DEGREE SEQUENCES AND SPLIT GRAPHS

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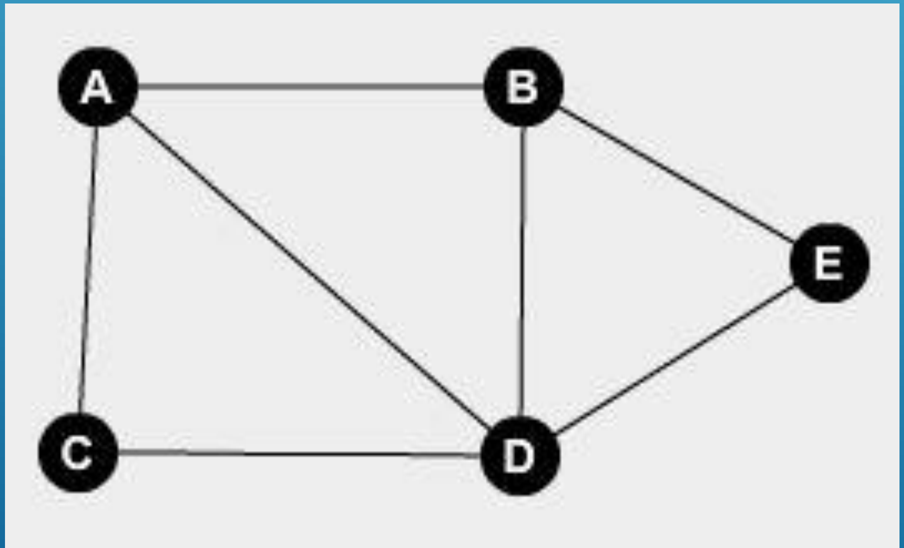
DEGREE SEQUENCES

A sequence of integers d1,...,dn is said to be a *degree sequence* (or *graphic* sequence) if there exists a graph in which vertex i is of degree di It is often required to be *non-increasing*, i.e. that d1≥...≥dn.



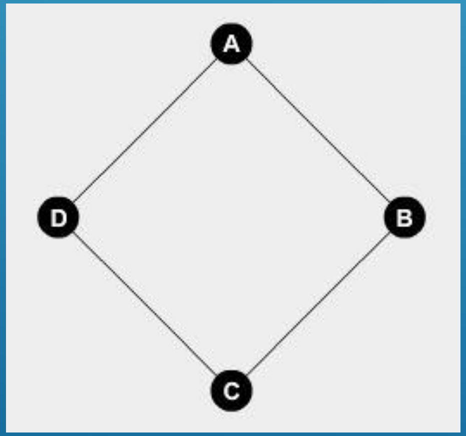
deg. seq.=[4,3,3,2,2]

DEGREE SEQUENCES

- Finding a graph with given degree sequence is known as ***graph realization problem***.

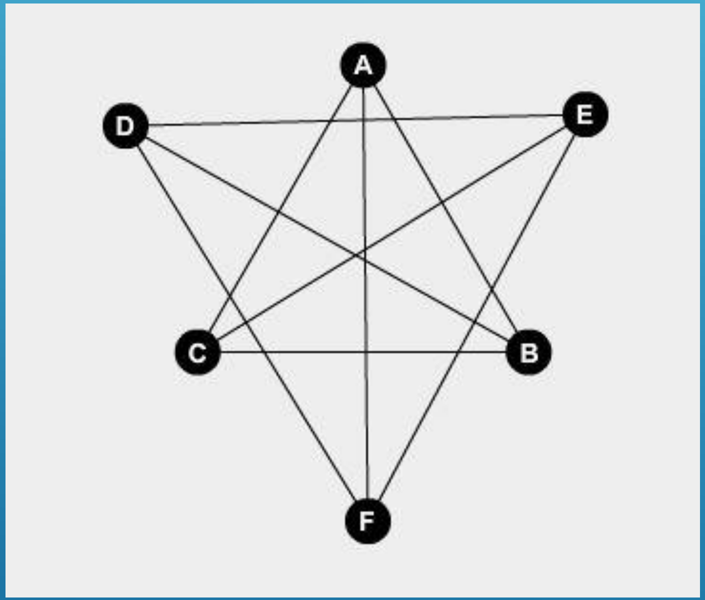
- Example:

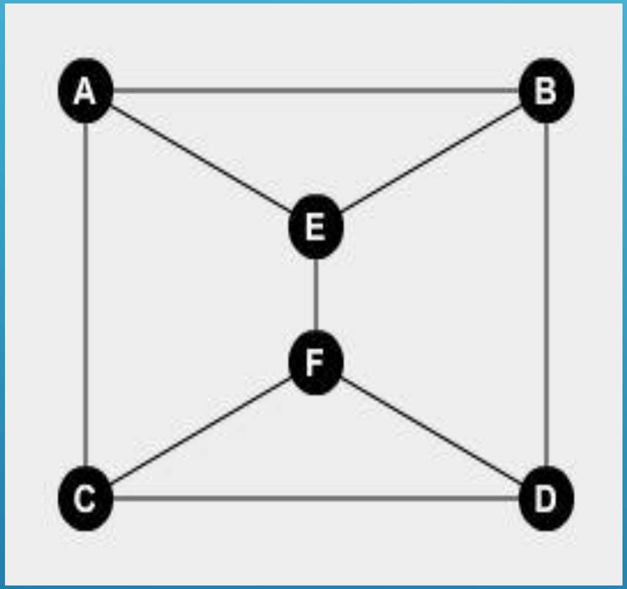
  The sequence [2,2,2,2] corresponds to the cordless 4-cycle C4



- The degree sequence does not uniquely identify a graph

this sequence [3,3,3,3,3,3] corresponds to both:





CHARACTERIZATIONS

1- Handshaking lemma

- The **degree sum formula** states that, given a graph G=(V,E):

$$\sum_{v \in V} \deg(v) = 2|E|$$

- The formula implies that in any graph.

To **prove** that in any group of people the number of people who have shaken hands with an odd number of other people from the group is even.

CHARACTERIZATIONS

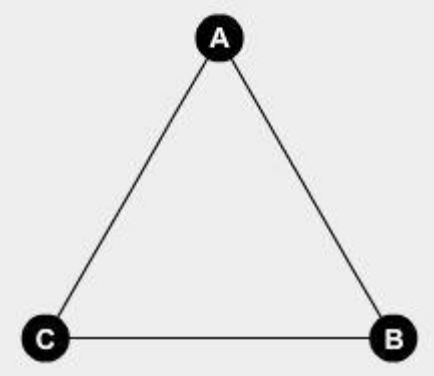
2- in a degree sequence of length ***n*** no integer can be larger than ***n-1.***

n= 8

deg. Seq.=[7,6,5,5,5,4,4,2]

3- The sum of these integer numbers is at most ***n(n-1)***

4- If all the vertices with degree ***n-1***, the corresponding graph of this sequence is a clique.



n=3

deg. seq.=[2,2,2]

GRAPHIC DEGREE SEQUENCE RECOGNITION PROBLEM

★ An integer sequence is not necessarily a degree sequence (graphic degree sequence), such as [1,1,1] and [4,4,2,1,1].

Theorem (Havel [1955], Hakimi [1962]). A sequence A of integers $n-1 \geq d_1 \geq d_2 \geq \ldots \geq d_n \geq 0$ is graphic if and only if the modified sequence :

$$\Delta' = [d_2 - 1, d_3 - 1, \ldots, d_{d_1+1} - 1, d_{d_1+2}, \ldots, d_n]$$

(sorted into decreasing order) is graphic.

7,6,5,5,5,4,4,2	<----- Order
5,4,4,4,3,3,1	<----- remove 7. Subtract 1 to 7 below
3,3,3,2,2,1	<----- remove 5. Subtract 1 to 5 below
2,2,1,2,1	<----- remove 3. Subtract 1 to 3 below
2,2,2,1,1	<----- Order
1,1,1,1	<----- remove 2. Subtract 1 to 2 below
0,1,1	<----- remove 1. Subtract 1 to 1 below
1,1,0	<----- Order
0,0	<----- remove 1. Subtract 1 to 1 below

THE SEQUENCE IS GRAPHIC

GRAPHIC DEGREE SEQUENCE RECOGNITION PROBLEM

Theorem (Erdos and Gallai [I960]). A sequence of integers $n-1 \geq d1 \geq d2 \geq \ldots \geq dn \geq 0$ *is* graphic if and only if:

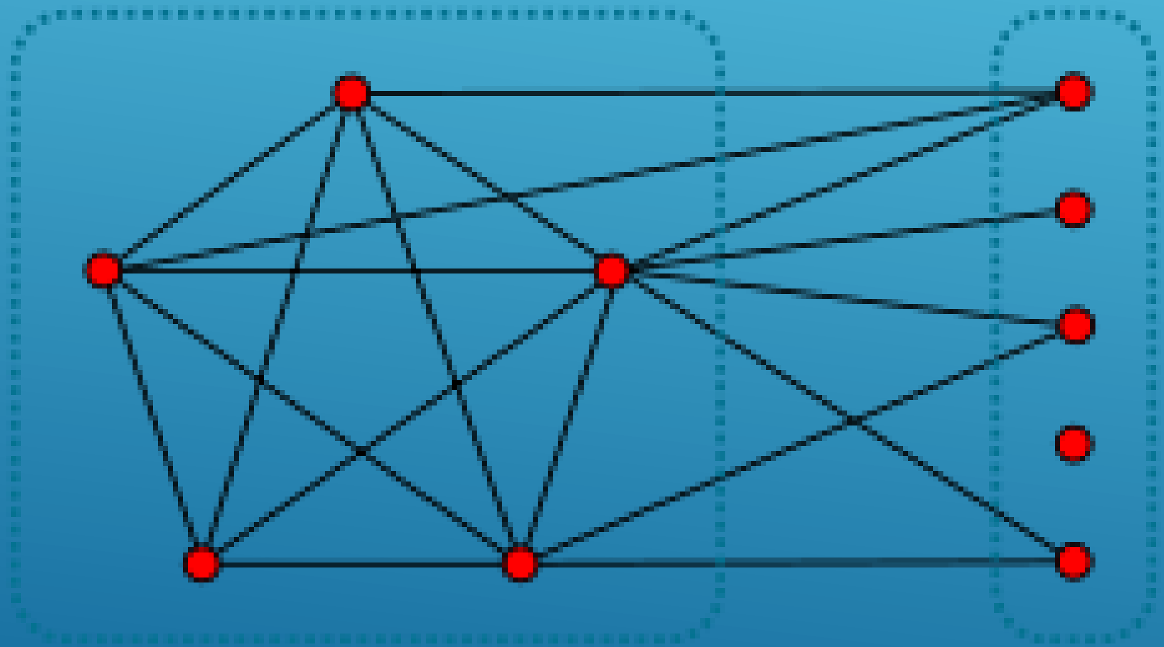
$$\text{(i)} \quad \sum_{i=1}^{n} d_i \text{ is even, } \quad \text{and}$$

$$\text{(ii)} \quad \sum_{i=1}^{r} d_i \leq r(r-1) + \sum_{i=r+1}^{n} \min\{r, d_i\},$$

for $r = 1, 2, \ldots, n-1$.

SPLIT GRAPHS

a **split graph** is a graph in which the vertices can be partitioned into a clique and an independent set.



GENERAL CHARACTERIZATIONS

- An undirected graph G is a split graph if and only if its complement G is a split graph.
- G contains no induced subgraph isomorphic toC4, or C5.
- G is chordal graph

SPLIT GRAPH CHARACTERIZATIONS

Theorem . Let G be a split graph whose vertices have been partitioned into a stable set *S* and a complete set *K*. Exactly one of the following conditions holds:

1- S= α(G) and K= ω(G)

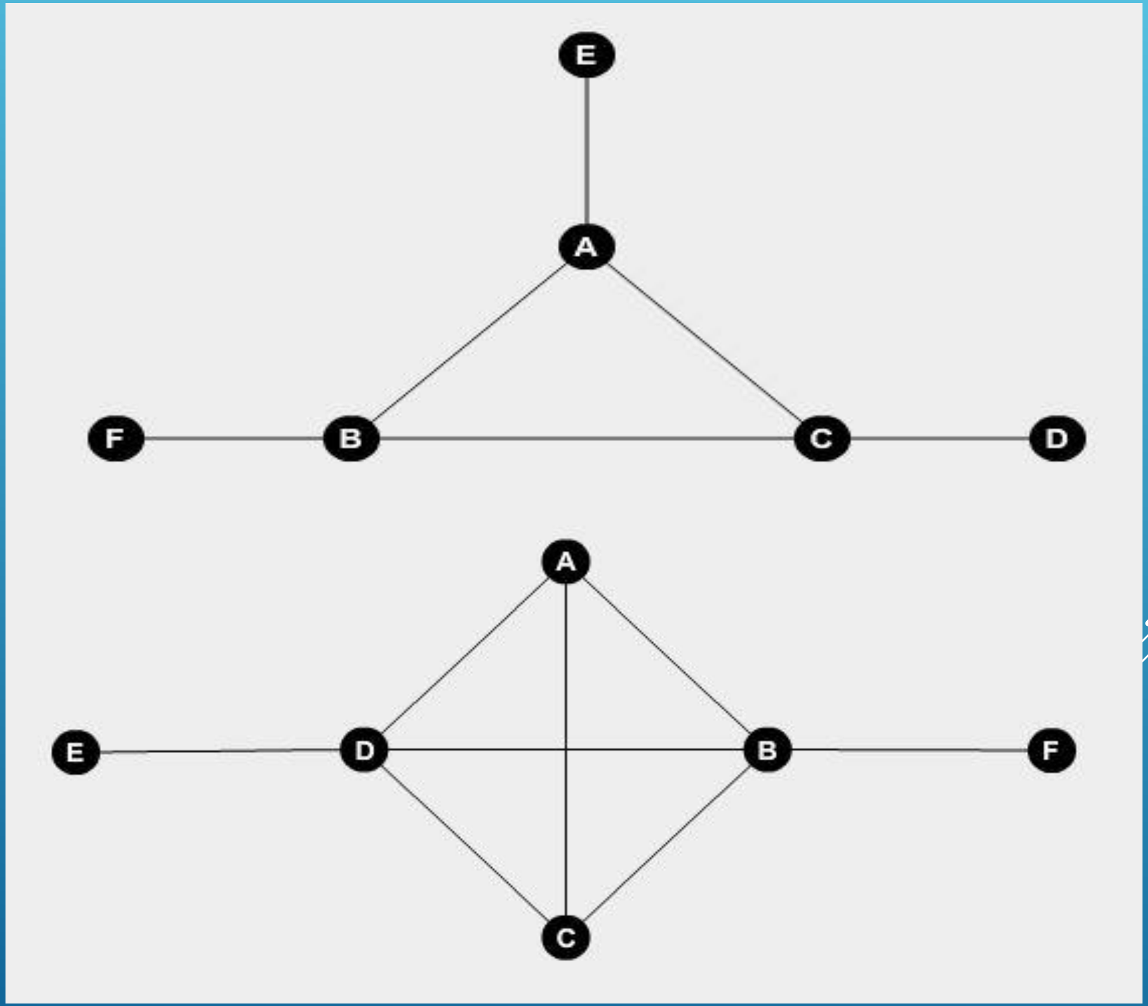
S+K is unique

2- S= α(G) and K= ω(G)-1

Exists x in S, K+{x} is complete

3- S= α(G)-1 and K= ω(G)

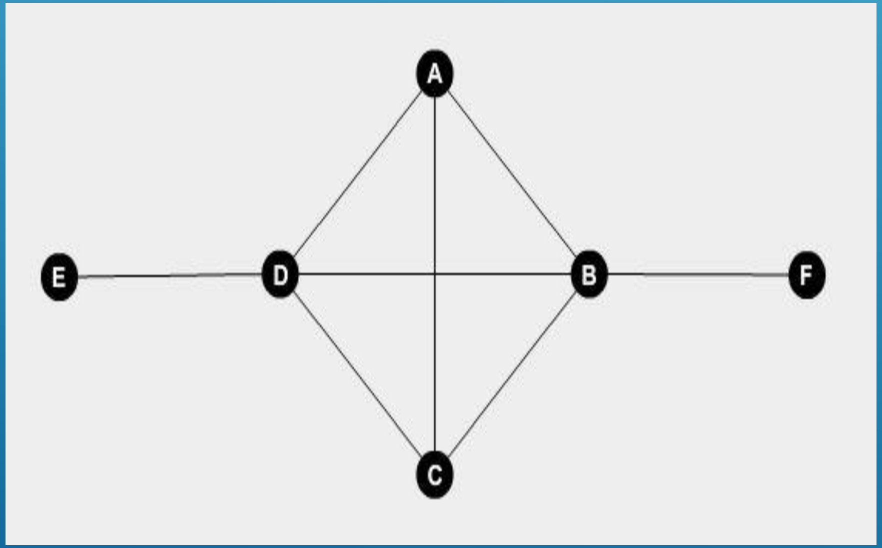
Exists y in K, S +{y} is stable

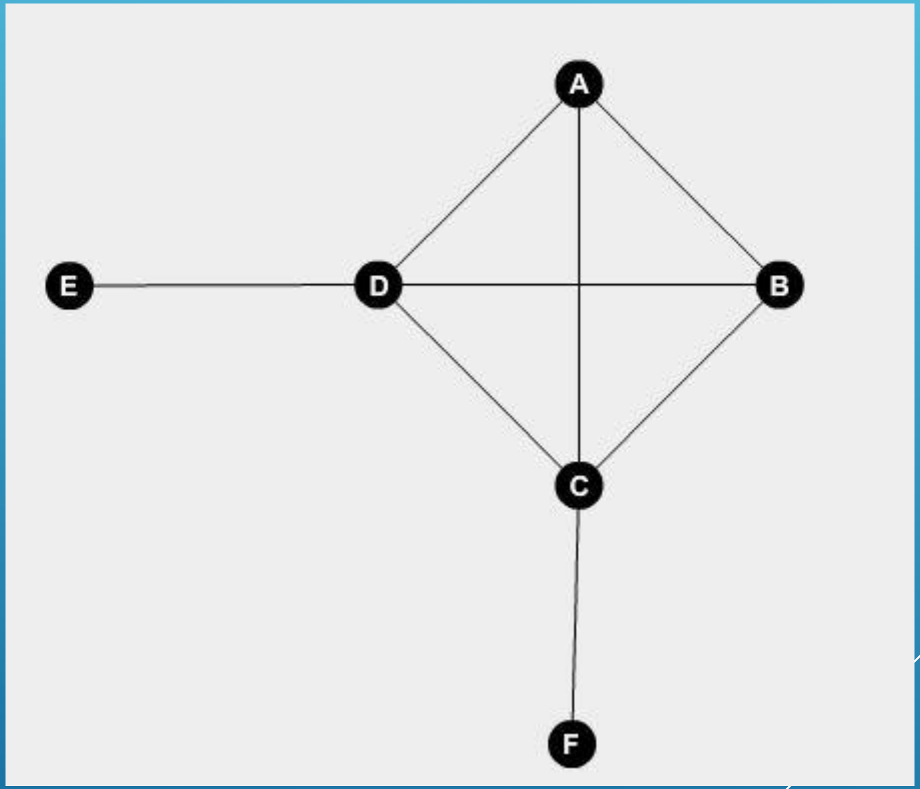


SPLIT GRAPH CHARACTERIZATIONS

- If G is a split graph, then every graph with the same degree sequence as G is also a split graph.
- Example:

deg. Seq.=[4,4,3,3,1,1]





SPLIT GRAPH AS REAL WORLD APPLICATION

Privacy Policy in Facebook

sharing contents with friends of friends represents a split graph.

PRIVACY POLICY IN FACEBOOK

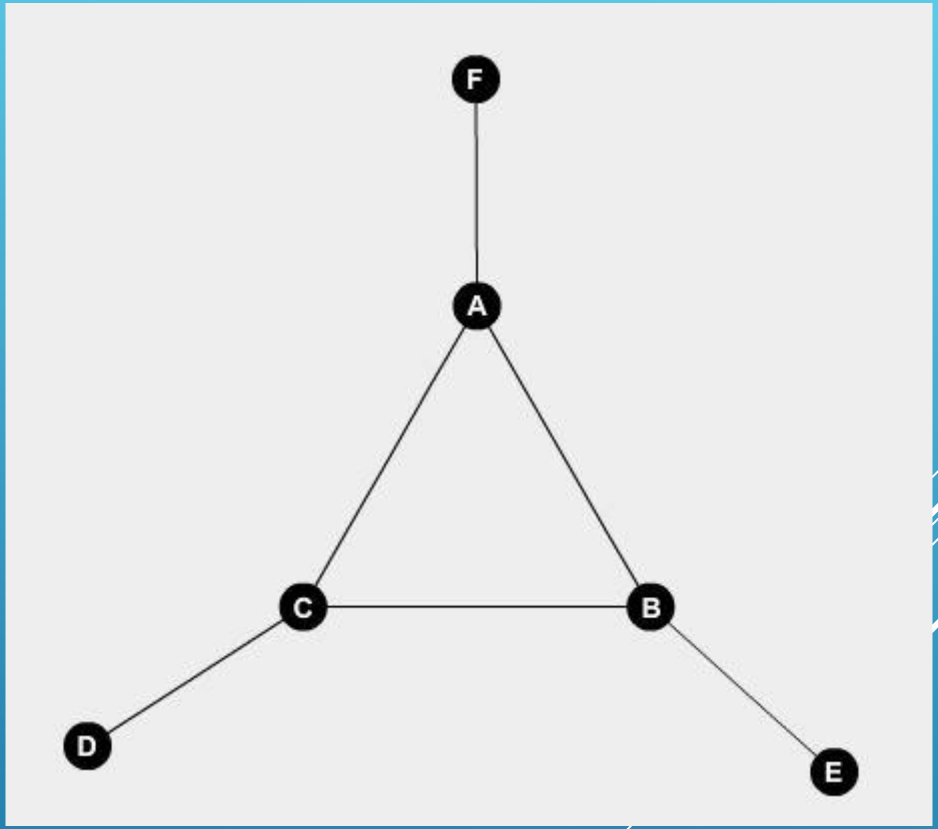
Assume *A*,*B*, and *C* are friends, and

F is friend of *A*

E is friend of *B*

D is friend of *C*

If F shared contents with friends of friends, these contents will be seen by *A*,*B*, and *C*, but not D and E. In this case, *K={F,A,B,C}*, and *S={D,E}*.



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