Distance-Based Location Update and Routing in Irregular Cellular Networks

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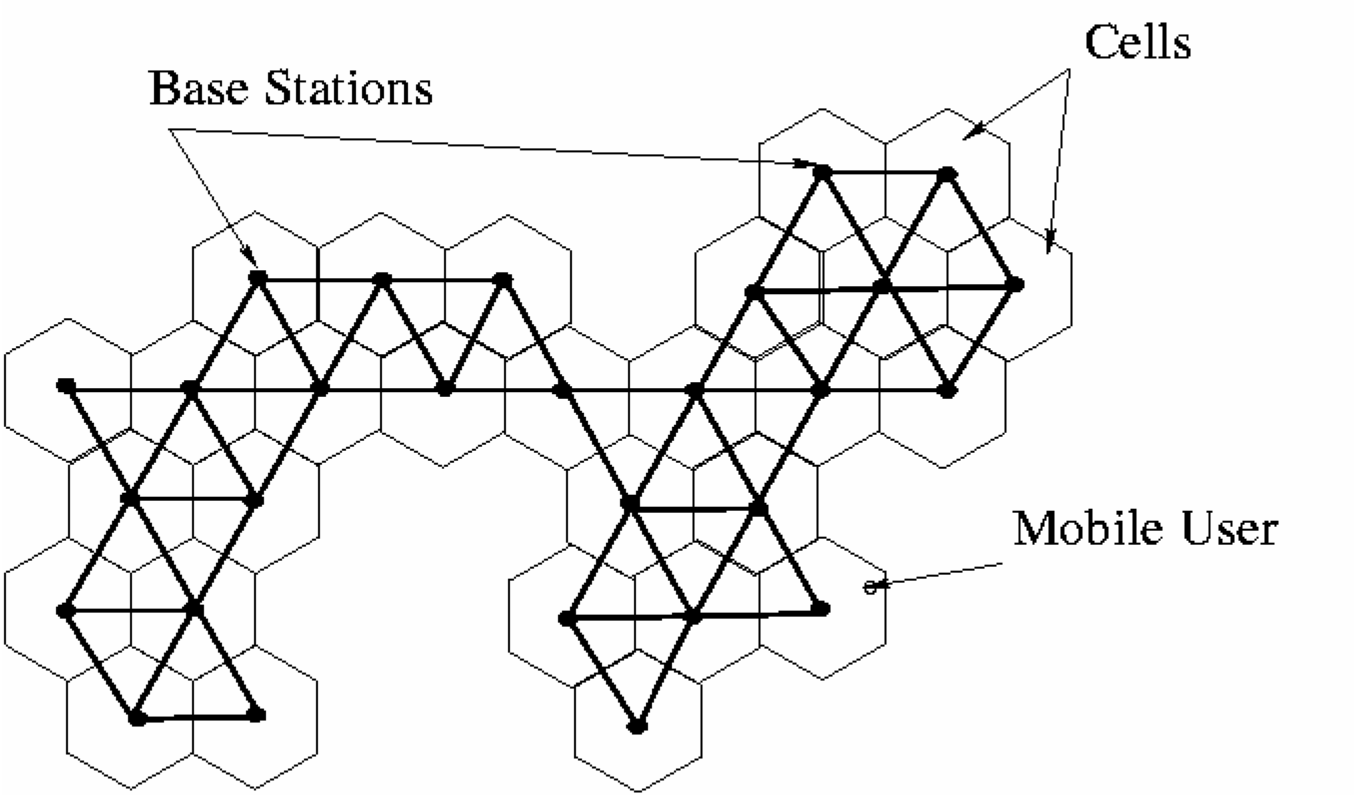
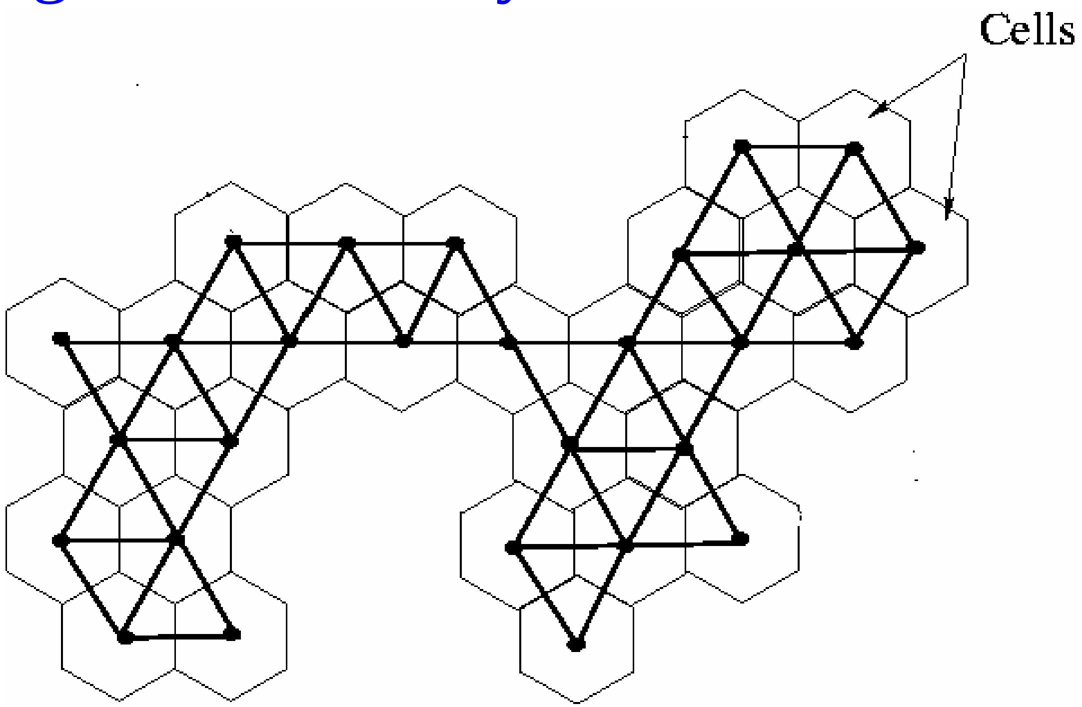
Regular Cellular Network



Regular Cellular Network as Benzenoid and Triangular Systems

- Benzenoid Systems: is a simple circuit of the hexagonal grid and the region bounded by this circuit.



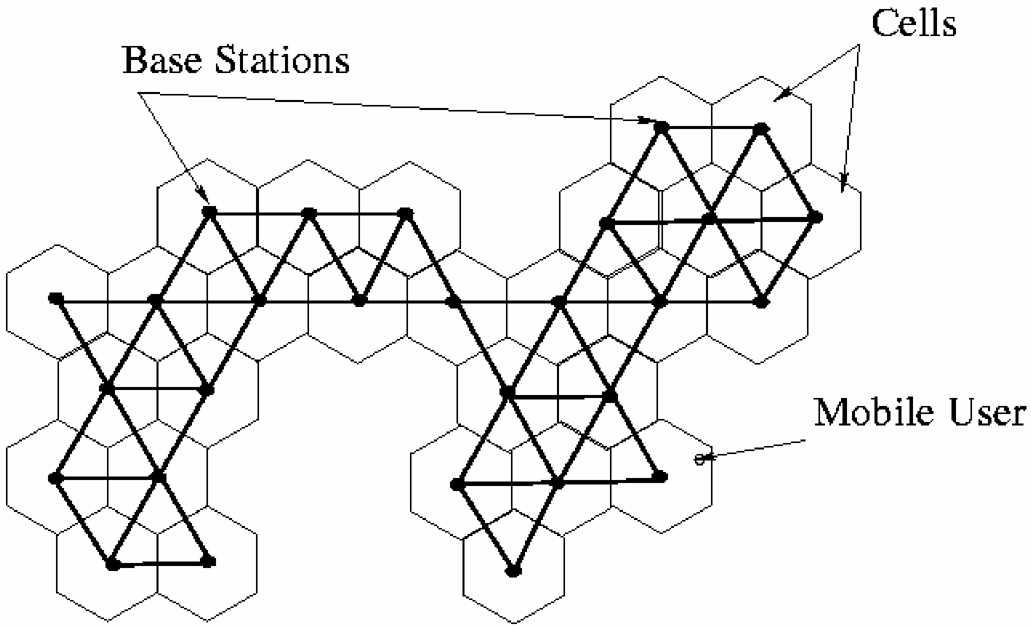
- The Duals to Benzenoid Systems are Triangular Systems

Addressing, Distances and Routing: Necessity

- Identification code (CIC) for tracking mobile users
- Dynamic location update (or registration) scheme
  - time based
  - movement based
  - distance based

(cell-distance based is best, according to [Bar-Noy&Kessler&Sidi'94])

- → Distances
- Routing protocol

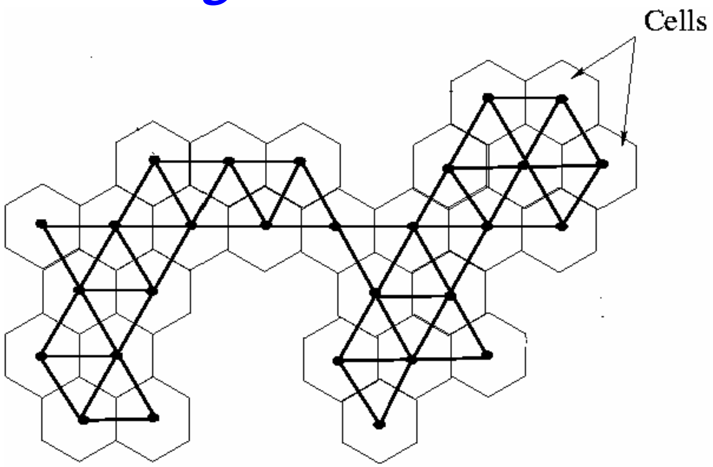


Current situation

- Current cellular networks do not provide information that can be used to derive cell distances
  - It is hard to compute the distances between cells (claim from [Bar-Noy&Kessler&Sidi'94])
  - It requires a lot of storage to maintain the distance information among cells (claim from [Akyildiz&Ho&Lin'96] and [Li&Kameda&Li'00])

Our WMAN'04 results for triangular systems

- Scale 2 isometric embedding into Cartesian product of 3 trees
  - ➔ cell addressing scheme using only three small integers
  - ➔ distance labeling scheme with labels of size $O(\log^2 n)$ -*bits per node and constant time distance decoder*
  - ➔ routing labeling scheme with labels of size $O(\log n)$*-bits per node and constant time routing decision.*



Distance Labeling Scheme

Goal: Short labels that encode distances and distance decoder, an algorithm for inferring the distance between two nodes only from their labels (in time polynomial in the label length)

- **Labeling:** *v* → *Label(v)*

  (for trees, $O(\log^2 n)$ *bits per node* [Peleg'99]*)*

- **Distance decoder:** *D(Label(v), Label(u))* → *dist(u,v)*

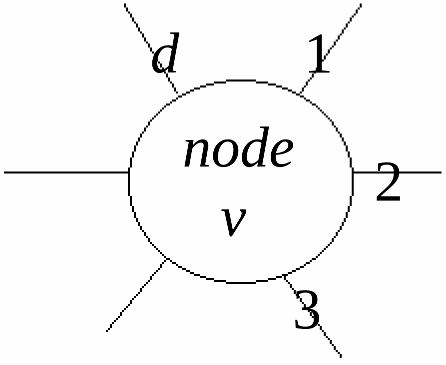
  (for trees, constant decision time)

Routing Labeling Scheme

Goal: Short labels that encode the routing information and routing protocol, an algorithm for inferring port number of the first edge on a shortest path from source to destination, giving only labels and nothing else

- **Labeling:** *v* → *Label(v)*
- **Distance decoder:** *R(Label(v), Label(u))* → *port(v,u)*

(for trees, $O(\log n)$ *bits per node and constant time decision* [Thorup&Zwick'01]*)*



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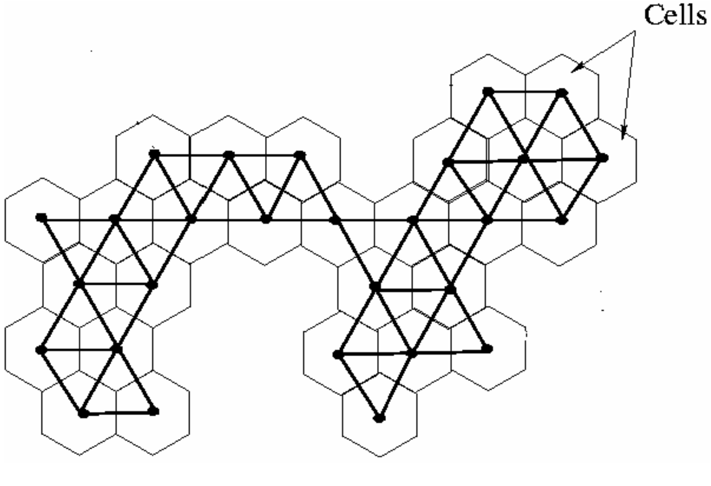
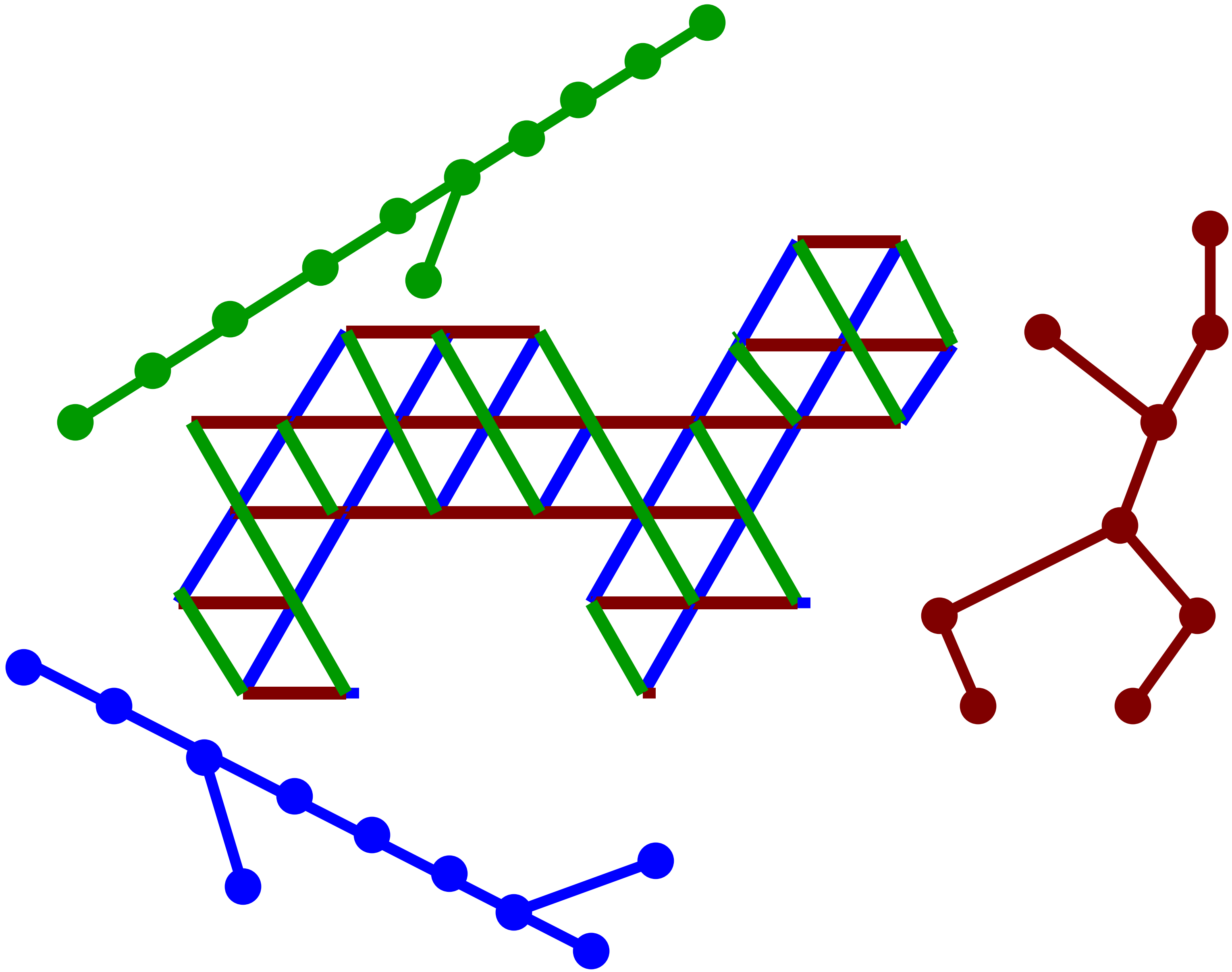
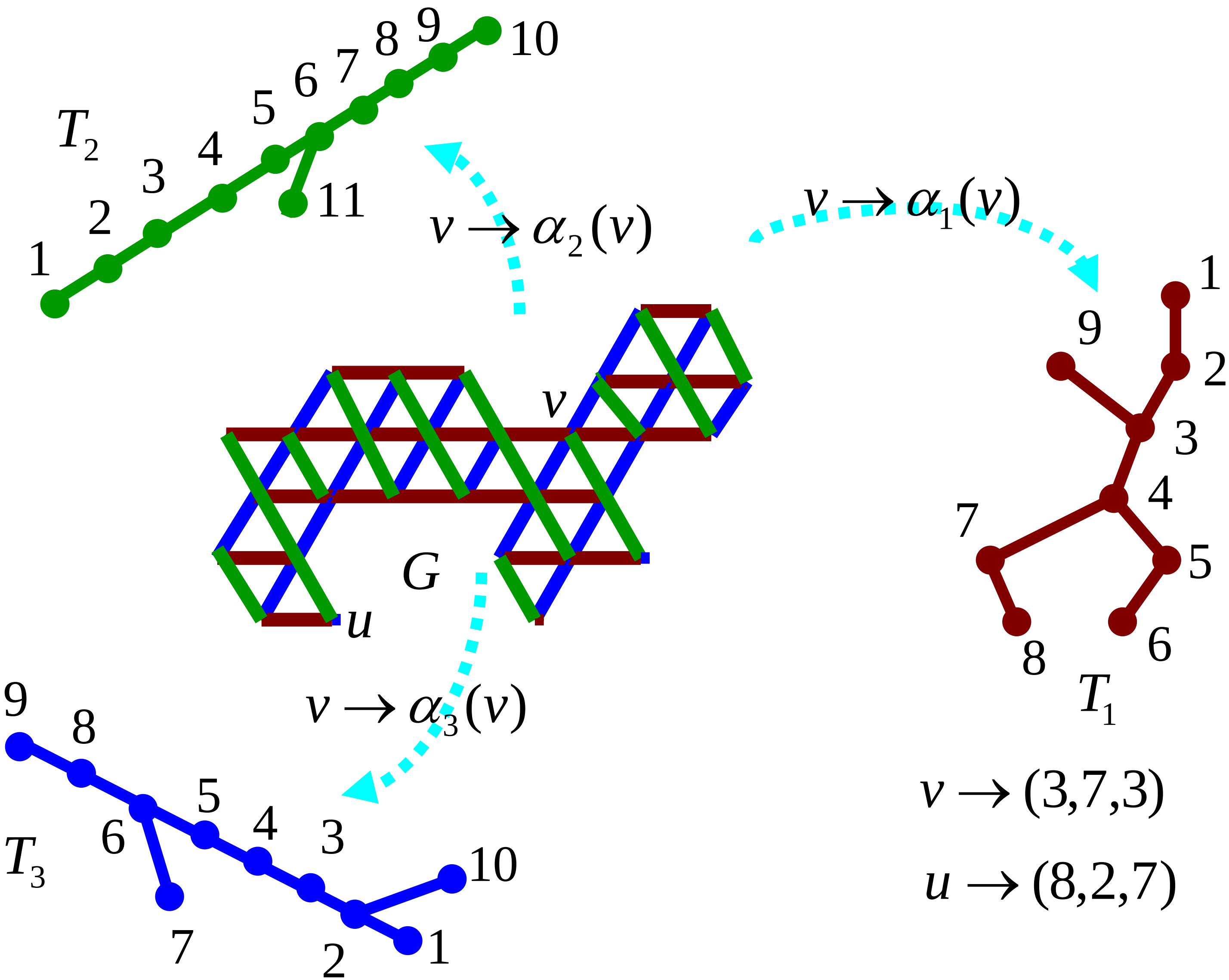


Three edge directions → three trees



Addressing

$$v \to (\alpha_1(v), \alpha_2(v), \alpha_3(v))$$

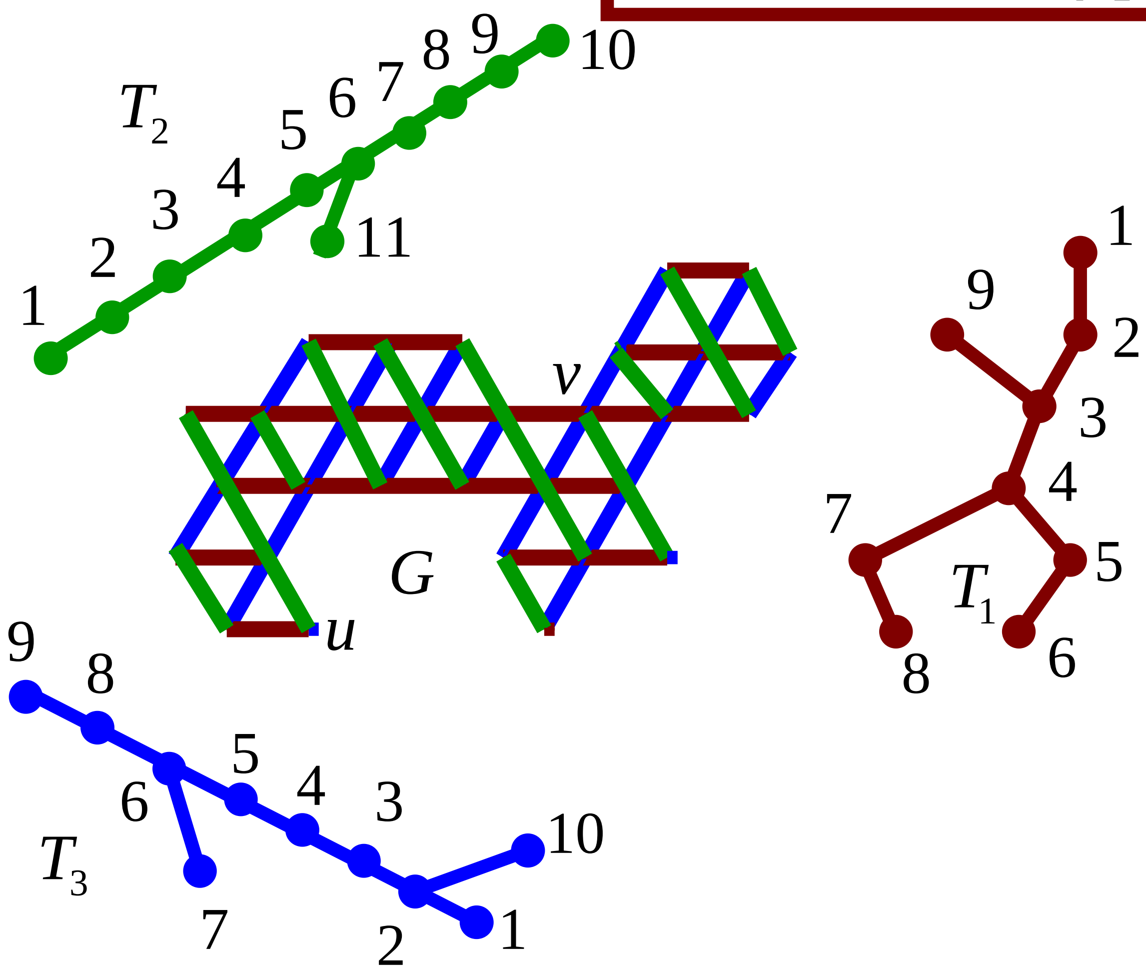


Scale 2 embedding into 3 trees

$$v \to (\alpha_1(v), \alpha_2(v), \alpha_3(v))$$

$$u \to (\alpha_1(u), \alpha_2(u), \alpha_3(u))$$

$$2dist_G(v,u) = \sum_{i=1}^{3} dist_{T_i}(\alpha_i(v), \alpha_i(u))$$



$$v \to (3,7,3)$$

$$u \to (8,2,7)$$

$$dist_{T_1}(\alpha_1(v), \alpha_1(u)) +$$
$$dist_{T_2}(\alpha_2(v), \alpha_2(u)) +$$
$$dist_{T_3}(\alpha_3(v), \alpha_3(u)) =$$
$$3 + 5 + 4 = 12 =$$
$$2dist_G(v,u)$$

Distance labeling scheme for triangular systems

- Given *G*, find three corresponding trees T_1, T_2, T_3 and addressing $v \to (\alpha_1(v), \alpha_2(v), \alpha_3(v))$ (*O(n) time*)

- Construct distance labeling scheme for each tree
  $\alpha_i(v) \to Label(\alpha_i(v))$ (*O(n*log*n) time*)

- Then, set $Label(v) = (Label(\alpha_1(v)), Label(\alpha_2(v)), Label(\alpha_3(v)))$

- $O(\log^2 n)$ -bit labels and constructible in total time $O(n \log n)$

Distance decoder for triangular systems

Given *Label(u)* and *Label(v)*

Function distance_decoder_triang_syst(Label(u),Label(v))

- Output ***½(distance_decoder_trees(*** $Label(\alpha_1(v)), Label(\alpha_1(u))$ ***)***

  +(distance_decoder_trees($Label(\alpha_2(v)), Label(\alpha_2(u))$ ***)***

  +(distance_decoder_trees($Label(\alpha_3(v)), Label(\alpha_3(u))$ ***))***

Thm: The family of *n*-node triangular systems enjoys a distance labeling scheme with $O(\log^2 n)$-bit labels and a constant time distance decoder.

Routing labeling scheme for triangular systems

- Given *G*, find three corresponding trees T_1, T_2, T_3 and addressing $v \rightarrow (\alpha_1(v), \alpha_2(v), \alpha_3(v))$

- Construct routing labeling scheme for each tree using Thorup&Zwick method (*log n* bit labels)

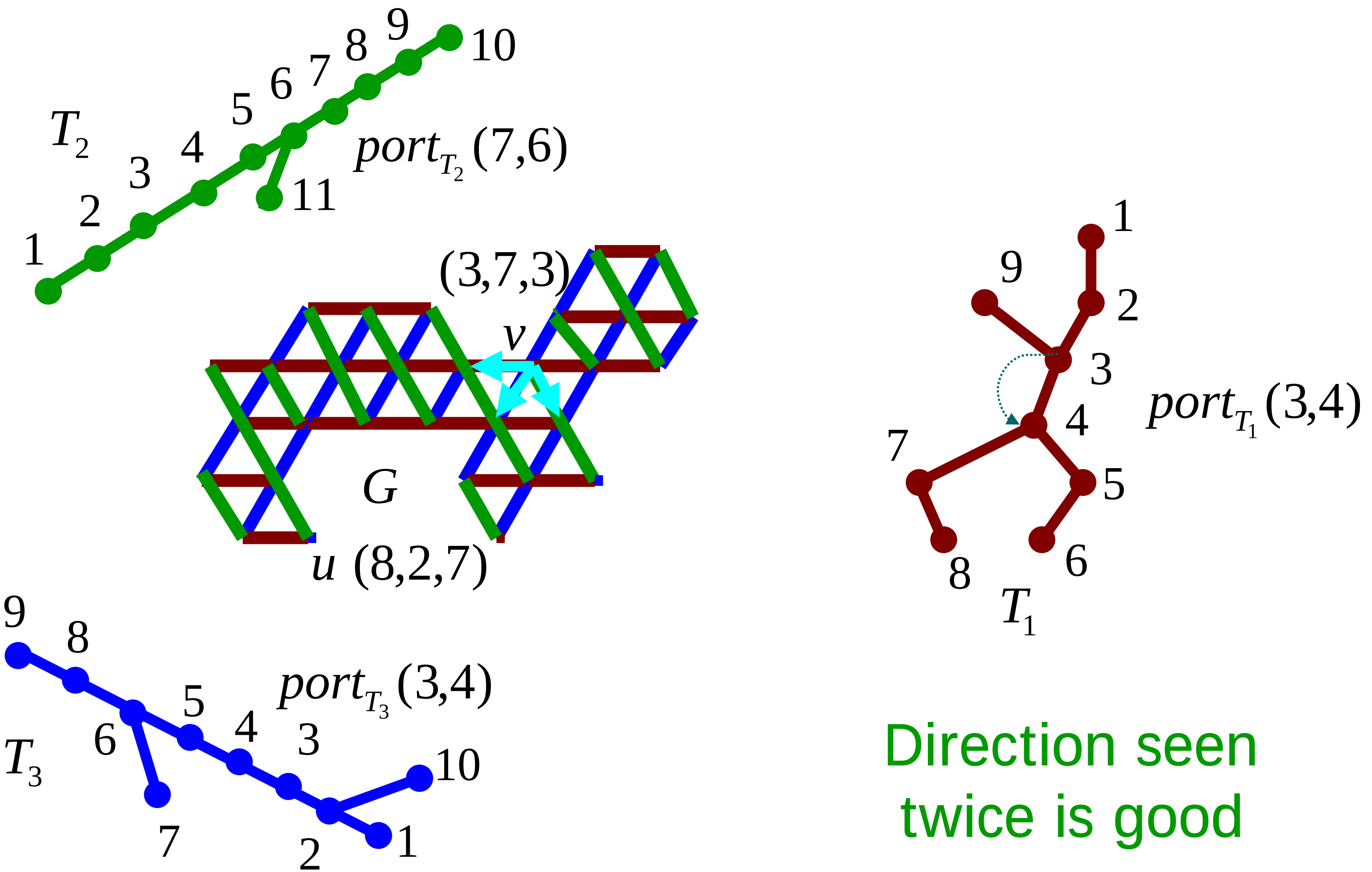
$$\alpha_i(v) \rightarrow Label(\alpha_i(v))$$

- Then, set

$$Label(v) = (Label(\alpha_1(v)), Label(\alpha_2(v)), Label(\alpha_3(v)), \ldots.)$$

Something more

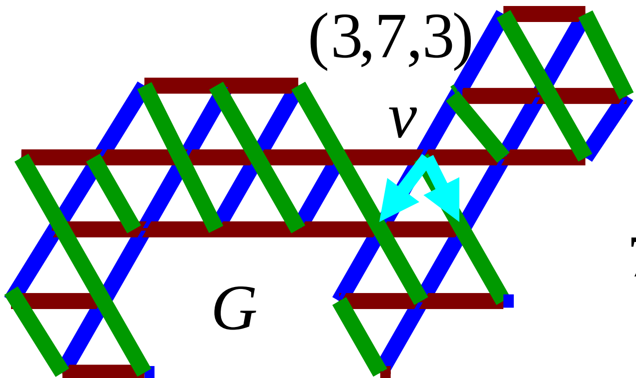
Choosing direction to go from v

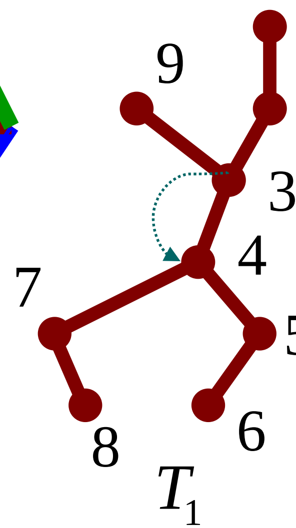


Direction seen twice is good

Mapping tree ports to graph ports

$$\Delta_i(v) = (\alpha_i^1(v),\ldots,\alpha_i^j(v)) = (2,4) \quad (j \le 4)$$





$port_{T_1}(3,4)$

$$O_i(v) = ((port_{T_i}(\alpha_i(v),\alpha_i^1(v)), Q_i^1(v)),\ldots,(port_{T_i}(\alpha_i(v),\alpha_i^j(v)), Q_i^j(v)) =$$

$$((port_{T_1}(3,2), Q_1^1(v)),\ldots,(port_{T_1}(3,4), Q_1^2(v))$$

$$Q_i^j(v) = (port_G^1, port_G^2)$$

Then, $Label(v) = (Label(\alpha_1(v)), Label(\alpha_2(v)), Label(\alpha_3(v)),\ldots.)$

(i.e., *3xlog n+3x4x3xlogn* bit labels)

Routing Decision for triangular systems

Given *Label(u)* and *Label(v)*

```
function routing_decision_triang_syst(L(x), H(y))

if (α1(x), α2(x), α3(x)) = (α1(y), α2(y), α3(y)) then return "packet reached its destination";
set A ← 0;
for each i ∈ {1,2,3} do
   p ← routing_decision_trees(L_Ti(αi(x)), H_Ti(αi(y)));
   for each j ∈ {1,...,|Oi(x)|} do
      if p = Oi(x)[j] then
         for each entry port_G of the array Q_i^j(x) do
            A[port_G] ← A[port_G] + 1;
            if A[port_G] = 2 then
               return port_G.
```

Thm: The family of *n*-node triangular systems enjoys a routing labeling scheme with $O(\log n)$-bit labels and a constant time routing decision.

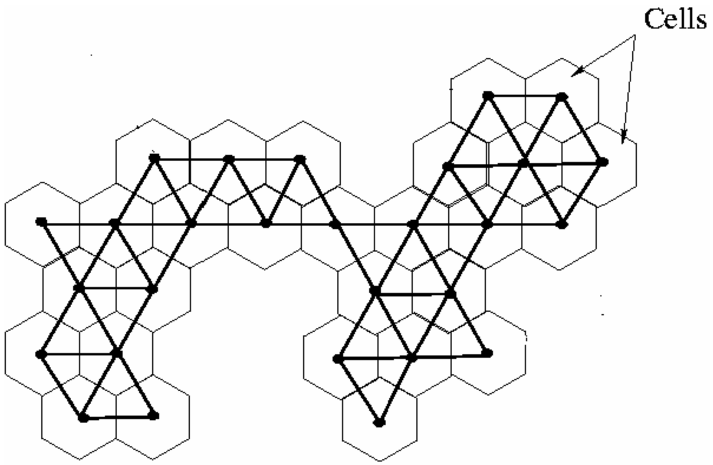
Cellular Networks in Reality

- Planned as uniform configuration of BSs, but in reality BS placement may not be uniformly distributed (← obstacles)

- To accommodate more subscribers, cells of previously deployed cellular network need to be split or rearranged into smaller ones.

- The cell size in one area may be different from the cell size in another area (dense/sparse populated areas)

- Very little is known for about cellular networks with non-uniform distribution of BSs and non-uniform cell sizes

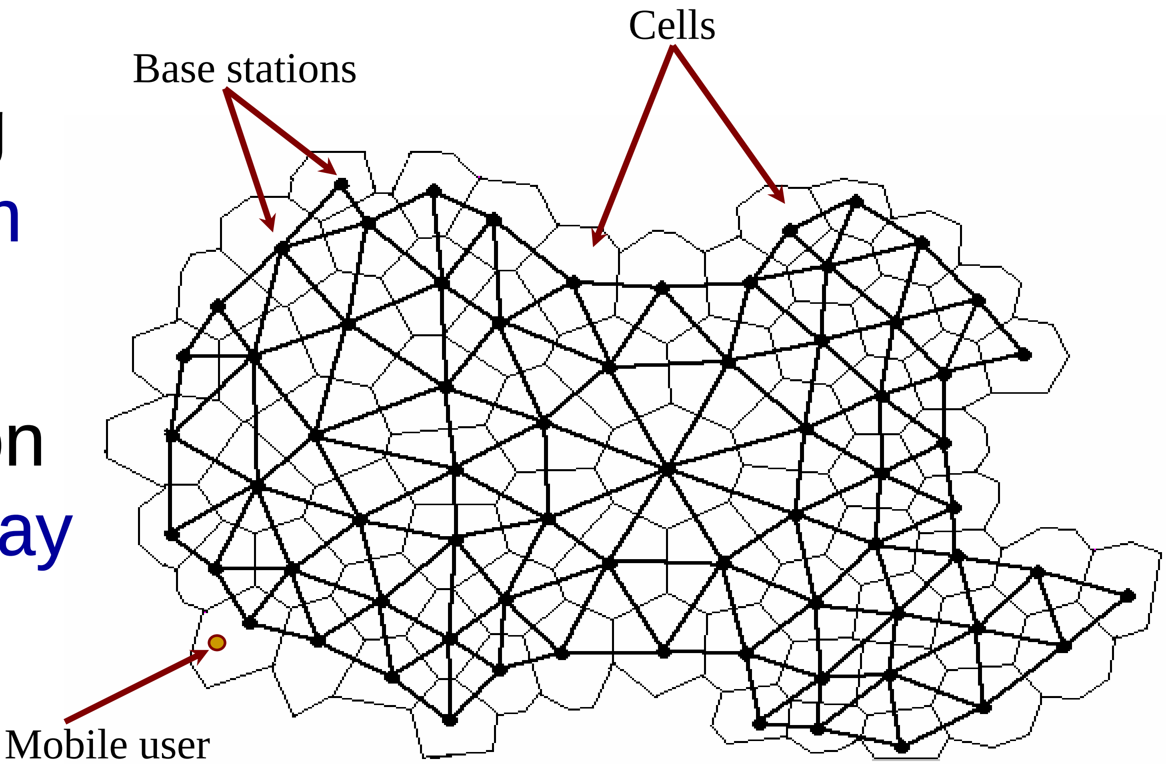


Our Irregular Cellular Networks

- We do not require from BSs to be set in a very regular pattern (➔ more flexibility in designing)

- Cells formed using the Voronoi diagram of BSs

- The communication graph is the Delaunay triangulation

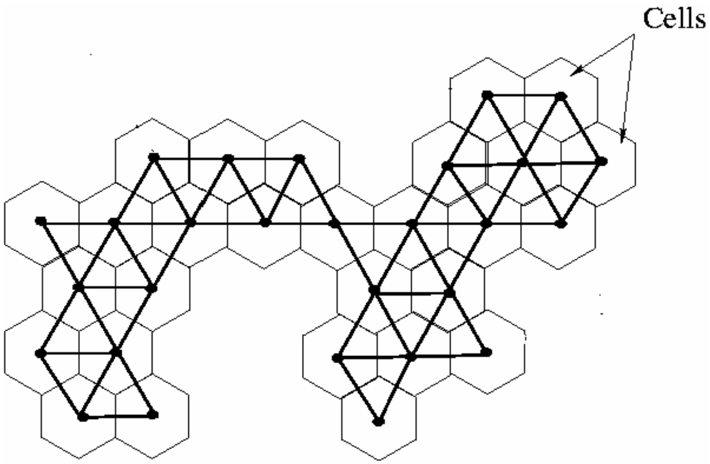


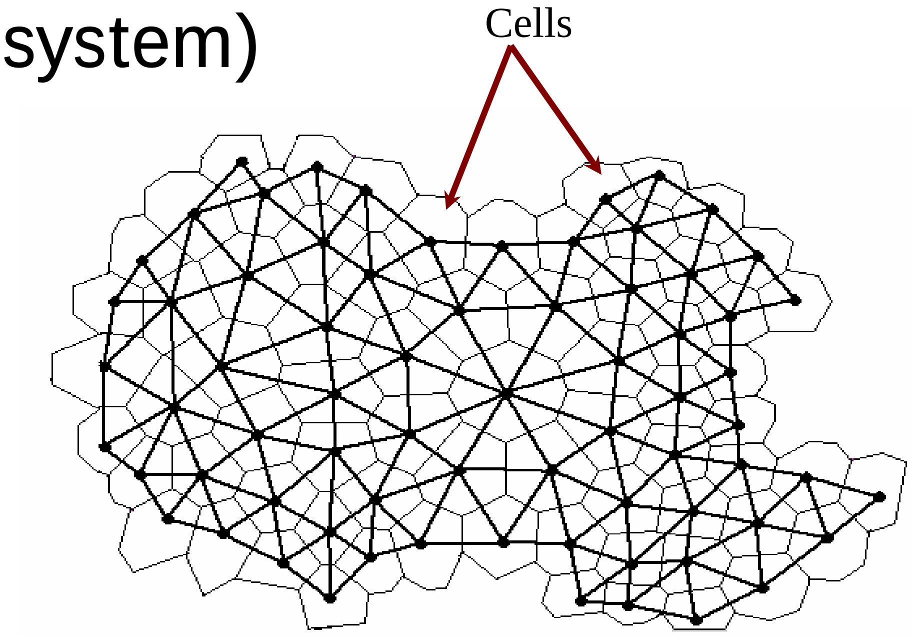
- Our only requirement: each inner cell has at least six neighbor cells (=6 in regular cellular networks)

Trigraphs

- If in the Voronoi diagram of BSs each inner cell has at least six neighbor cells (=6 in regular cellular networks)

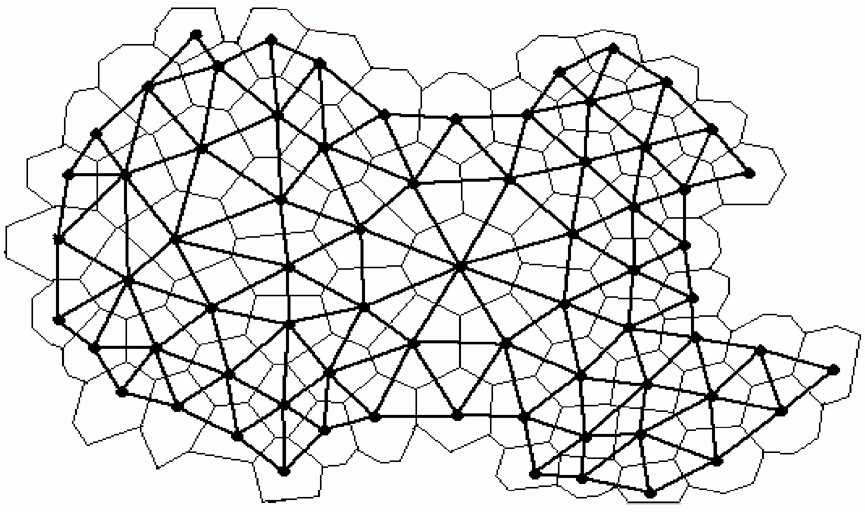
- ➔ (the Delaunay graphs=) Trigraphs are planar triangulations with inner vertices of degree at least six (if all =6 ➔ triangular system)



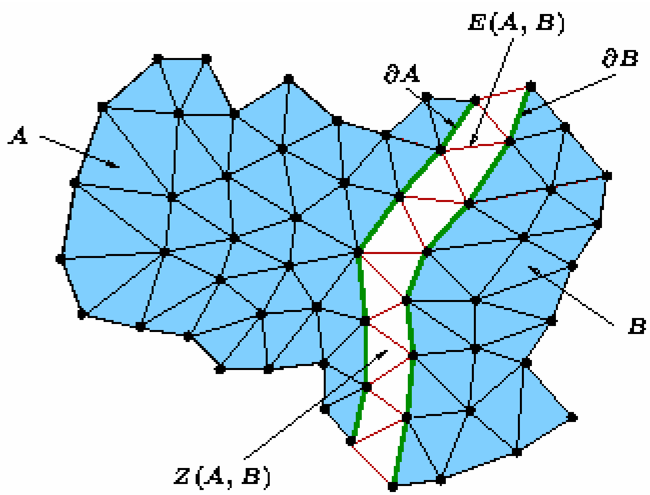


Our results for trigraphs

- Low depth hierarchical decomposition of a trigraph
  - ➔ distance labeling scheme with labels of size $O(\log^2 n)$ - *bits per node and constant time distance decoder*
  - ➔ routing labeling scheme with labels of size $O(\log^2 n)$ - *bits per node and constant time routing decision.*



Cuts in Trigraphs

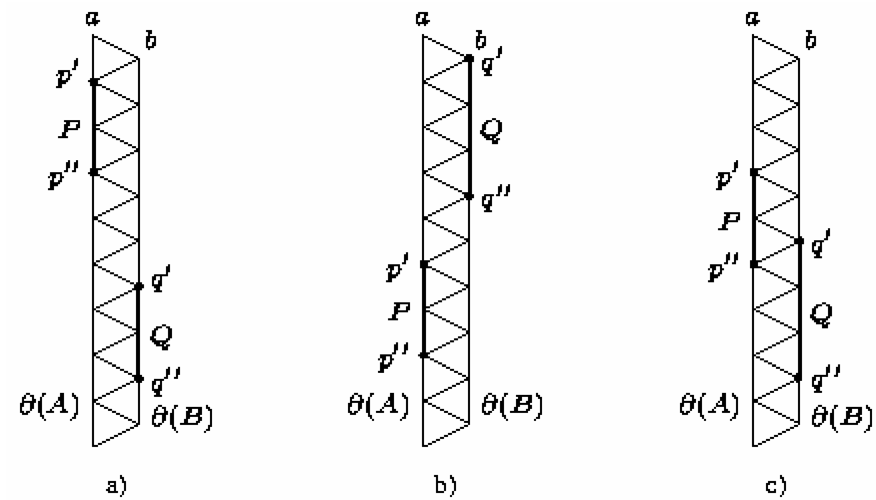


A *convex* cut $\{A, B\}$, its zone $Z(A, B)$, its border lines ∂A and ∂B and the set $E(A, B)$ of edges crossed by this cut.

- Border lines are shortest paths
- A and B parts are convex
- Projections are subpaths

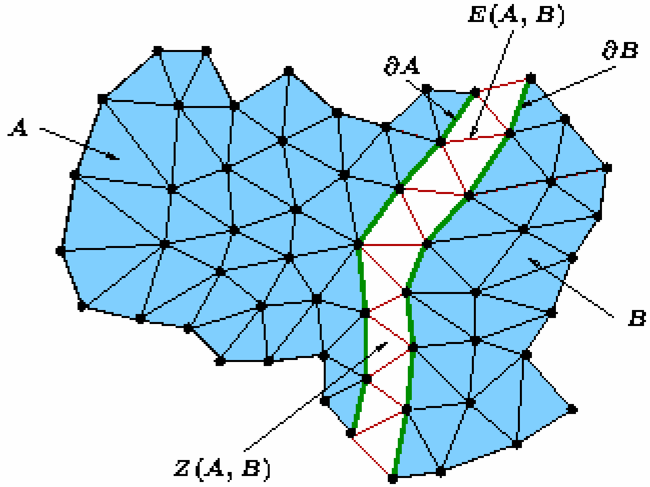
- Distance formula

$$d(x, y) = d(x, P) + d(P, Q) + d(y, Q)$$

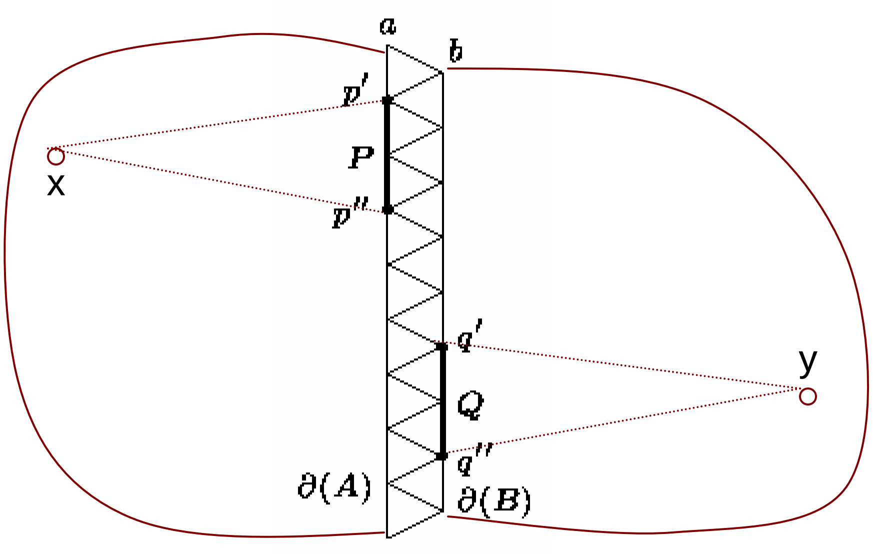


a) $d(P,Q) = d(p'', q') = d(b, q') - d(a, p'') + 1$; **b)** $d(P,Q) = d(p', q'') = d(a, p') - d(b, q'')$; **c)** $d(P,Q) = 1$.

Distances via cut



A *convex* **cut** $\{A, B\}$**, its zone** $Z(A, B)$**, its border lines** ∂A **and** ∂B **and the set** $E(A, B)$ **of edges crossed by this cut.**



- Distance formula

$$d(x, y) = d(x, P) + d(P, Q) + d(y, Q)$$

- Necessary information

$$D_x := \overset{1}{(1,} \quad \overset{2}{d(x, P),} \quad \overset{3}{d(p', a),} \quad \overset{4}{d(p'', a))}$$

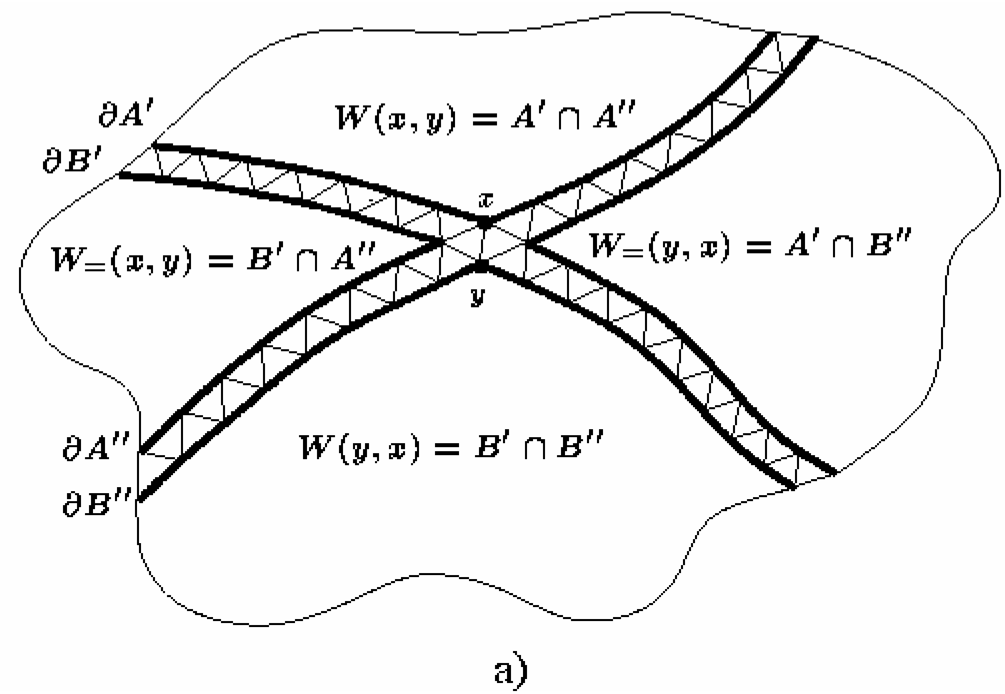
$$D_y := \overset{1}{(0,} \quad \overset{2}{d(y, Q),} \quad \overset{3}{d(q', b),} \quad \overset{4}{d(q'', b))}$$

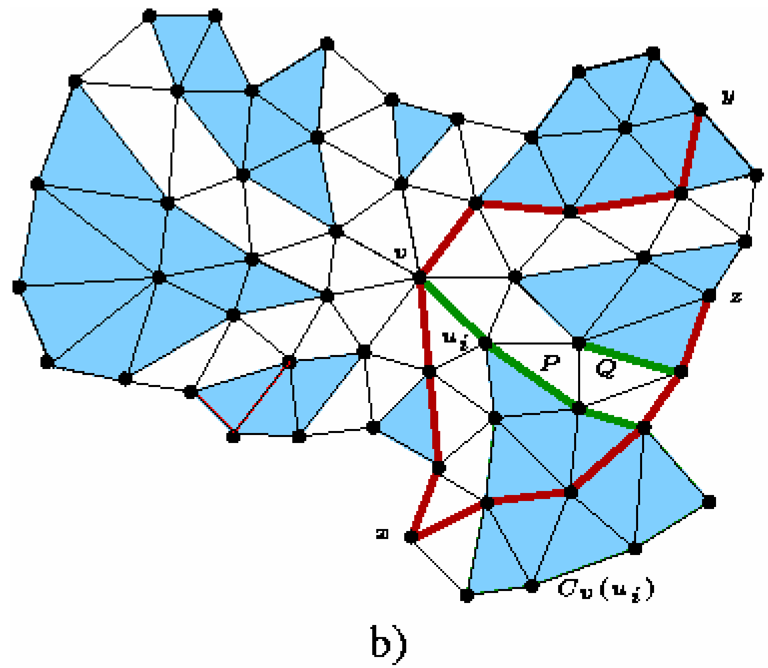
- Decoder

function distance_graphs(D_x, D_y)

if $D_x(1) = 0$ then /* rename inputs */ set
$C := D_x, D_x := D_y, D_y := C$
if $D_x(4) \leq D_y(3)$ then
return $D_x(2) + (D_y(3) - D_x(4) + 1) + D_y(2)$
else if $D_x(3) > D_y(4)$ then
return $D_x(2) + (D_x(3) - D_y(4)) + D_y(2)$
else return $D_x(2) + 1 + D_y(2)$

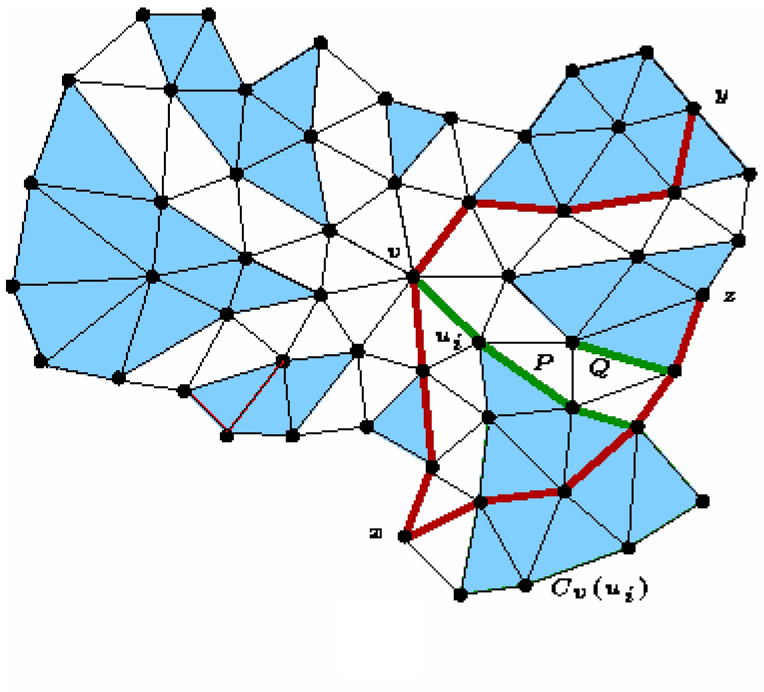
Decomposition: partition into cones

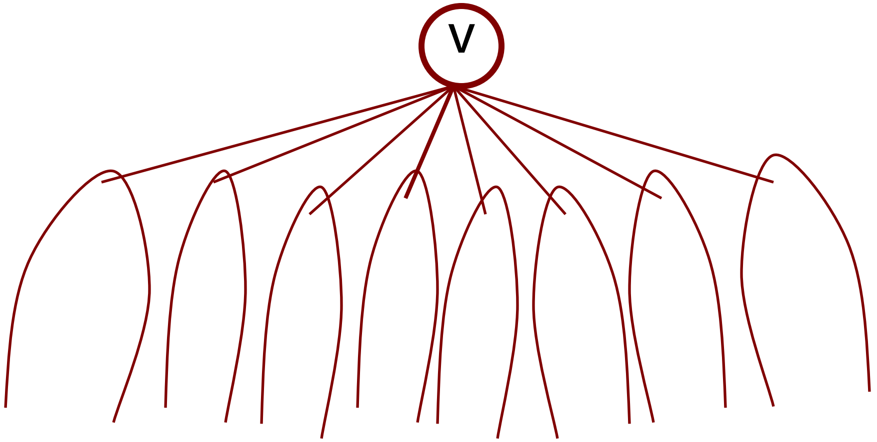




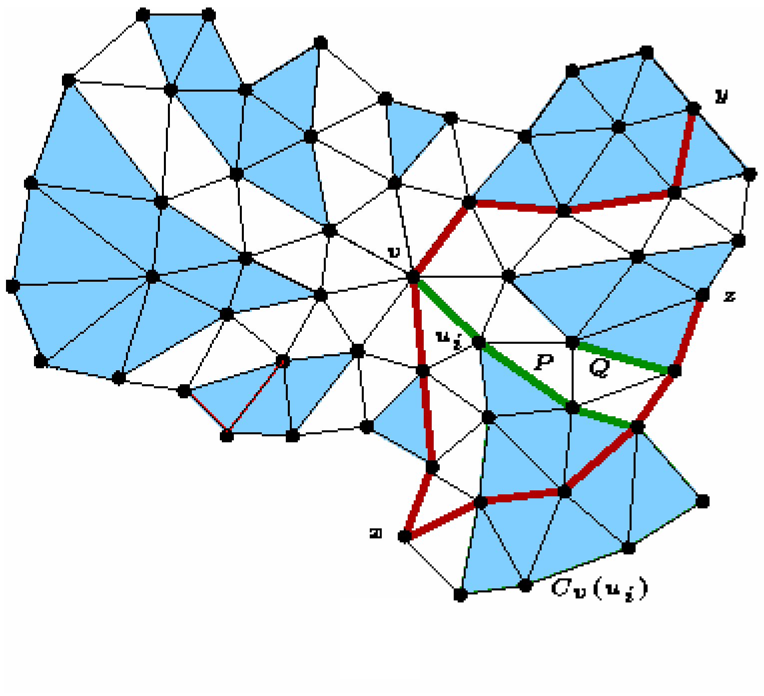
a) The pair of alternating cuts crossing the edge $e = xy$; b) The partition into cones around the vertex v. Since x and z lie in 2-neighboring cones, we have $d(x, z) = d(x, P) + d(P, Q) + d(Q, z)$. On the other hand, x and y lie in 3-neighboring cones implying $d(x, y) = d(x, v) + d(v, y)$.

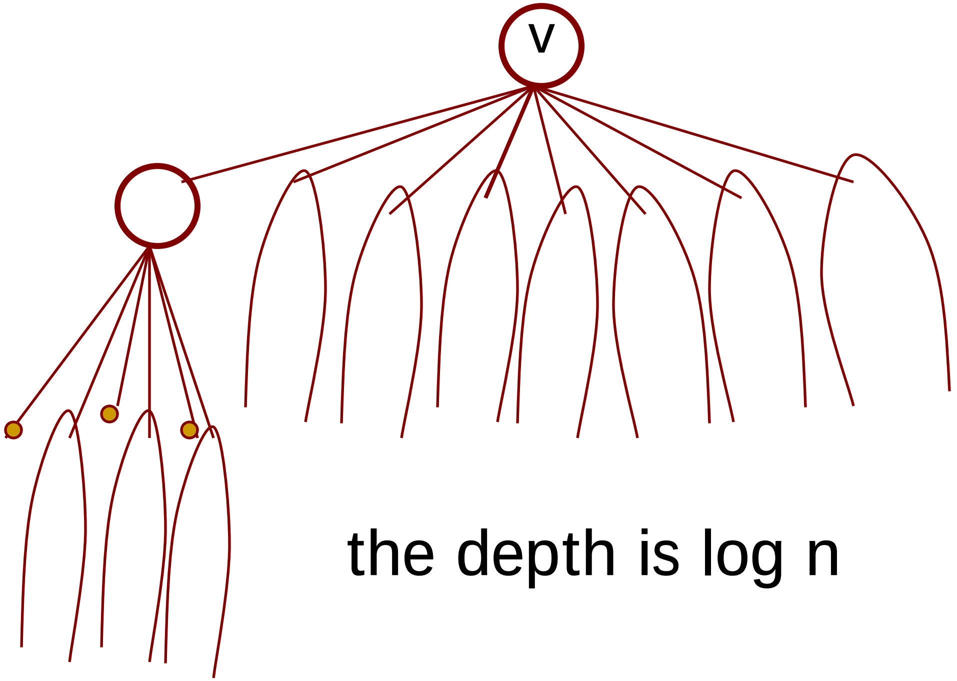
Decomposition tree





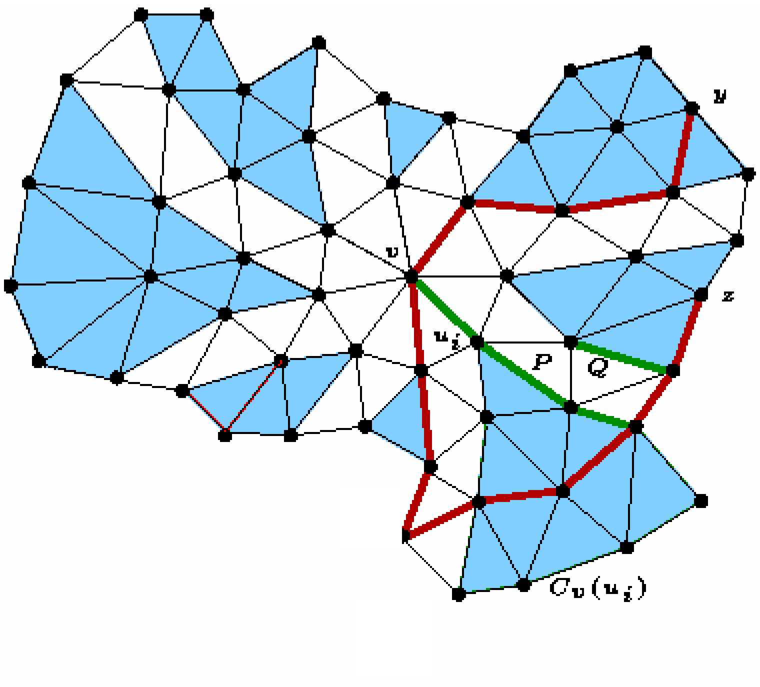
Decomposition tree

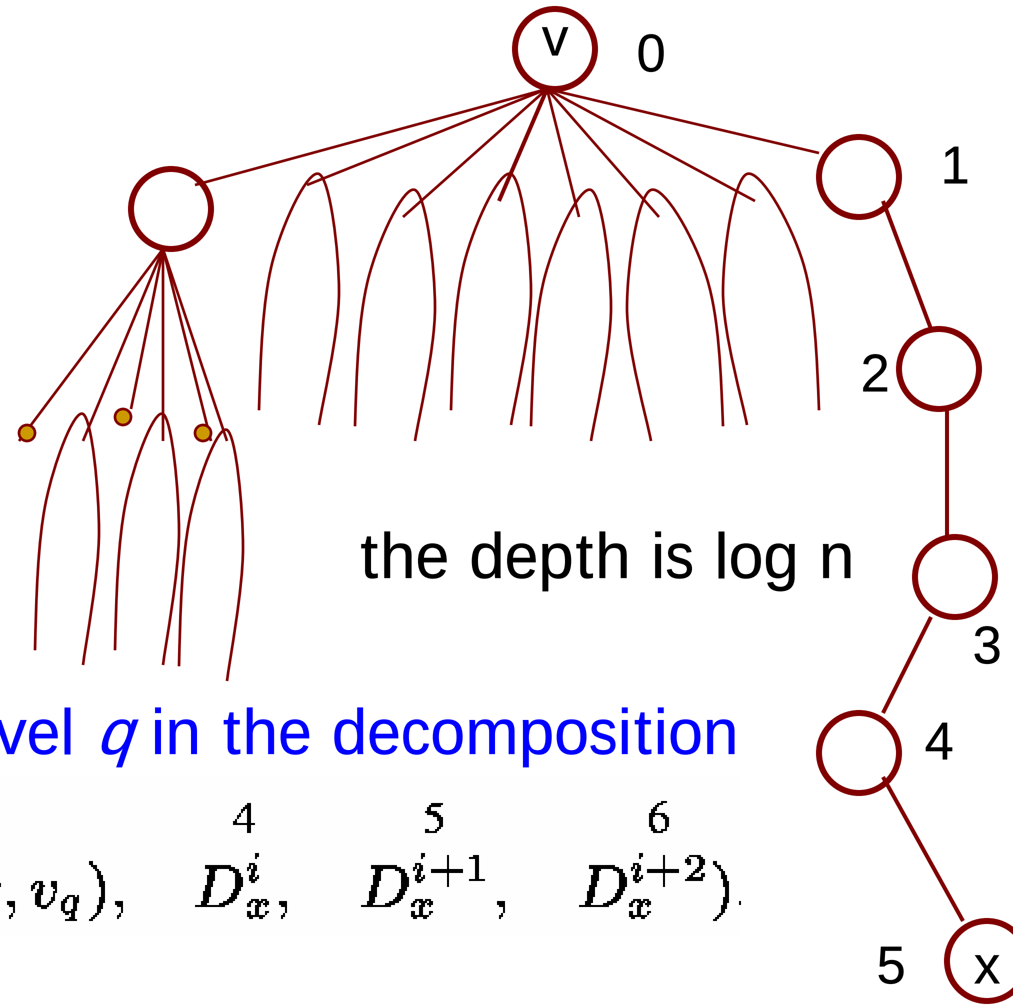




the depth is log n

The decomposition tree and the labels





- Necessary information for level *q* in the decomposition

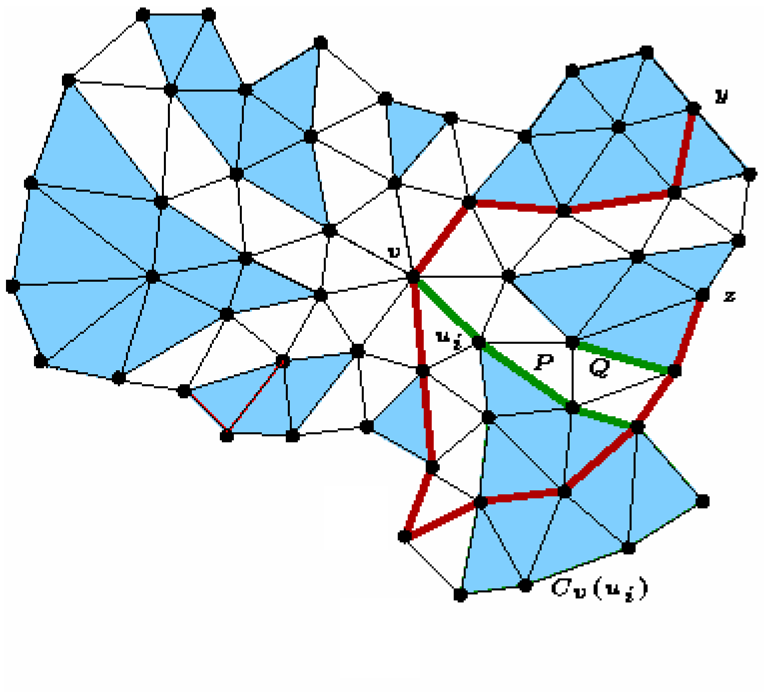
$$\tau_q^x := \overset{1}{(i,}\ \overset{2}{\delta_{G_q}(v_q),}\ \overset{3}{d_{G_q}(x, v_q),}\ \overset{4}{D_x^i,}\ \overset{5}{D_x^{i+1},}\ \overset{6}{D_x^{i+2})}$$

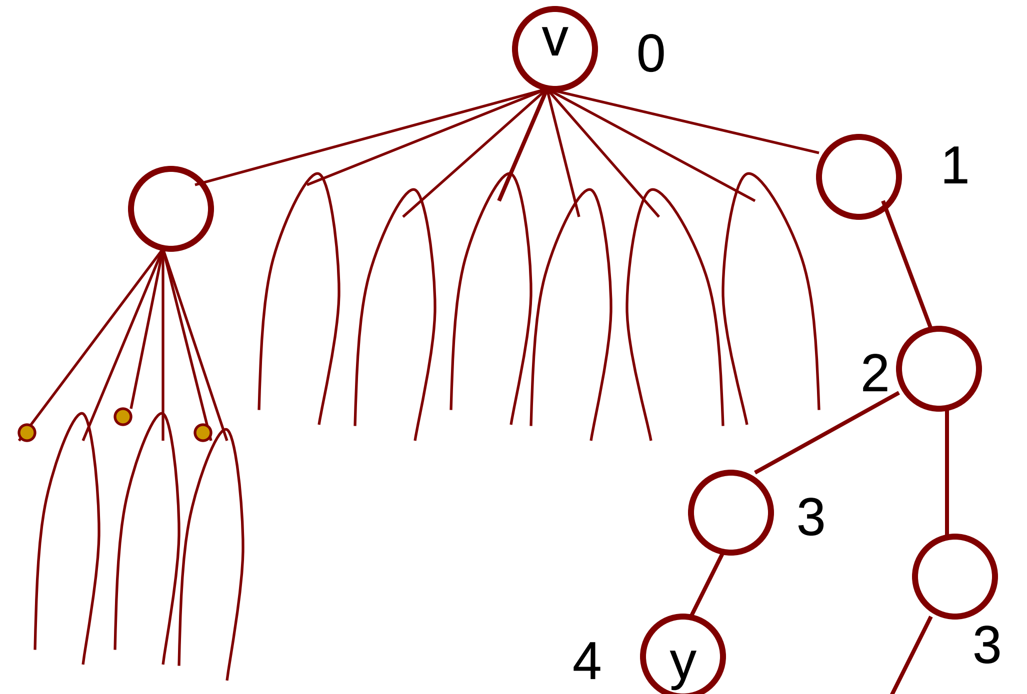
- The Labels

$$L(x) = A_x \circ \tau_0^x \circ \tau_1^x \circ \cdots \circ \tau_h^x$$

($O(\log^2 n)$ bits)

The decomposition tree and the labels





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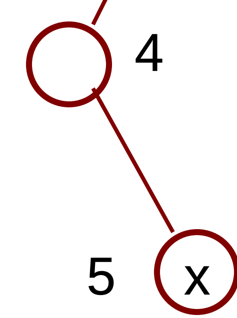


- The Labels

$$L(x) = A_x \circ \tau_0^x \circ \tau_1^x \circ \cdots \circ \tau_h^x$$

($O(\log^2 n)$ bits)

$$L(y) = A_y \circ \tau_0^y \circ \tau_1^y \circ \cdots \circ \tau_q^y$$

Distance decoder

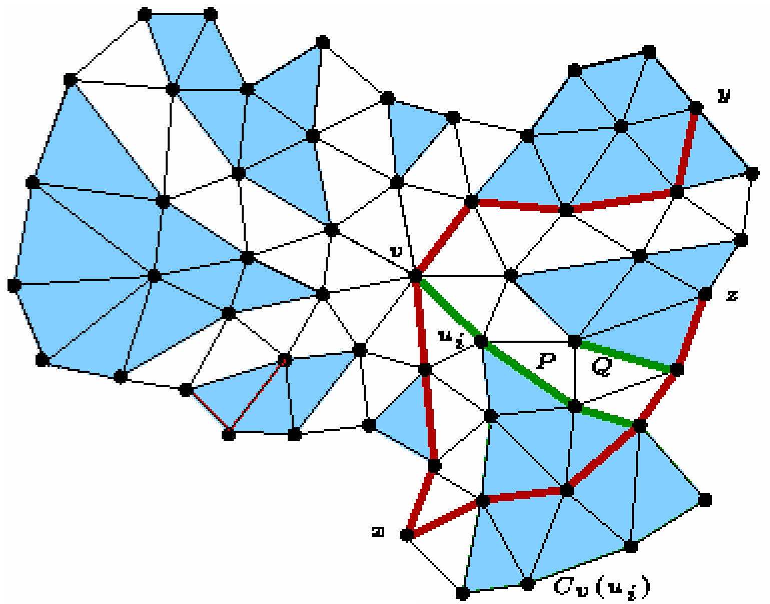
Algorithm DISTANCE_DECODER: Distance decoder for tri-graphs.

Input: two labels $L(x) = A_x \circ \tau_0^x \circ \tau_1^x \circ \cdots \circ \tau_h^x$ and $L(y) = A_y \circ \tau_0^y \circ \tau_1^y \circ \cdots \circ \tau_q^y$.

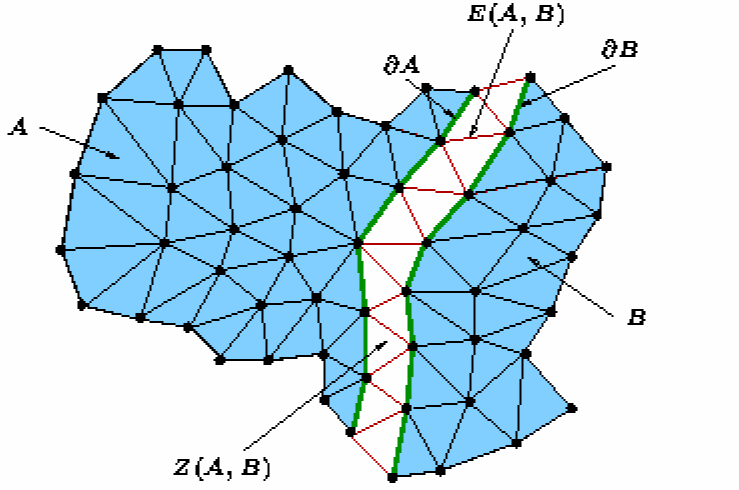
Output: $d(x, y)$, the distance between x and y in G.

Method:

use A_x and A_y to find the depth l in $T(G)$ of
  the nearest common ancestor of $S(x)$ and $S(y)$;
extract from $L(x)$ and $L(y)$ the tuples τ_l^x and τ_l^y;
if $\tau_l^x(1) = \tau_l^x(2)$ then output $\tau_l^y(3)$ and stop; /* $x = v_q$ */
if $\tau_l^y(1) = \tau_l^y(2)$ then output $\tau_l^x(3)$ and stop; /* $y = v_q$ */
/* if the cones are 1-neighboring */
if $\tau_l^x(1) = \tau_l^y(1) - 1$ or $\tau_l^y(1) = 0$ and $\tau_l^x(1) = \tau_l^x(2) - 1$
  then output distance_graphs($\tau_l^x(5), \tau_l^y(4)$) and stop;
if $\tau_l^y(1) = \tau_l^x(1) - 1$ or $\tau_l^x(1) = 0$ and $\tau_l^y(1) = \tau_l^x(2) - 1$
  then output distance_graphs($\tau_l^y(5), \tau_l^x(4)$) and stop;
/* if the cones are 2-neighboring */
if ($\tau_l^x(1) = \tau_l^y(1) - 2$ or $\tau_l^y(1) = 0$ and $\tau_l^x(1) = \tau_l^x(2) - 2$
  or $\tau_l^y(1) = 1$ and $\tau_l^x(1) = \tau_l^x(2) - 1$) then output
  distance_graphs($\tau_l^x(6), \tau_l^y(4)$) and stop;
if ($\tau_l^y(1) = \tau_l^x(1) - 2$ or $\tau_l^x(1) = 0$ and $\tau_l^y(1) = \tau_l^x(2) - 2$
  or $\tau_l^x(1) = 1$ and $\tau_l^y(1) = \tau_l^x(2) - 1$) then output
  distance_graphs($\tau_l^y(6), \tau_l^x(4)$) and stop;
else output $\tau_l^x(3) + \tau_l^y(3)$.



Routing via cut



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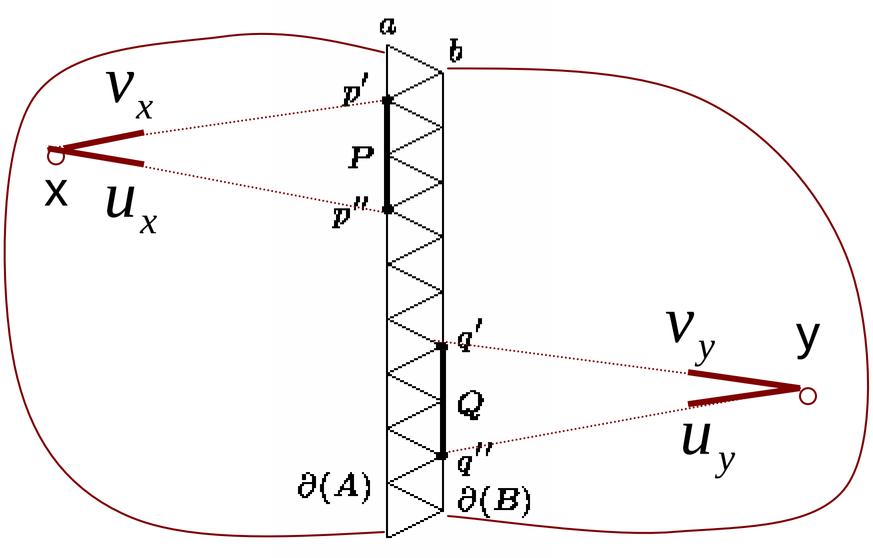


- Necessary routing information

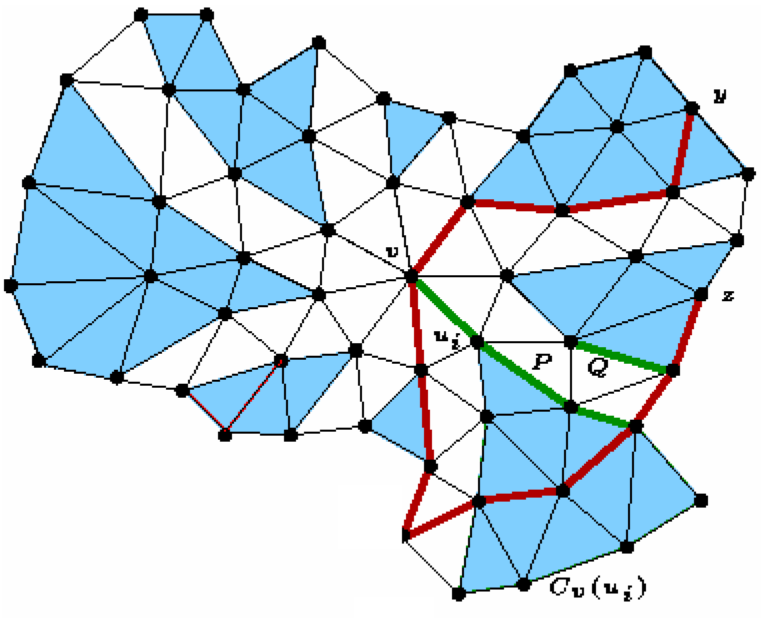
$$R_x := (\overset{1}{D_x}, \overset{2}{D_{v_x}}, \overset{3}{D_{u_x}}, \overset{4}{port(x, v_x)}, \overset{5}{port(x, u_x)}, \overset{6}{help(v_x)}, \overset{7}{help(u_x)})$$

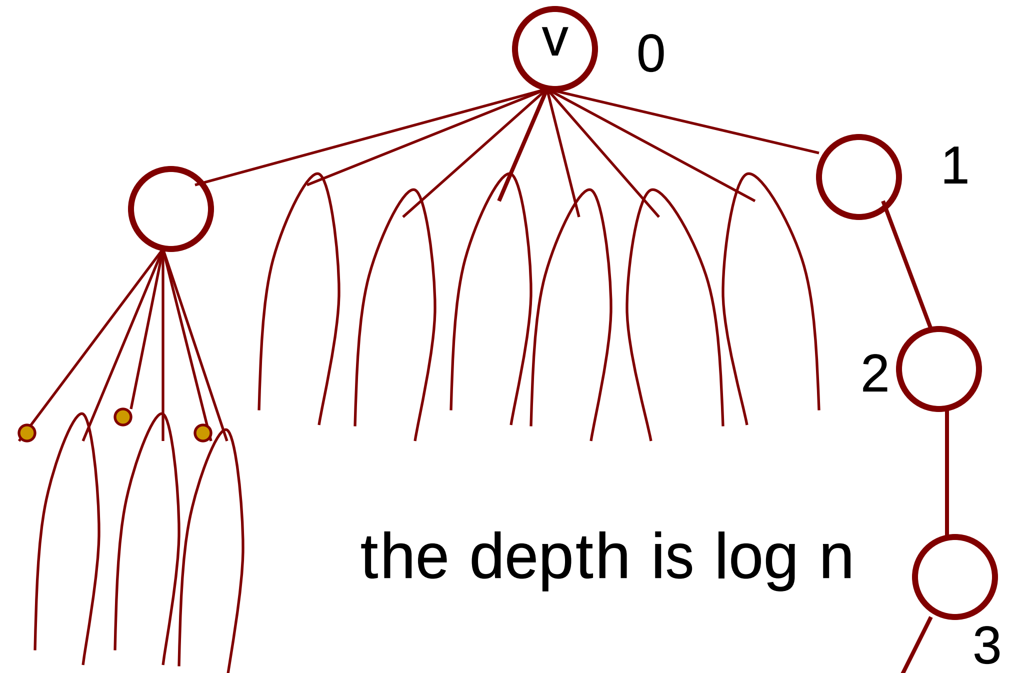
$$R_y := (\overset{1}{D_y}, \overset{2}{D_{v_y}}, \overset{3}{D_{u_y}}, \overset{4}{port(y, v_y)}, \overset{5}{port(y, u_y)}, \overset{6}{help(v_y)}, \overset{7}{help(u_y)})$$

- Decoder

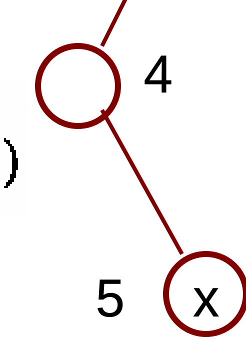
function routing_decision(R_x, R_y)
if $R_x(6) \neq 1$ then
  if distance_graphs($R_x(1), R_y(1)$) = distance_graphs($R_x(2), R_y(1)$) + 1 then output $R_x(4)$ else output $R_x(5)$
else if $R_x(7) \neq 1$ then
    if distance_graphs($R_x(1), R_y(1)$) = distance_graphs ($R_x(3), R_y(1)$) + 1 then output $R_x(5)$ else output $R_x(4)$
  else extract $D_y(4)$ from $R_y(1)$
    extract $D_{v_x}(3)$ from $R_x(2)$
    extract $D_{u_x}(3)$ from $R_x(3)$
    if $D_{u_x}(3) \leq D_{v_x}(3)$ then
      if $D_y(4) \leq D_{u_x}(3)$ then output $R_x(5)$
      else output $R_x(4)$
    else if $D_y(4) \leq D_{v_x}(3)$ then output $R_x(4)$
      else output $R_x(5)$

Routing labels





- Necessary information for level *q* in the decomposition



$$\mu_q^x := \overset{1}{(i,}\ \ \overset{2}{\delta_{G_q}(v_q)},\ \ \overset{3}{port_{G_q}(x, v_q)},\ \ \overset{4}{R_x^i},\ \ \overset{5}{R_x^{i+1}},\ \ \overset{6}{R_x^{i+2}},\ \ \overset{7}{port_{G_q}(v_q, x))}$$

- The Labels

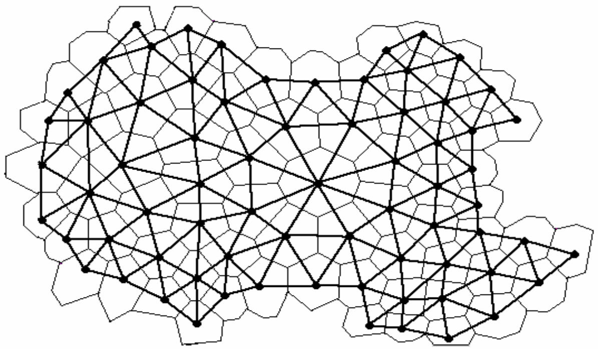
$$L(x) = A_x \circ \mu_0^x \circ \mu_1^x \circ \cdots \circ \mu_h^x$$

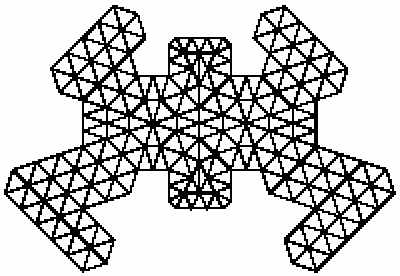
$$L(y) = A_y \circ \mu_0^y \circ \mu_1^y \circ \cdots \circ \mu_q^y$$

($O(\log^2 n)$ bits)

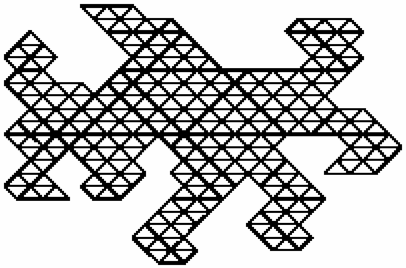
Main Result and Forthcomings

Thm: The family of n-vertex trigraphs enjoy distance and routing labeling schemes with $O(\log^2 n)$ -bit labels and constant time distance decoder and routing decision.

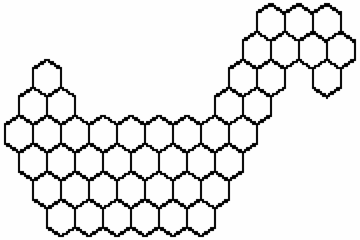




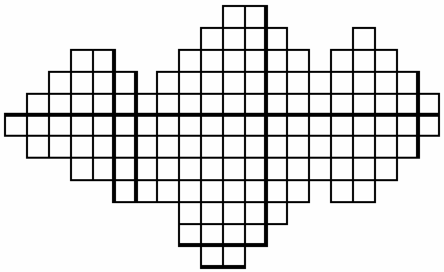
trigraph



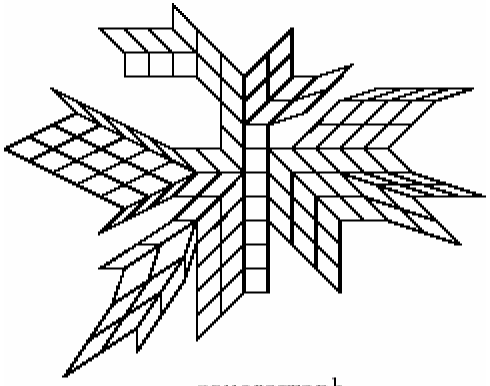
triangular system



hexagonal system



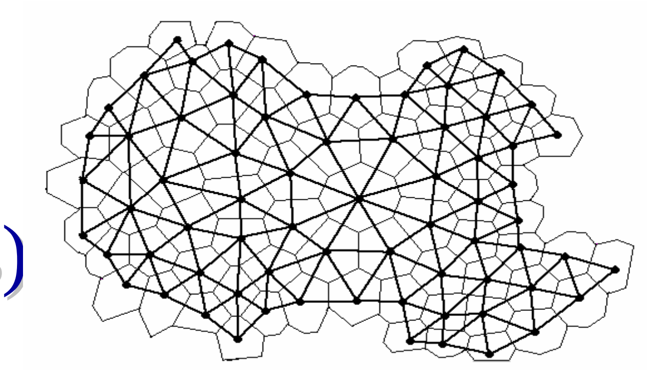
square system



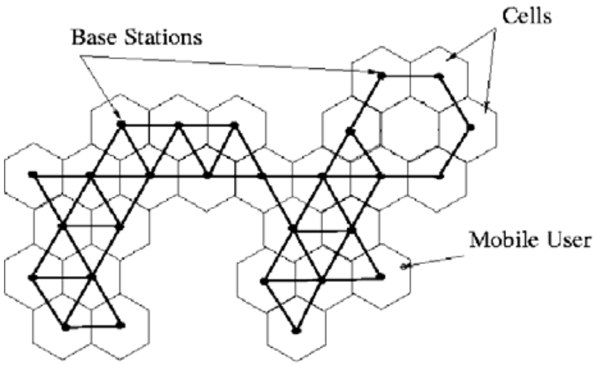
squaregraph

Open Problems

- Channel Assignment Problem in Irregular Cellular Networks ($L(p_1,\ldots,p_k)-$coloring in Trigraphs)



- BSs Placement Problem (resulting in a Trigraph)
  - Service area with demands, obstacles
  - Deploy min. # of BSs to cover area



- Not-Simply Connected Regular Cellular Networks (with holes)

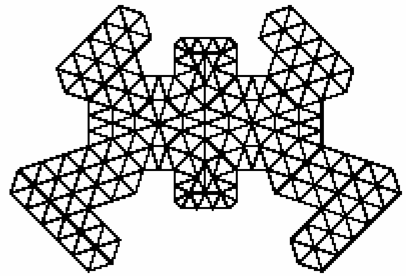
Thank you

Other Results

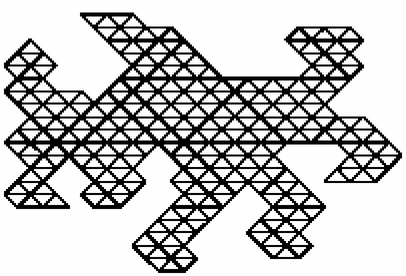
Thm: The families of *n*-node (6,3)-,(4,4)-,(3,6)-planar graphs enjoy distance and routing labeling schemes with $O(\log^2 n)$ -bit labels and constant time distance decoder and routing decision.

(p,q)-planar graphs:

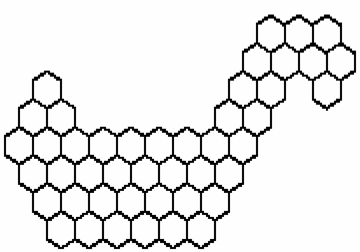
- inner faces of length at least p
- inner vertices of degree at least q



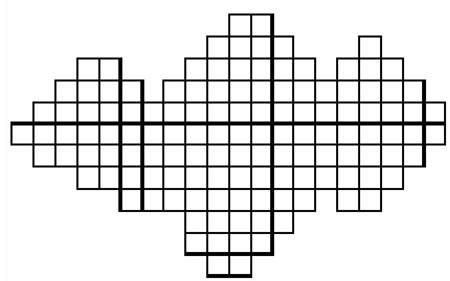
trigraph



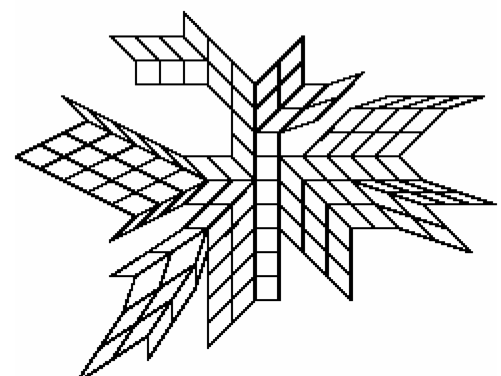
triangular system



hexagonal system



square system



squaregraph