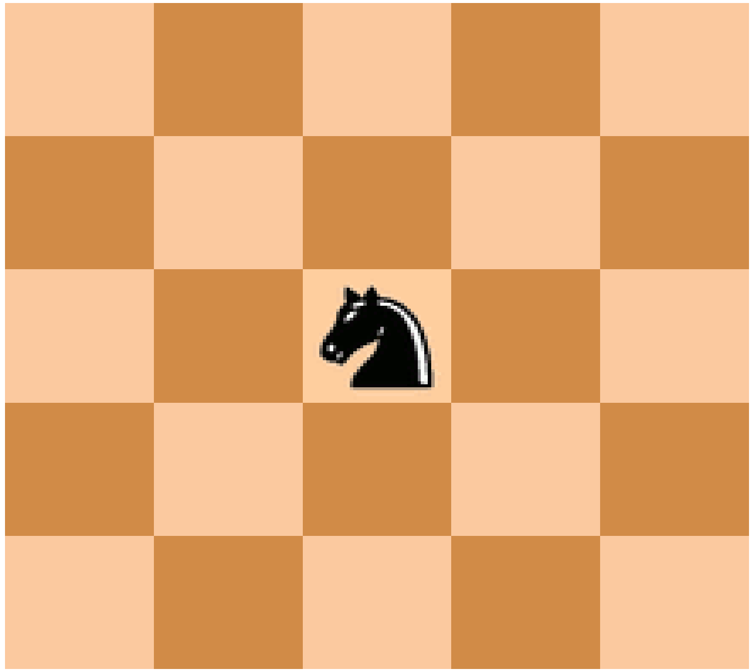
Knight's Tour Problem and its Graph Analysis



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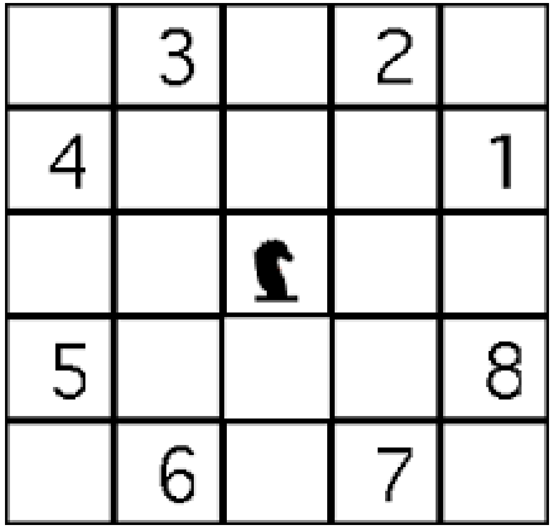
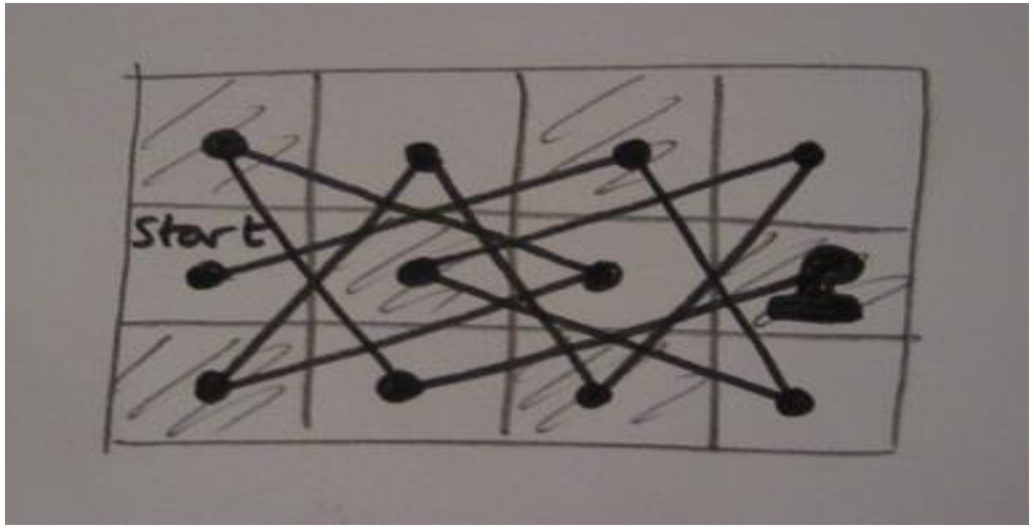
In this Presentation:

- Knight’s Tour and its Problem discussion
- Building Knight’s Graph
- Analysis on Knight Graph
- Special Properties of the Knight Graph

Knight Tour Problem

• The knight is placed on any block of an empty board and is move according to the rules of chess, must visit each square exactly once.
• If the knight ends on a square that is one knight's move from the beginning square, the tour is closed otherwise it is open tour. It is also called as Hamiltonian path.
• A cycle that uses each graph vertex of a graph exactly once is called a Hamiltonian cycle
• Knight's tour can be defined on any grid pattern.





There are few questions we can ask:

1. Is it possible for a knight to start on some square and, by a series of valid knight moves, visit each square on an 8×8 chessboard or any other grid once?
2. Is the graph will be connected? Can I start my knight at a vertex and get to any vertex by only making valid knight's moves?
3. What is the degree of each vertex?
4. How many colors does it take to color this graph such that no two vertices of the same color are connected by an edge?

Open and Closed Tour

1	60	15	24	47	36	13	26
16	23	64	59	14	25	38	35
63	2	61	46	37	48	27	12
22	17	56	49	58	51	34	39
3	62	21	52	45	40	11	28
18	55	44	57	50	31	8	33
43	4	53	20	41	6	29	10
54	19	42	5	30	9	32	7

1	24	39	36	11	22	49	34
40	37	12	23	50	35	10	21
13	2	25	38	57	64	33	48
26	41	60	63	54	51	20	9
3	14	53	56	61	58	47	32
42	27	62	59	52	55	8	19
15	4	29	44	17	6	31	46
28	43	16	5	30	45	18	7

Building Knight Graph

• Each square on the chessboard can be represented as a node in the graph.
• Each legal move by the knight can be represented as an edge in the graph.
• Legal move is shift one square along one axis and two square along the other axis

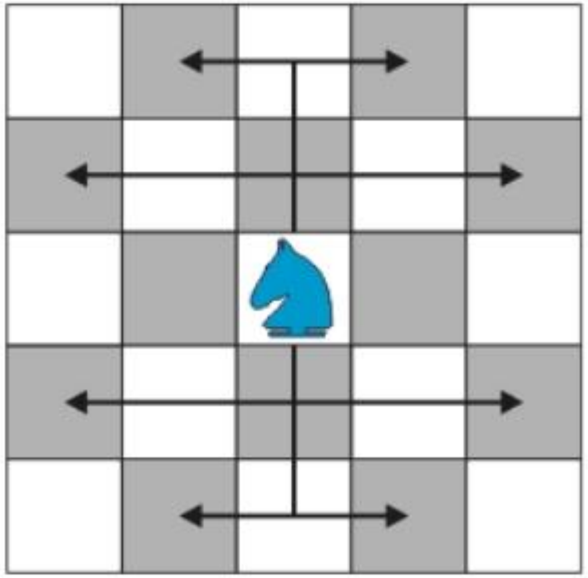
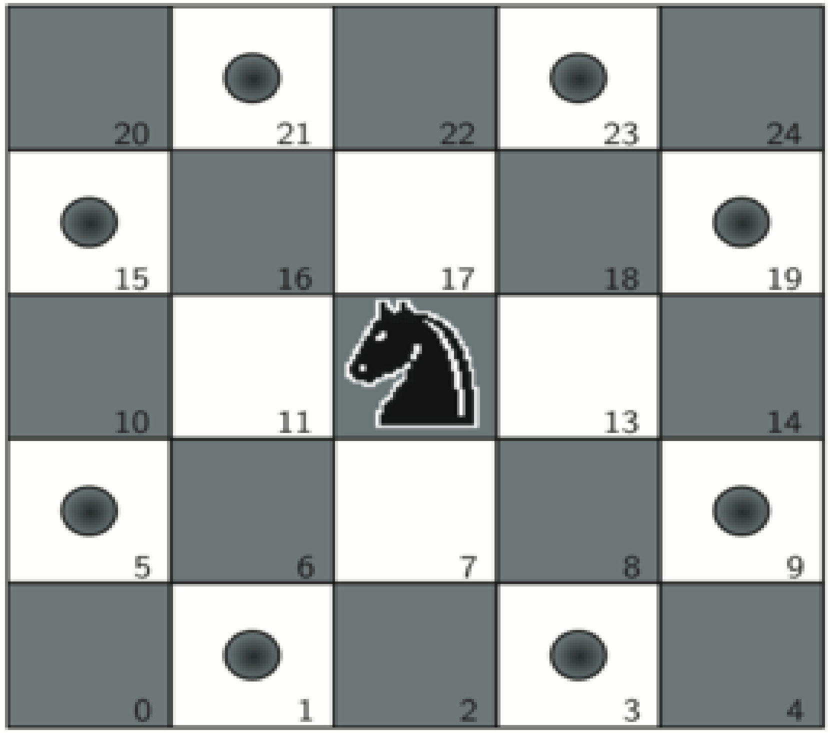
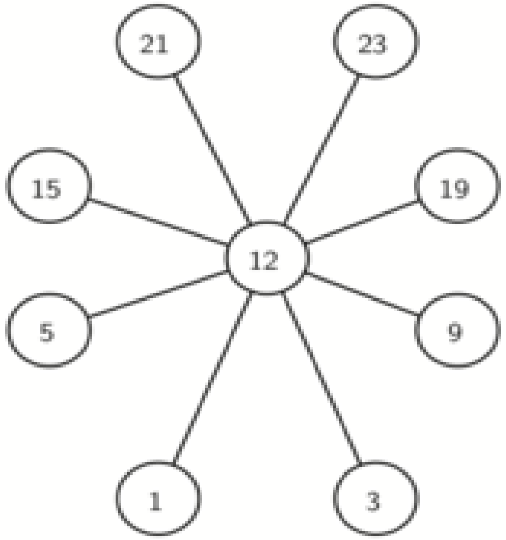


Figure 1: Legal moves for a knight

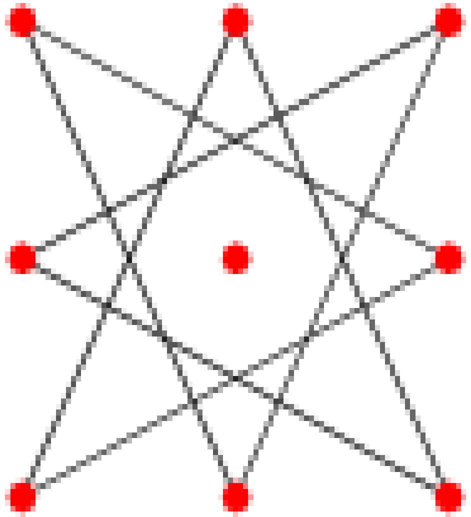


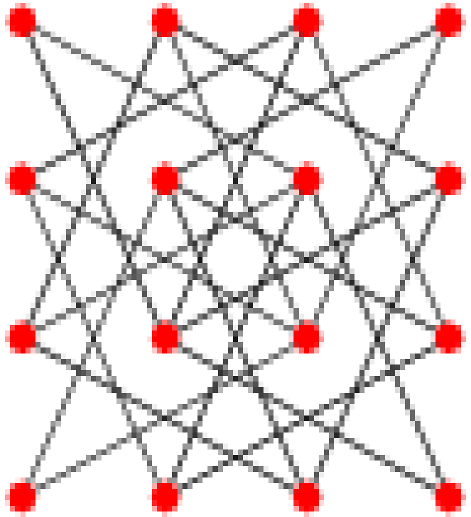


- For a n*m knight's tour graph the total number of vertices is simply n*m.

- For a n*n knight's tour graph the total number of vertices is simply n^2 and the total number of edges is 4(n-2)(n-1).

- Knight's tour path exists on an n*n board if n >= 5





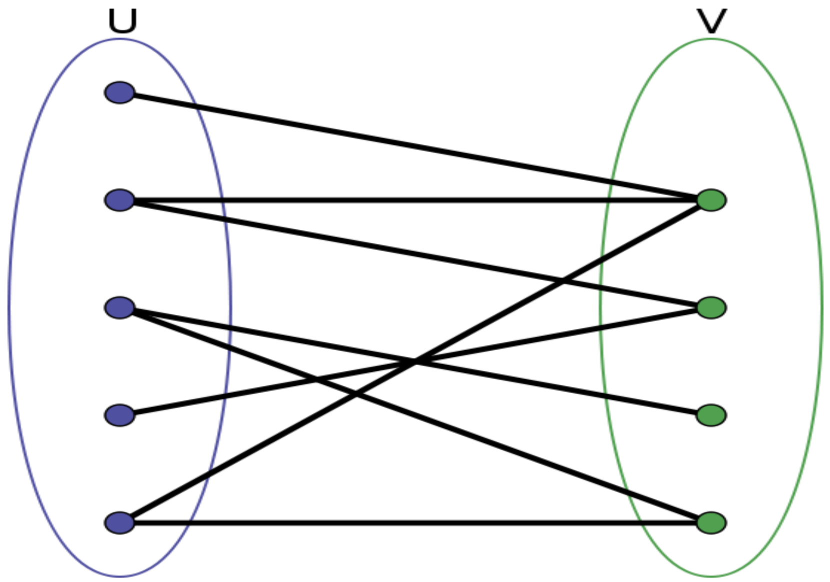
6	22	16	3	24
15	2	23	10	21
27	9	20	14	1
19	13	5	26	8
4	25	7	18	17

Bipartite Graph

- A bipartite is a graph whose vertices can be divided into two disjoint sets U and V (that is, U and V are each independent sets) such that every edge connects a vertex in U to one in V.
- The two sets U and V may be thought of as a coloring of the graph with two colors: if one colors all nodes in U blue, and all nodes in V green, each edge has endpoints of differing colors.
- All acyclic graph is bipartite, and a cyclic graph is bipartite if all its cycles are of even length.
- A graph is bipartite if and only

✓ if it does not contain an odd cycle.

✓ if it is 2-colorable.

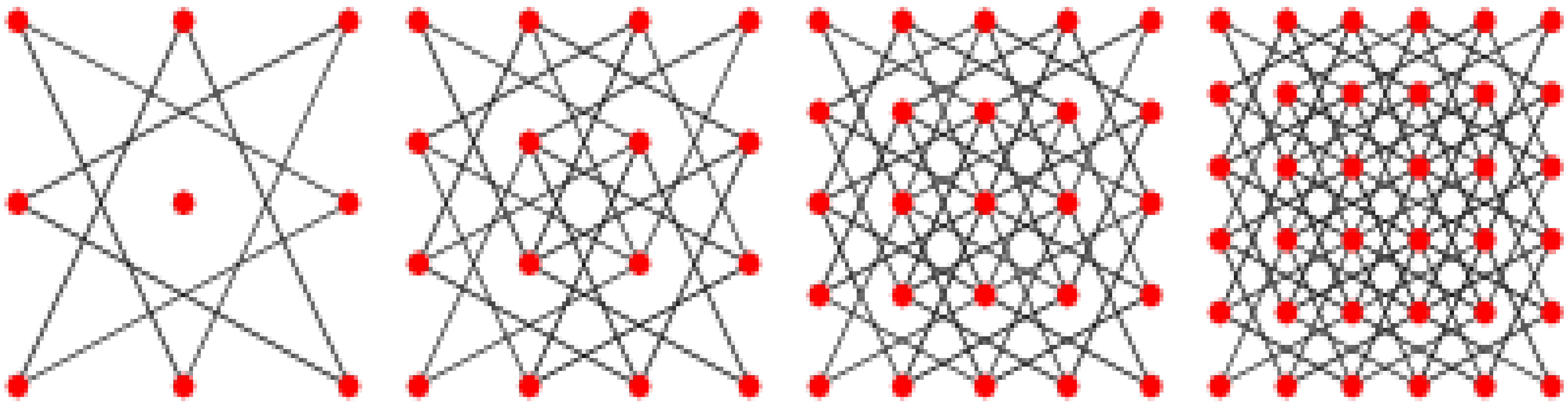


Special Properties of Knight Graph

- Knight Graph are bipartite graph.
- In Knight graph, no two graph vertices within the same set are adjacent.
- A knight move always alternates between white and black squares.
- Knight graph are equivalent to two-colorable graph. Its chromatic number is 2.

- Knight graph is a special case of k-partite graph with k=2.
- Knight graph are perfect Graph.
- A perfect graph is a graph G such that for every induced subgraph of G, the clique number equals the chromatic number, i.e., $\omega(G)=\chi(G)$.

For Knight graph, $\omega(G)=\chi(G) = 2$.



- A Knight graph is perfect as neither the graph G nor its graph complement has a chord less cycle of odd order.
- The perfect graph theorem states that the graph complement of a perfect graph is itself perfect. A Knight graph is therefore perfect as its complement is also perfect.
- Degree of each vertex in Knight graph is 2. It is also called as regular bipartite graph as degree is same for every vertex.
- In general, every bipartite graph is perfect.
- Every tree is a bipartite graph.

Solution Approach

- Brute force
- Neural networks
- Depth first search with backtracking:- Knight moves to a square that has the lowest number of next moves available. The idea is that at the end of the tour it will visit squares that have more move choices available.
- Ant colony optimization:- ACO has a good capability of finding better tour path, which not only uses the feedback principle to quicken evolution process of colonies but also is an essential parallel algorithm.
- The Knight’s tour problem can be solved in linear time.

Questions?