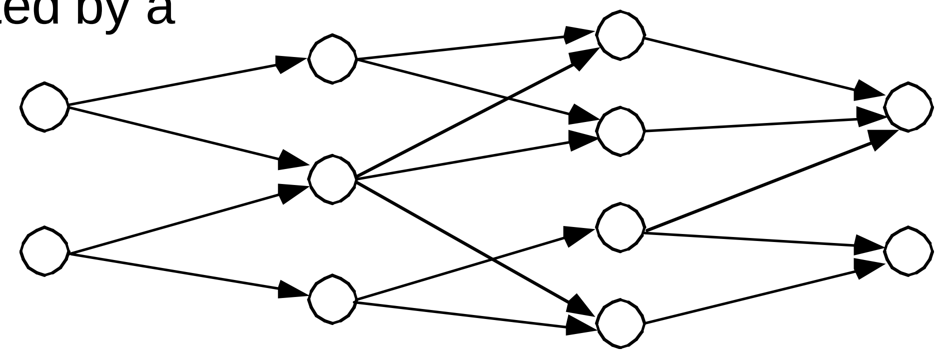
81 proofs in http://www.cut-the-knot.org/pythagoras/index.shtml

Acknowledgement

- Next slides are adapted from ones created by Professor Bart Selman at Cornell University.

Graphs and Networks

•Many problems can be represented by a graphical network representation.



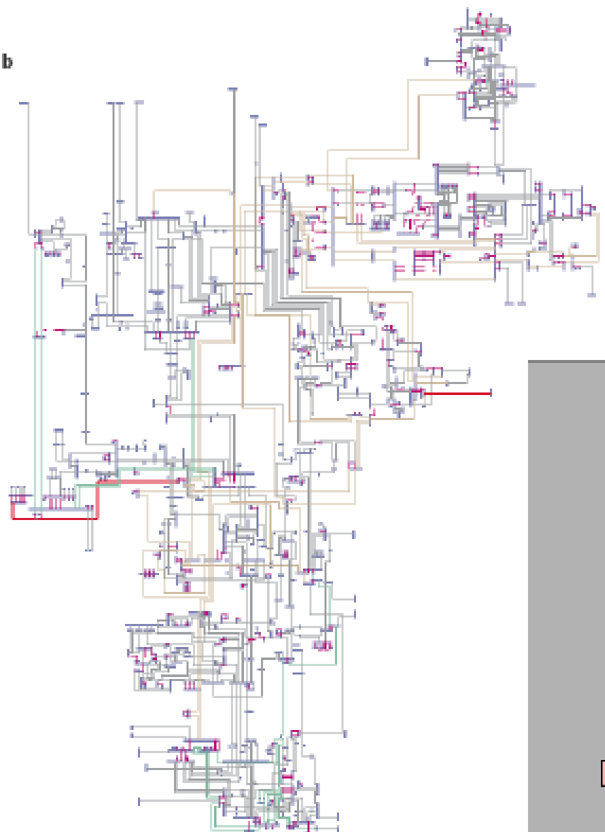
•Examples:

- Distribution problems
- Routing problems
- Maximum flow problems
- Designing computer / phone / road networks
- Equipment replacement
- And of course the Internet

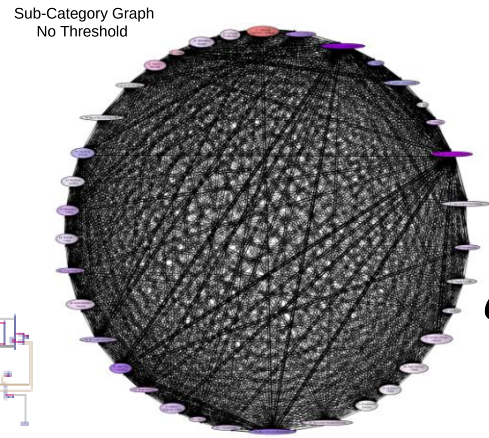
<u>Aside</u>: finding the right problem representation is one of the key issues.

New Science of Networks

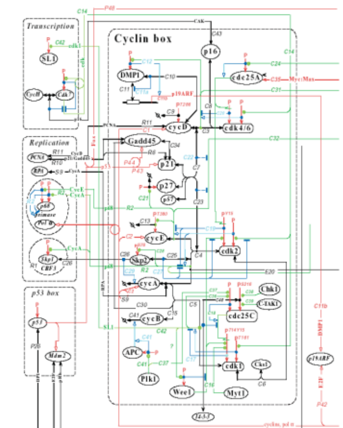
Networks are pervasive



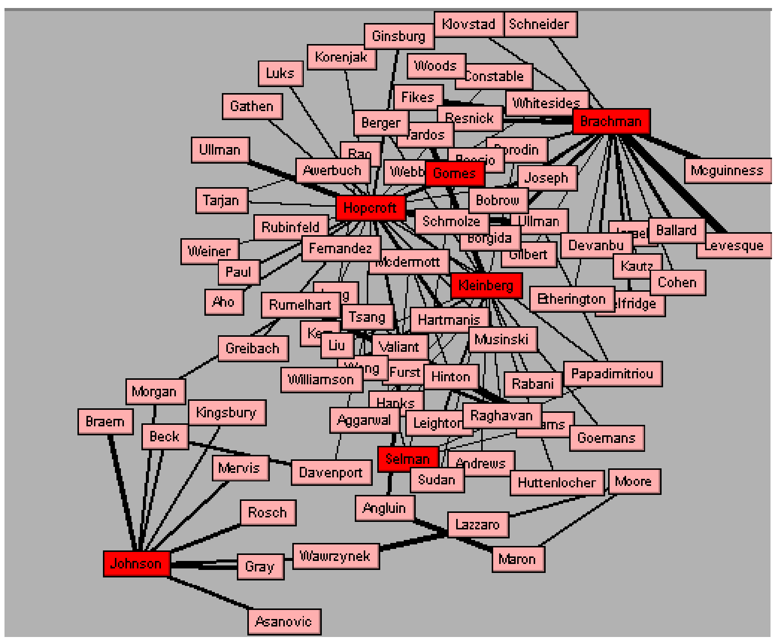
NYS Electric Power Grid (Thorp,Strogatz,Watts)



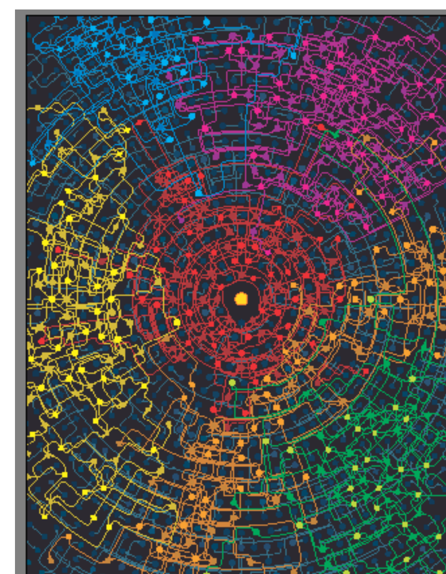
Utility Patent network 1972-1999 (3 Million patents) Gomes,Hopcroft,Lesser,Selman



Neural network of the nematode worm C- elegans (Strogatz, Watts)

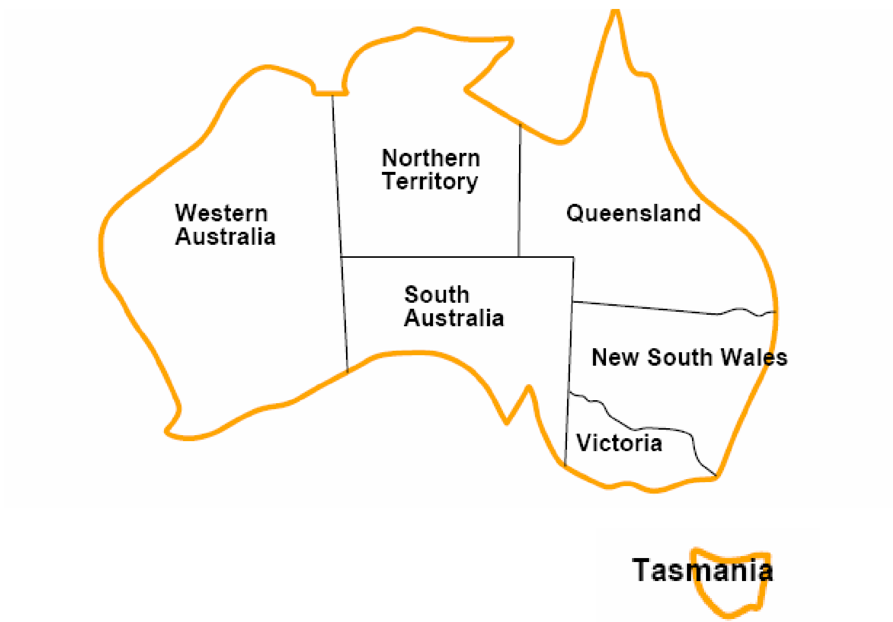


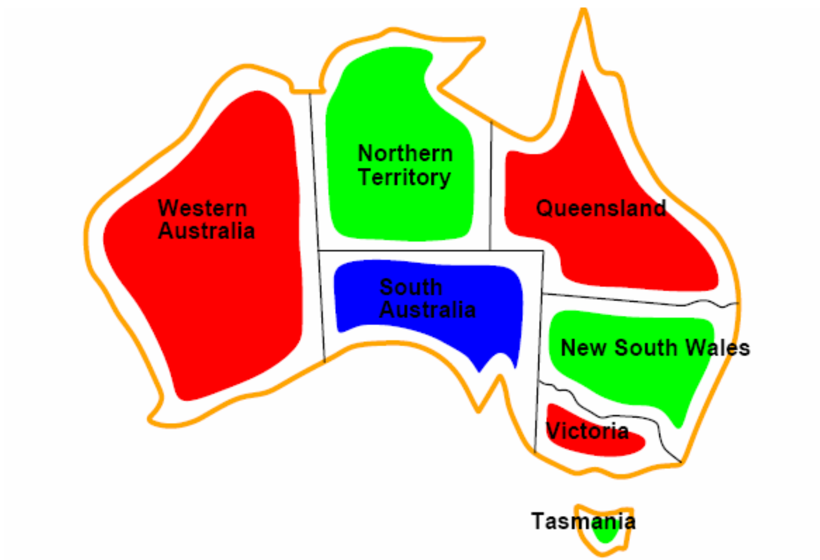
Network of computer scientists ReferralWeb System (Kautz and Selman)



Cybercommunities (Automatically discovered) Kleinberg et al

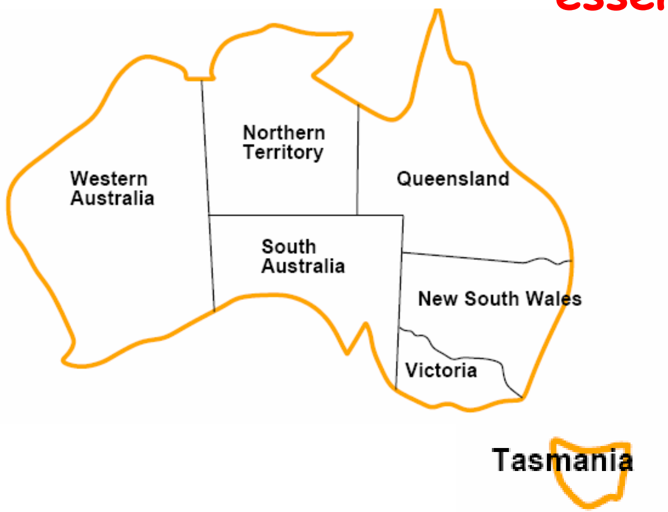
Example: Coloring a Map



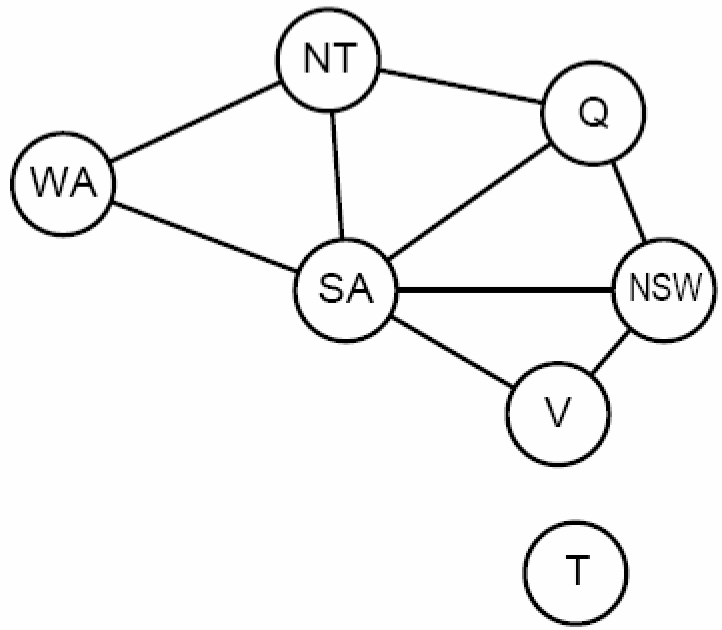


How to color this map so that no two adjacent regions have the same color?

Graph representation

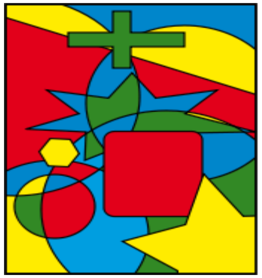


Abstract the essential info:



Coloring the nodes of the graph:
What's the minimum number of colors such that any two nodes connected by an edge have different colors?

Four Color Theorem



Four color map.

- The chromatic number of a graph is the least number of colors that are required to color a graph.

- The Four Color Theorem – ***the chromatic number of a planar graph is no greater than four. (quite surprising!)***

- Proof by Appel and Haken 1976;
- careful case analysis performed by computer;
- Proof reduced the infinitude of possible maps to 1,936 reducible configurations (later reduced to 1,476) which had to be checked one by one by computer.
- The computer program ran for hundreds of hours. The first significant *computer-assisted* mathematical proof. *Write-up was hundreds of pages including code!*

Examples of Applications of Graph Coloring

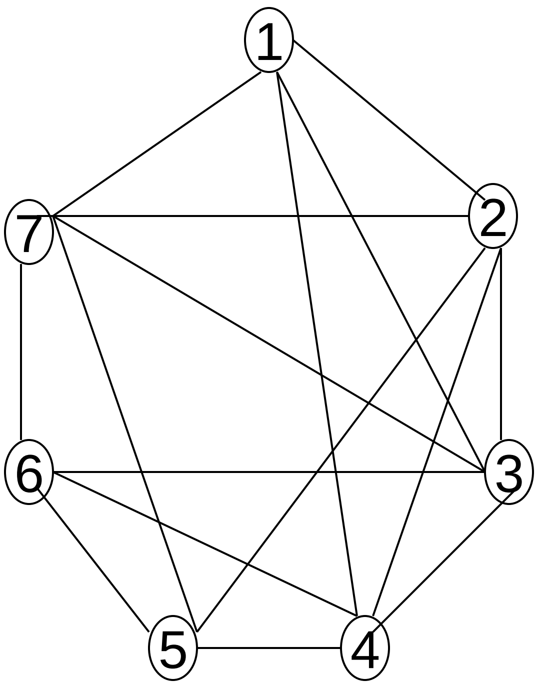
Scheduling of Final Exams

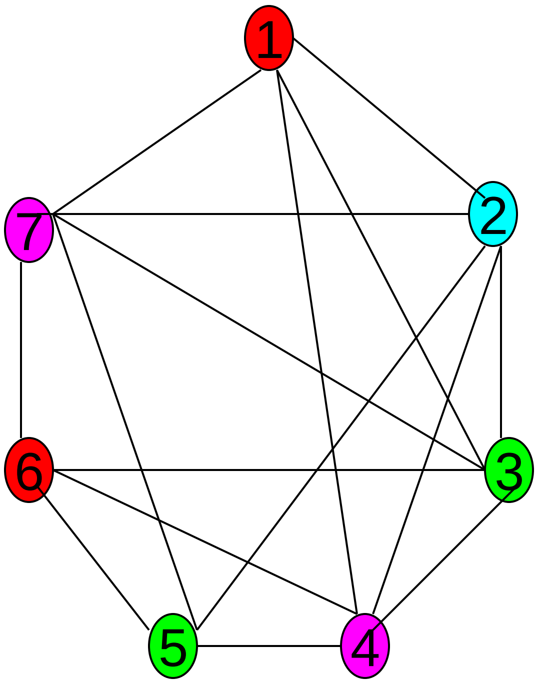
- How can the final exams at Kent State be scheduled so that no student has two exams at the same time? ***(Note not obvious this has anything to do with graphs or graph coloring!)***

Graph:
A vertex correspond to a course.
An edge between two vertices denotes that there is at least one common student in the courses they represent.
Each time slot for a final exam is represented by a different color.

A coloring of the graph corresponds to a valid schedule of the exams.

Scheduling of Final Exams





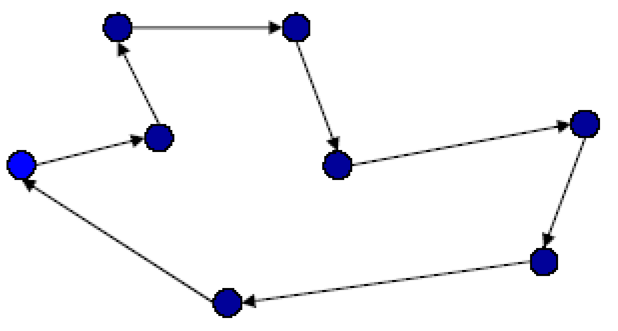
Time Period	Courses
I	1,6
II	2
III	3,5
IV	4,7

Why is mimimum number of colors useful?

What are the constraints between courses?
Find a valid coloring

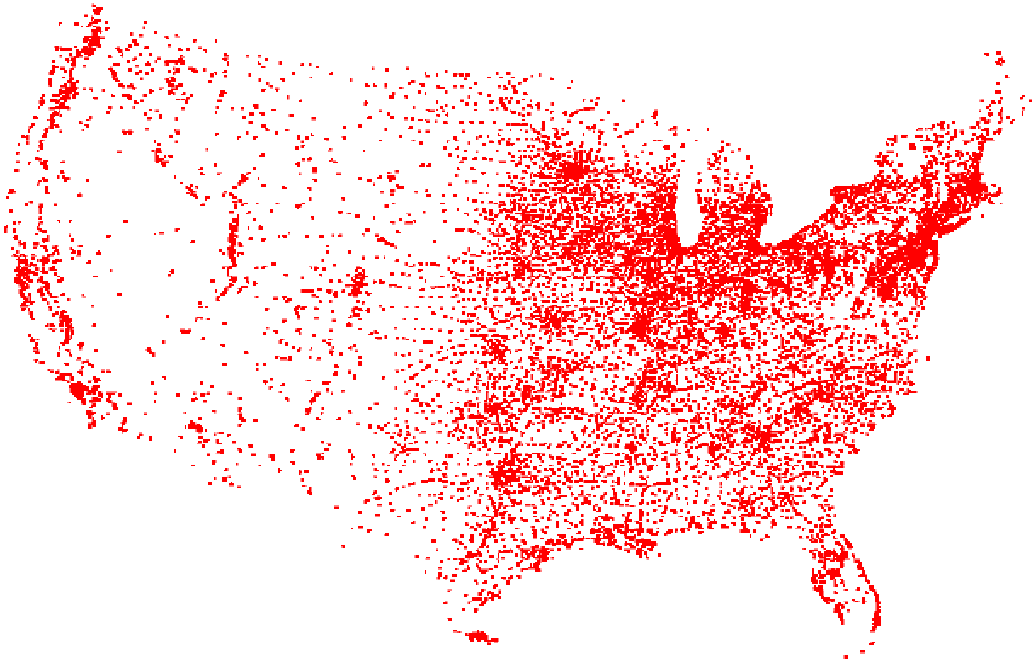
Example 2: Traveling Salesman

Find a closed tour of minimum length visiting all the cities.



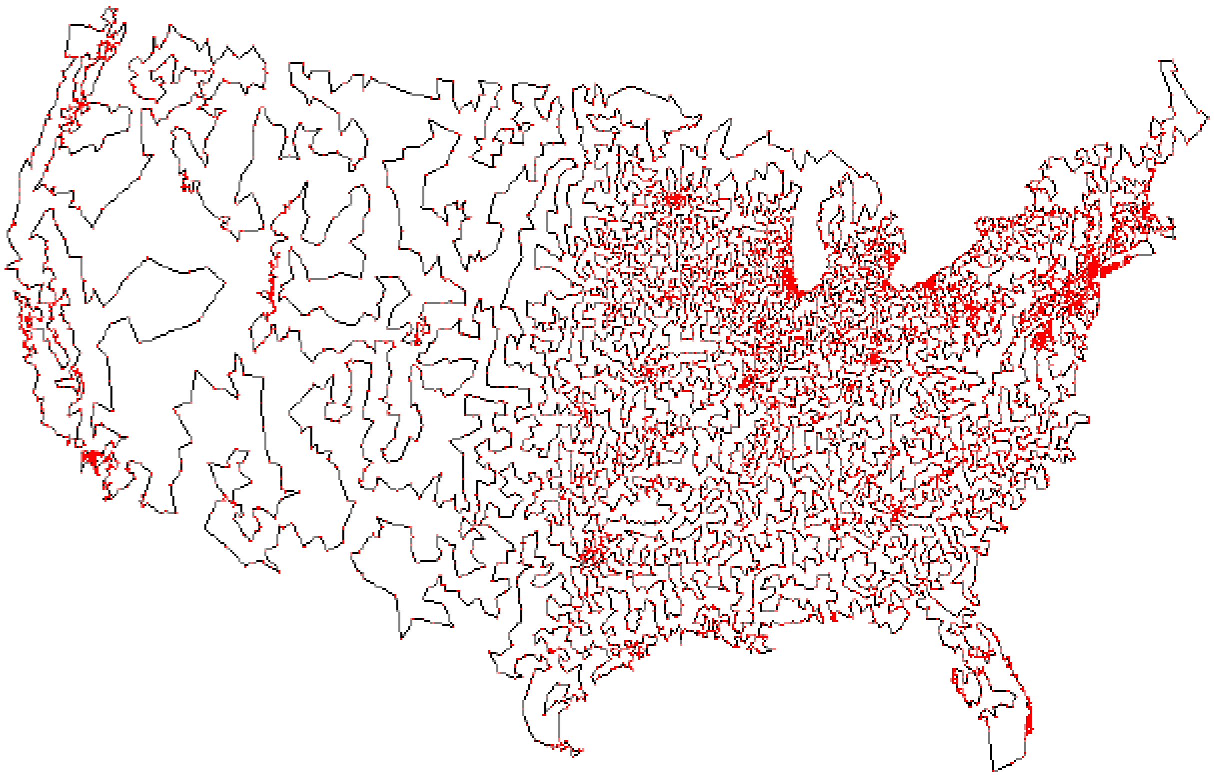
TSP → lots of applications:
Transportation related: scheduling deliveries
Many others: e.g., Scheduling of a machine to drill holes in a circuit board ;
Genome sequencing; etc

13,509 cities in the US



13508!= 1.4759774188460148199751342753208e+49936

13509 cities in the USA



(Applegate, Bixby, Chvatal and Cook, 1998)

The optimal tour!