Art Gallery Problem

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R-W Problem Statement

- Art Gallery needs to be guarded.
- Objective: Appoint minimum number of guards to efficiently guard the entire building.

Generic Solution

- Usually the buildings are rectangular in shape.
- Therefore it is easy to find the minimum number of guards.
- Consider the buildings with different shapes for which finding a general answer is difficult.

Constructing a graph

- Considering the object in the R-W problem is a building, the graph will be a simple polygon joining the vertices of the building.
- n - Number of Vertices
- e - Number of Edges

Types of Problems

- There are two types of problems that arise from this example based on what visibility is important:
  - Considering Museum Problem where the valuables are placed inside of it, Interior Visibility is preferred.
  - Consider a Fort where is it important to be vigilant and watchful, Exterior Visibility is preferred.

Interior Visibility

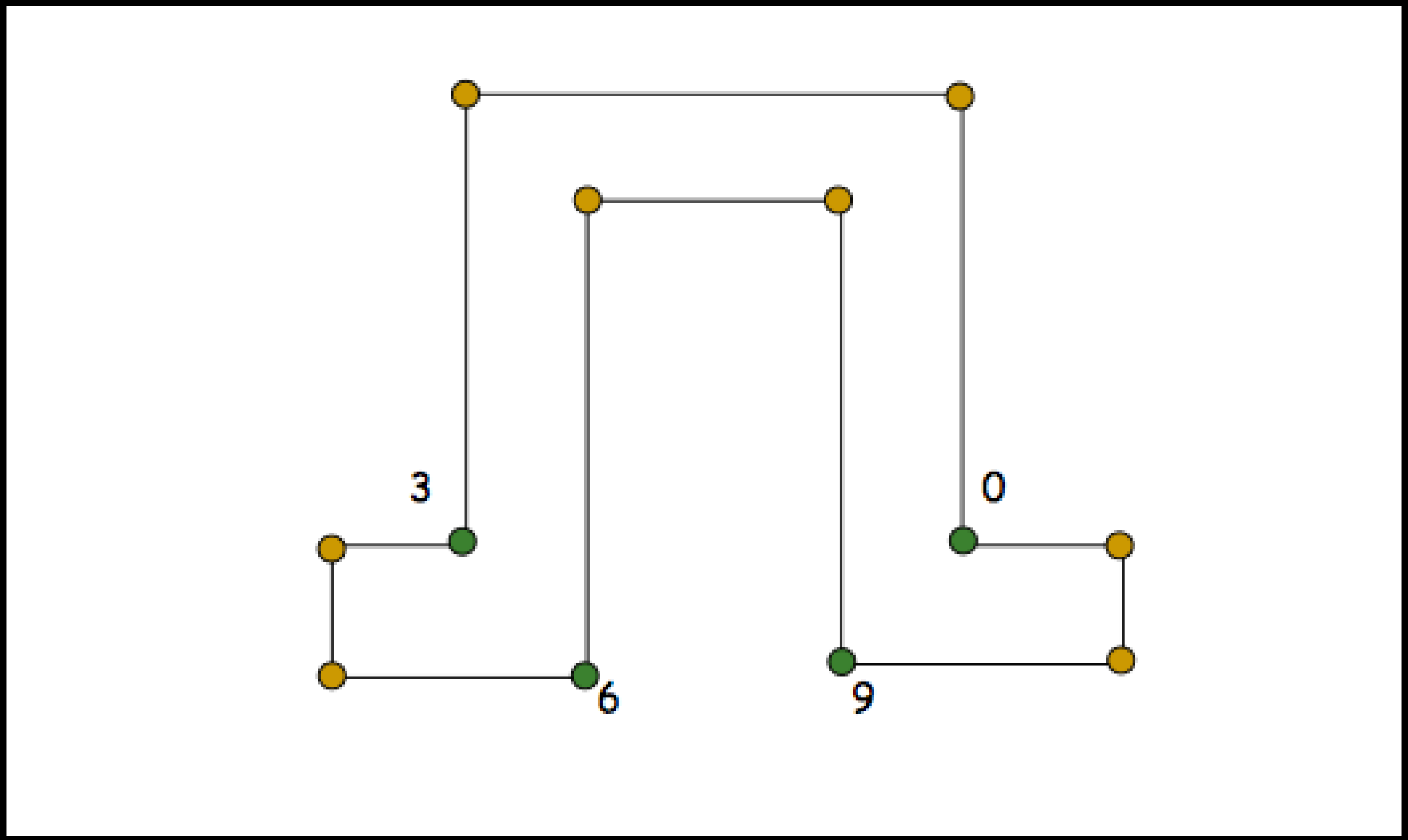
- Definition of Problem:
  - P - Simple Polygon
  - x, y are points in P
  - x "covers" y if the edge xy also belongs to P
  - G(P) - minimum number of point of P such that for any y in P, some x=x1, x2, ...xk covers y
  - g(n) be the max(G(P)) over all n-vertex polygons

Fisk's Clever Argument

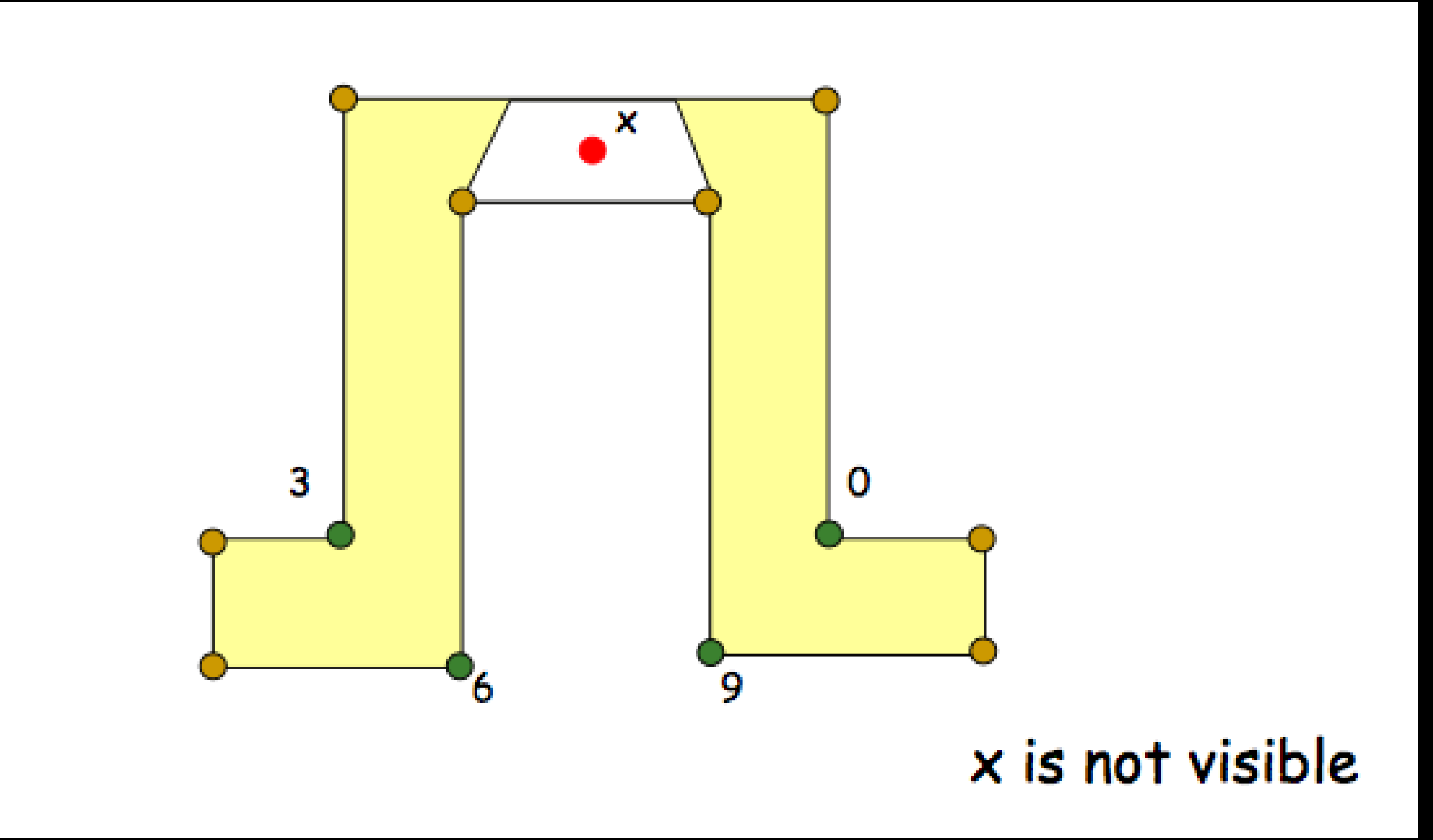
- Consider a simple polygon bounded by straight line segments that do not cross each other.
- Number of guards depend on number of vertices in the polygon.
- According to Fisk's Clever Argument, you will not need more than n/3 guards to guard the building where n is the number of vertices.
- This is equivalent to saying that a guard is needed for ever third vertex.

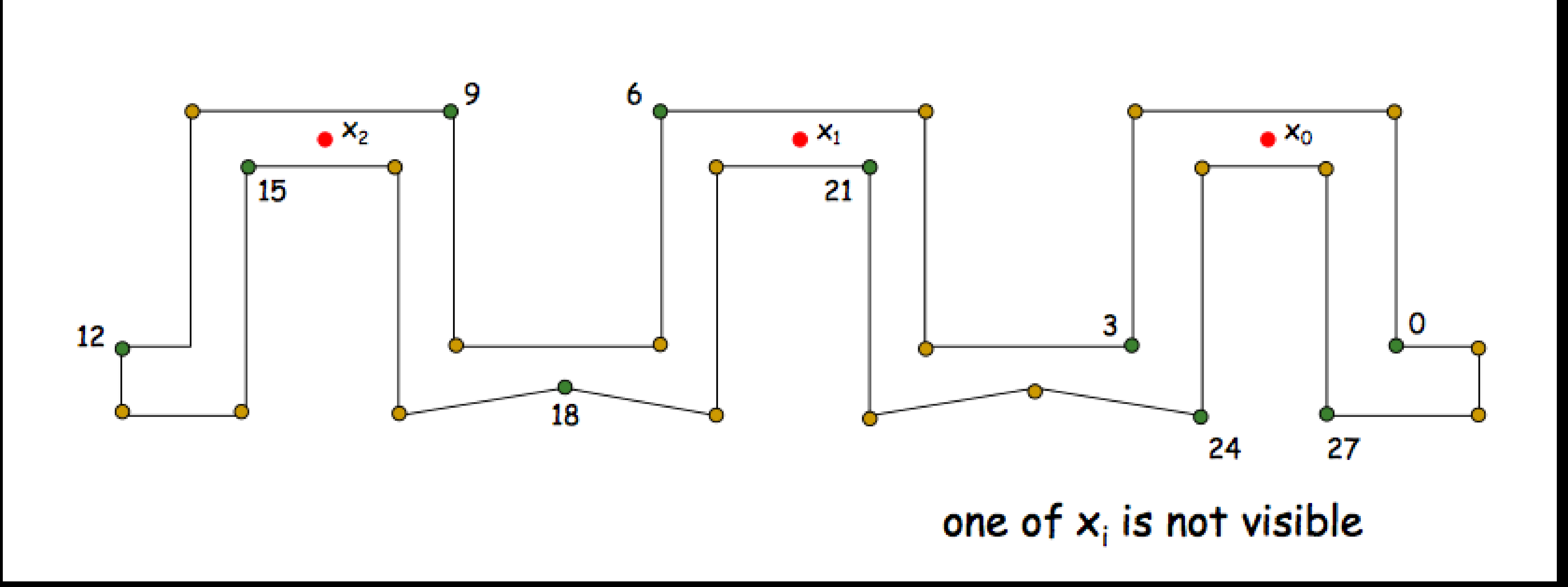
Is the Fisk's Proof always true?

- The answer is NO!



- The following examples will prove the same.


x
3
0
6
9
x is not visible


9
6
x2
x1
x0
15
21
12
3
0
18
24
27
one of xi is not visible

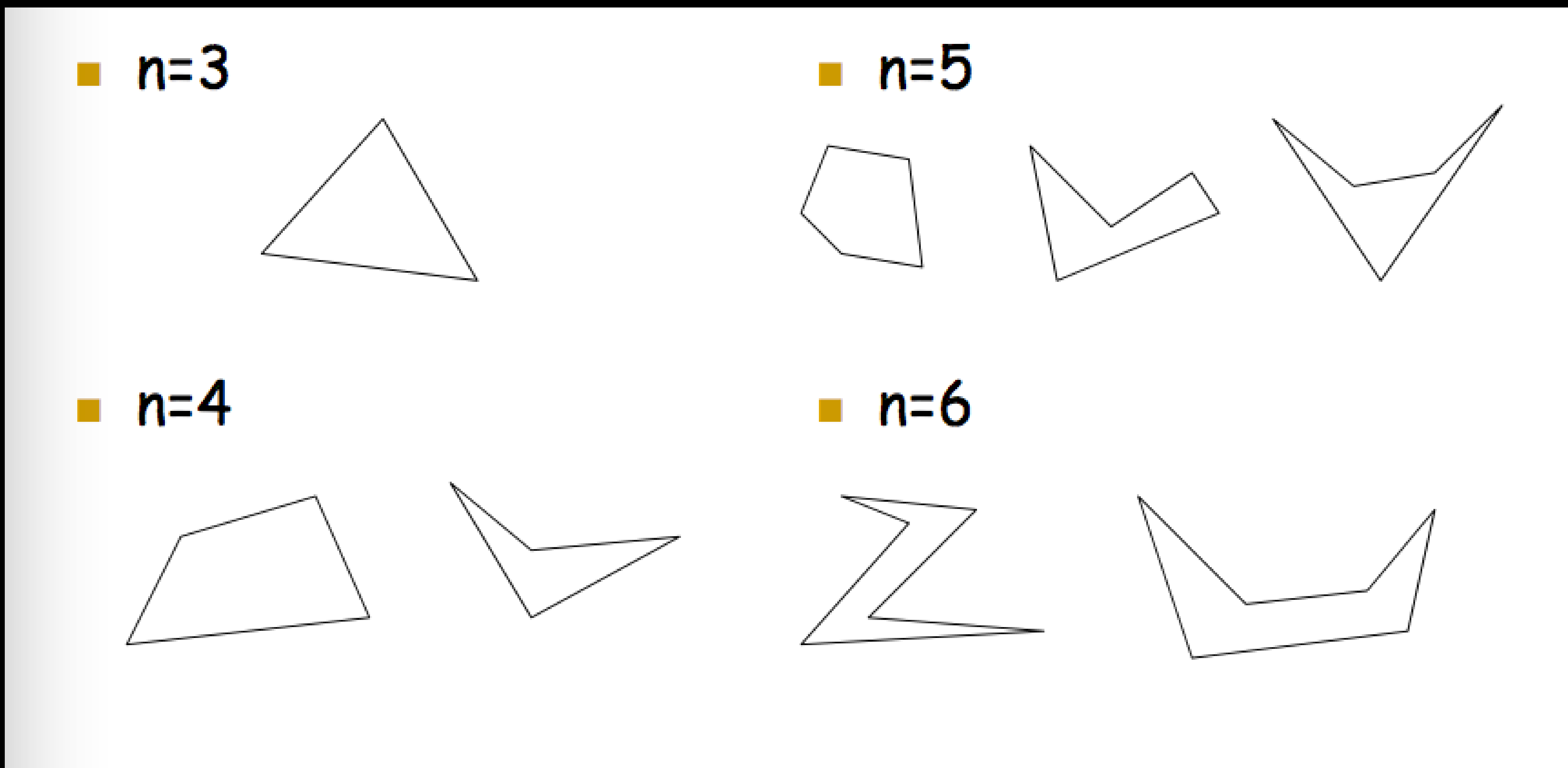
Constraints

- Unlike Exterior Visibility the placements of guards is efficient if they can see all the edges.
- They must also be able to see the interior.
- Decision to choose the vertices or edges to place the guards or a combination of both.

Fisk's Proof

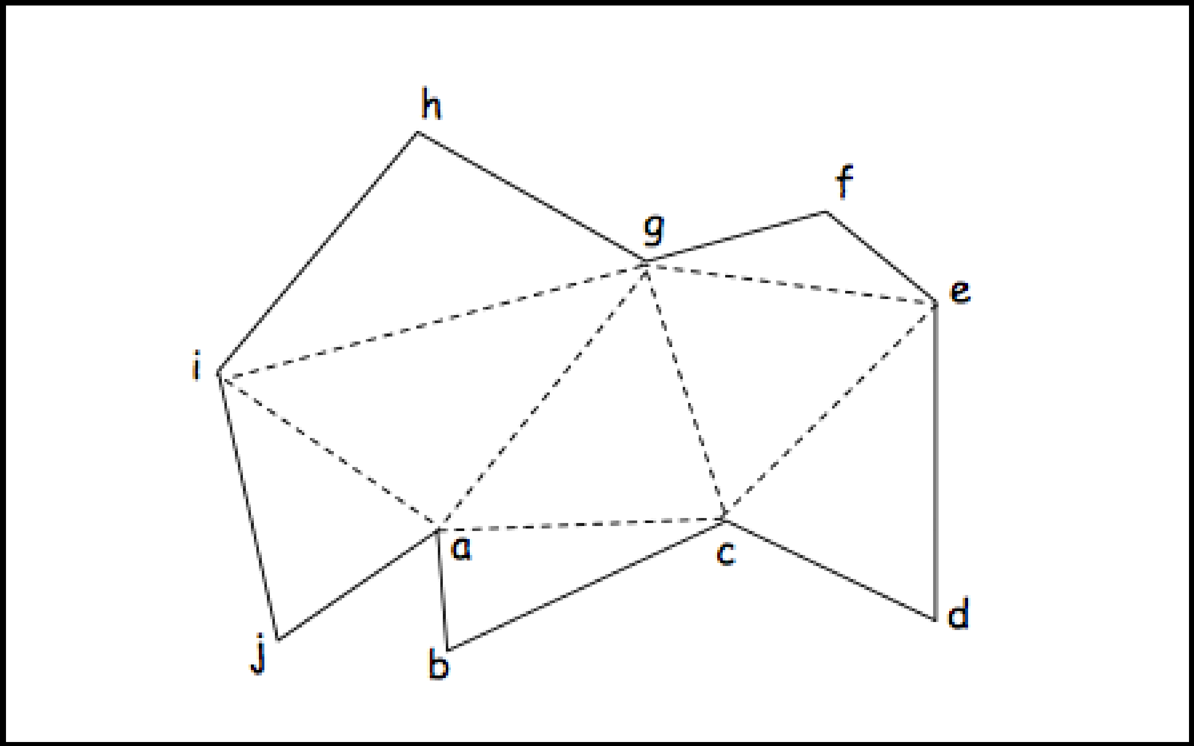
- Necessity:
  - g(n) is always greater than or equal to "floor(n/3)"
- Sufficiency:
  - g(n) is always less than or equal a to "floor(n/3)"
- To satisfy both it is said that g(n) = floor(n/3)

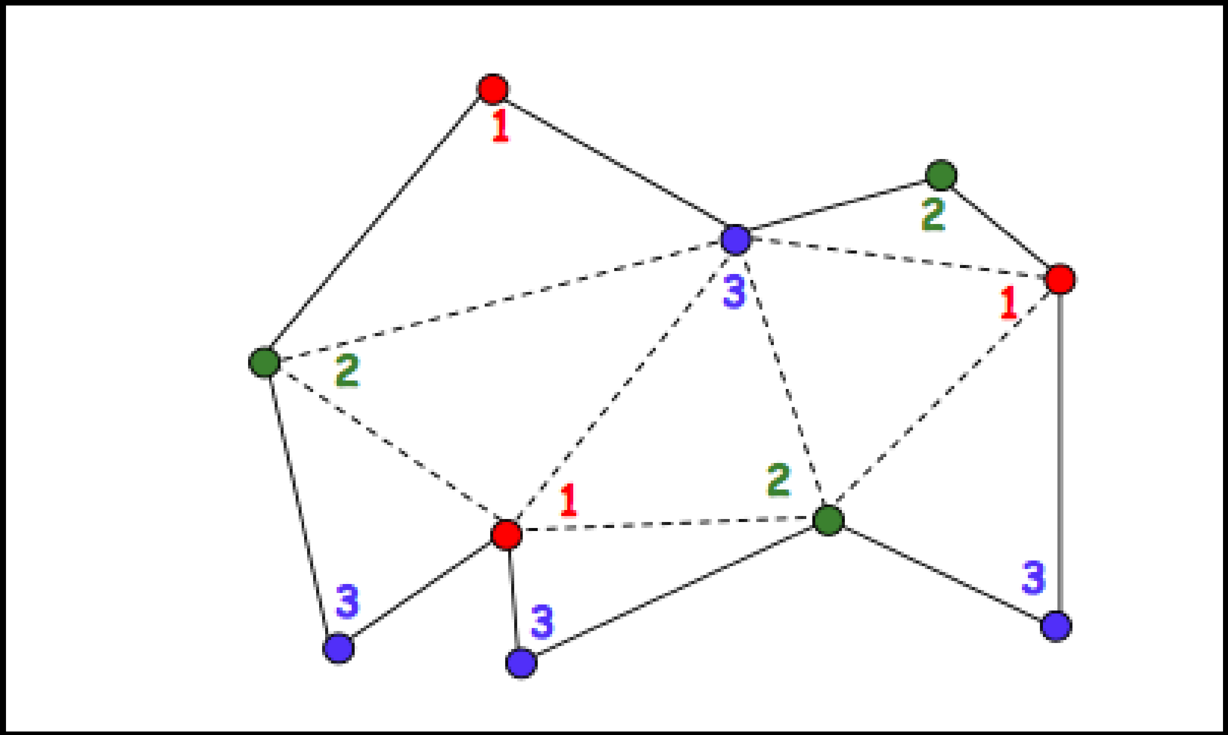
Proving Necessity



Proving Sufficiency

- Triangulation Theorem:
  - Triangulate (join the vertices to form triangles without overlapping) the polygon
  - If there are n vertices, the number of triangles formed are n-2
  - Color the vertices of a graph by ensuring that there are different colors in the graph.
  - Considering Fisk's algorithm, 1 guard can replace all the points in the triangle.


h
f
g
e
i
a
c
d
j
b


1
2
3
1
2
1
2
3
3
3

Contd..

- It is observed that every color is used no more than floor(1/3) times.
- As there are three colors, lets say a, b and c represent the number of nodes of each color.
- $a \leq b \leq c$
- $n = a+b+c$
- Therefore $a \leq \text{floor}(n/3)$.
- Assumption made here is that every triangle is treated as an individual polygon where a covers the other two nodes.

Thank You