



Interval Graphs - Berge Mystery

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Outline

- *The mystery story & few facts*
- *Interval Graphs*
- *Applications*
- *Properties*
- *Solving the Mystery*
- *References*

A Mystery in the Library

Six professors has been to the library on the day that the rare tractate was stolen.

Each has entered once, stayed for sometime, and then left.

If two were in the library in the same time , then at least one of them saw the other.

Detectives questioned the professors and gathered the following testimony :

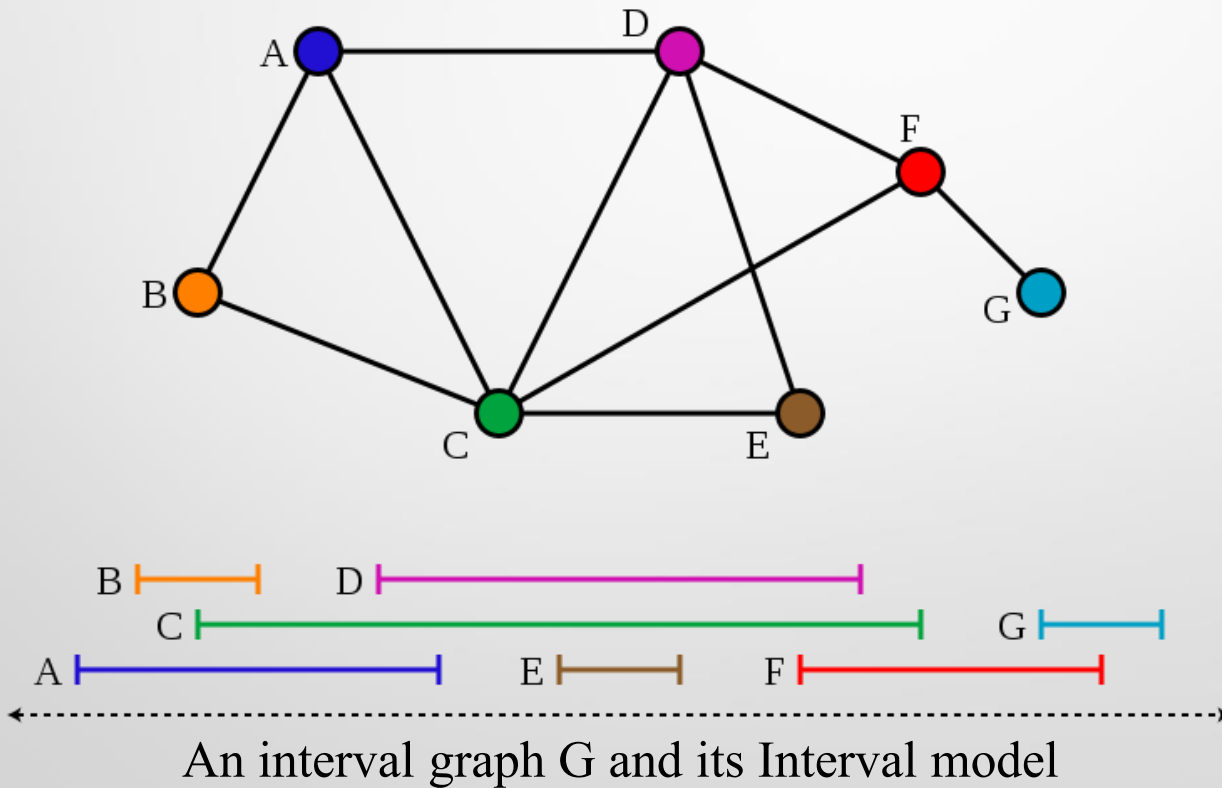
The Facts:

- Abe said that he saw Burt and Eddie
- Burt said that he saw Abe and Ida
- Charlotte claimed to see Desmond and Ida
- Desmond said that he saw Abe and Ida
- Eddie testified to seeing Burt and Charlotte
- Ida said that she saw Charlotte and Eddie.

**ONE OF THE PROFESSOR LIED.
WHO WAS IT?**

Interval Graph

A graph G is an interval graph if we can represent the intervals on a number line such that two vertices are joined by an edge if and only if their corresponding intervals overlap.



Applications of Interval Graphs

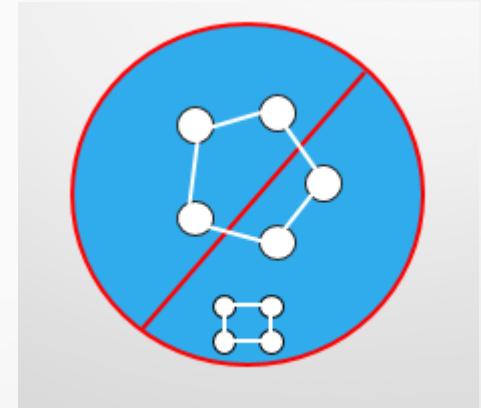
- Resource allocation problems in operations research
- Independent set problem
- Graph coloring

Other Applications include

- Scheduling
- VLSI Designs
- Frequency assignment
- More ...

Properties of Interval Graph

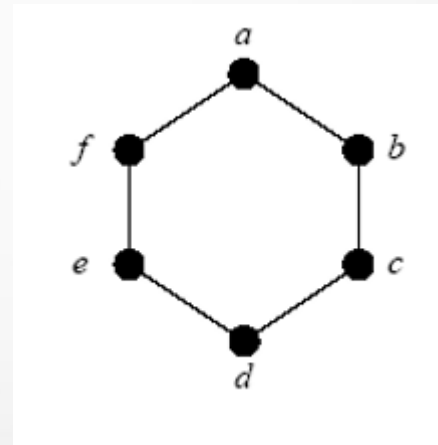
- Chordal Graph Property
- AT-Free
- C4 & S3 Free
- Co-TRO Property



Interval Graphs are Chordal


Interval graphs should not contain chordless cycles. Because we cannot construct an interval model with a cycle

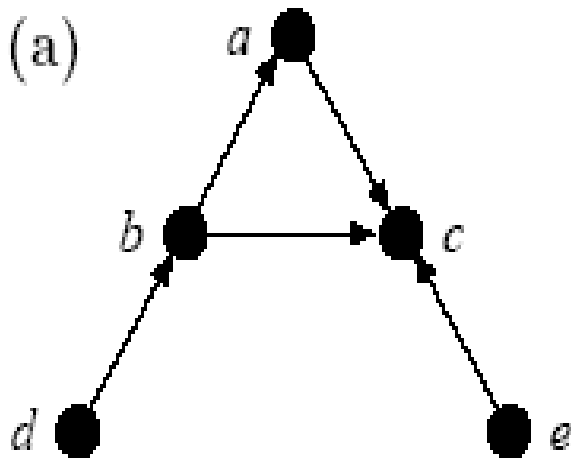
- i.e., they are chordal.



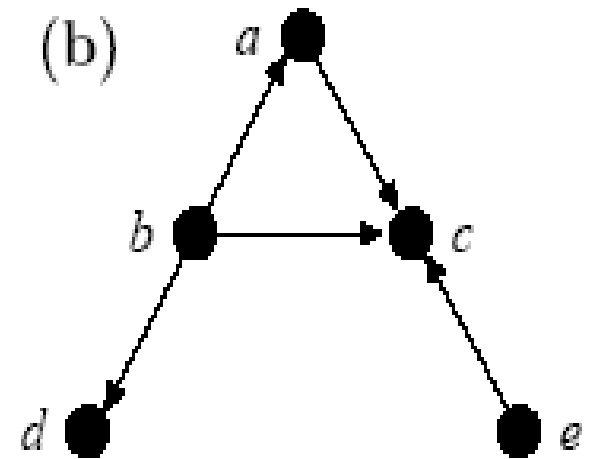
The co-TRO Property

An interval graph's complement must be transitively orientable i.e., the complement must have a Transitive Orientation

TRO Eg: $a \rightarrow b$, $b \rightarrow c$  $a \rightarrow c$



Not transitive !

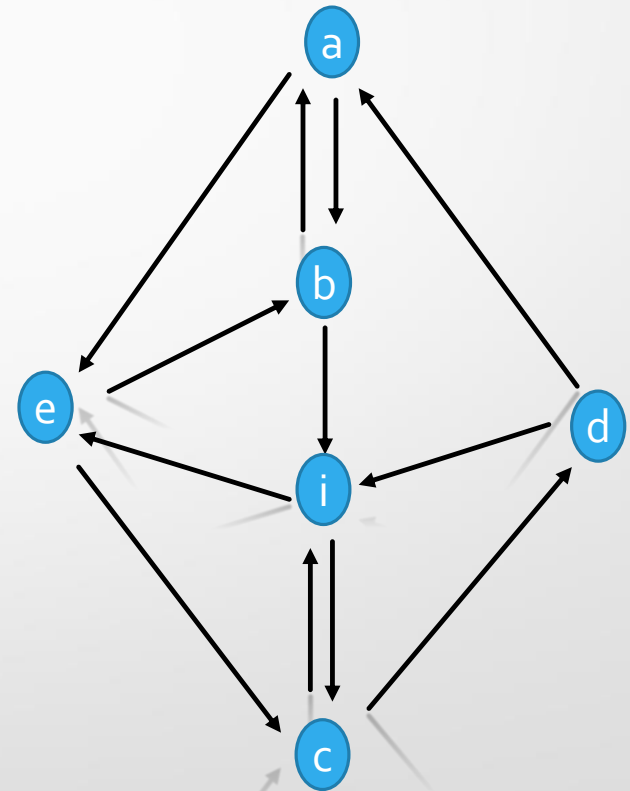


Transitive !

Solving the Mystery

The Testimony Graph

- Abe said that he saw Burt and Eddie
- Burt said that he saw Abe and Ida
- Charlotte claimed to see Desmond and Ida
- Desmond said that he saw Abe and Ida
- Eddie testified to seeing Burt and Charlotte
- Ida said that she saw Charlotte and Eddie.

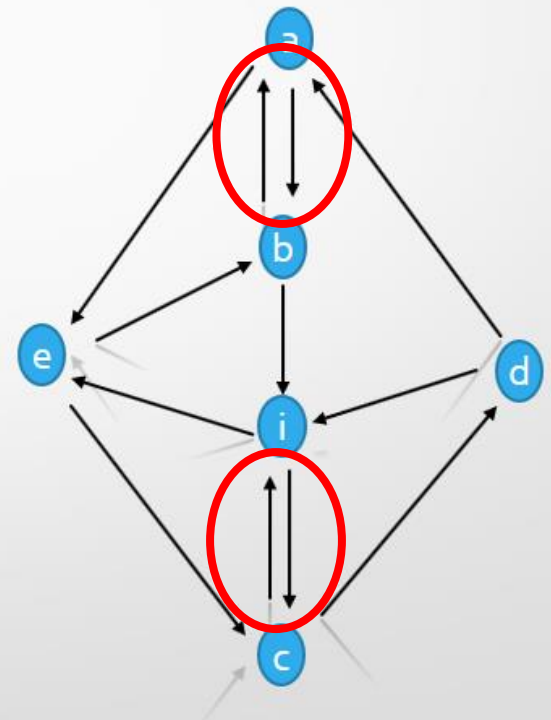


Solving the Mystery

The Testimony Graph

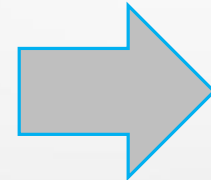
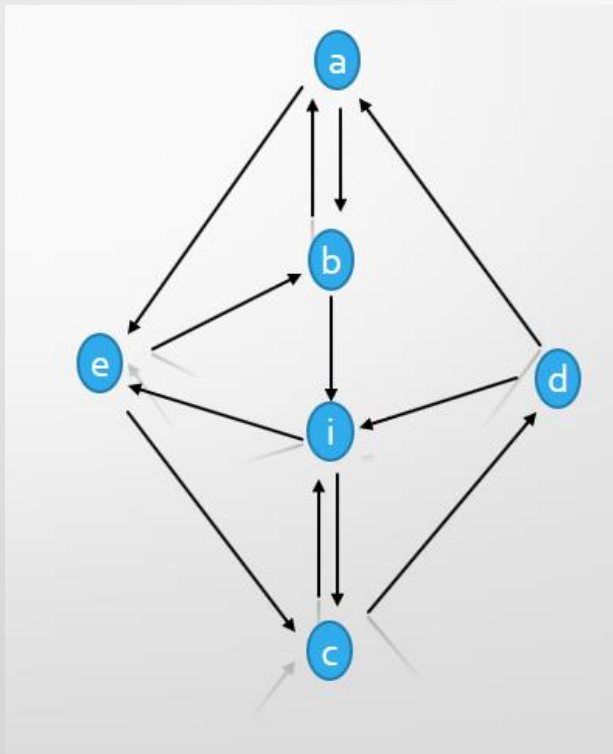
Hint!

Double arrows imply a TRUTH

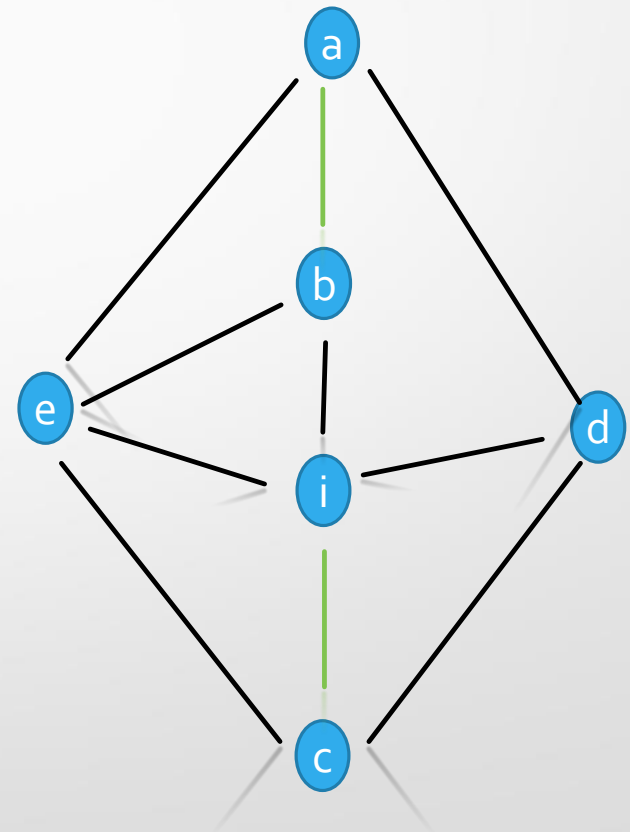


Solving the Mystery

Directed Testimony Graph



Undirected Testimony Graph



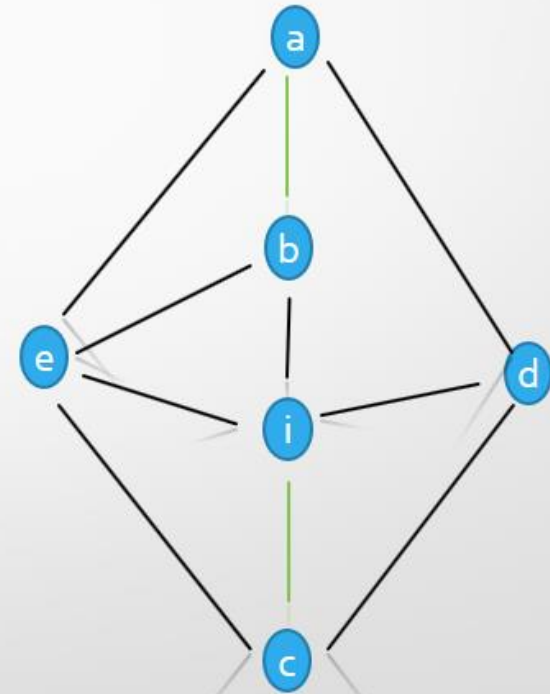
Intersecting Intervals cannot form Chordless Cycles

Burt

Desmond

Abe

Here No place for Ida's interval:
It must hit both B and D but cannot hit A.
which is Impossible!



Solving the Mystery

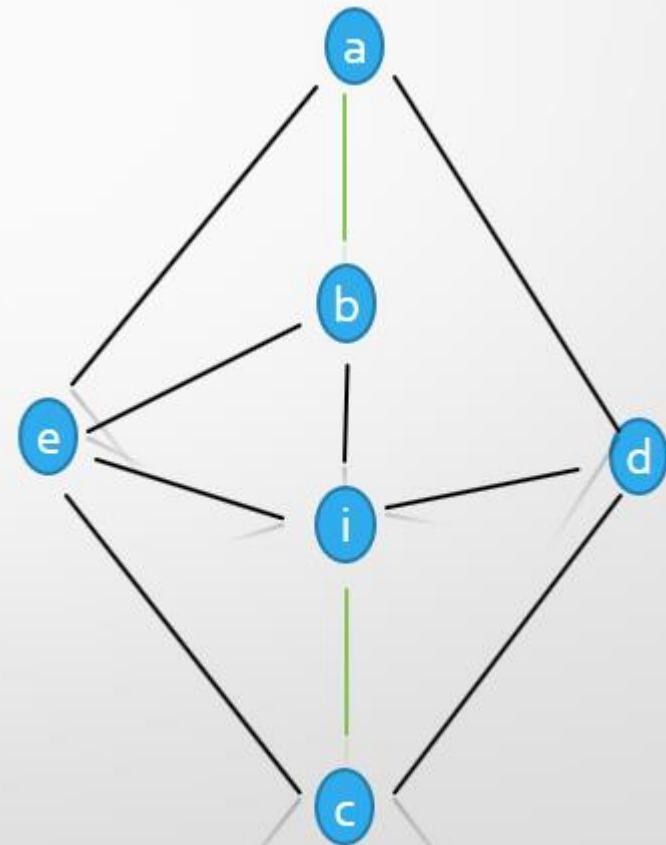
One professor from the chordless 4-cycle must be a liar.

There are three chordless 4-cycles:

{A, B, I, D}

{A, E, I, D}

{A, E, C, D}



Solving the Mystery

One professor from the chordless 4-cycle must be a liar.

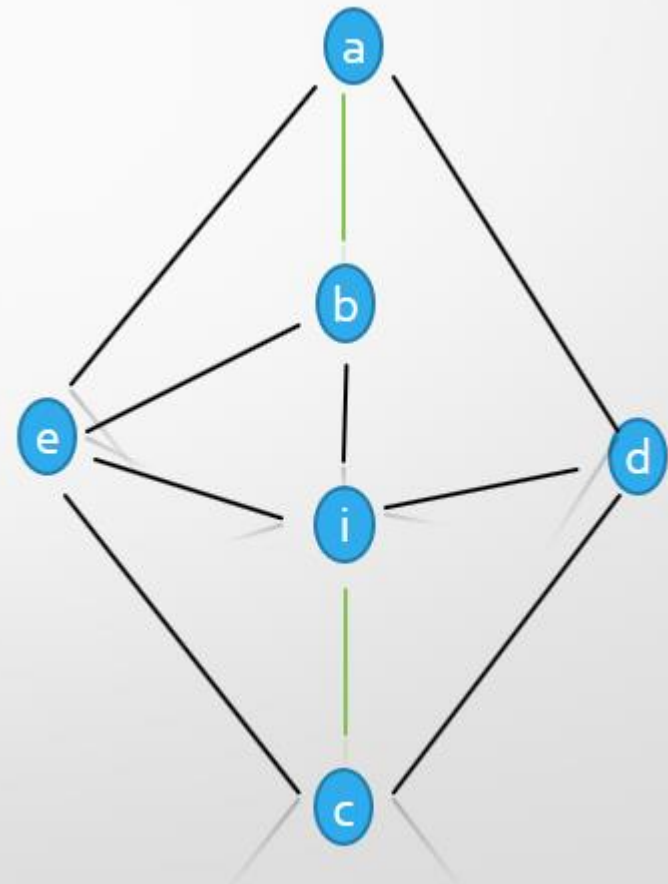
There are three chordless 4-cycles:

$\{A, B, I, D\}$

$\{A, E, I, D\}$

$\{A, E, C, D\}$

Burt is NOT a liar: He is missing from second cycle.



Solving the Mystery

One professor from the chordless 4-cycle must be a liar.

There are three chordless 4-cycles:

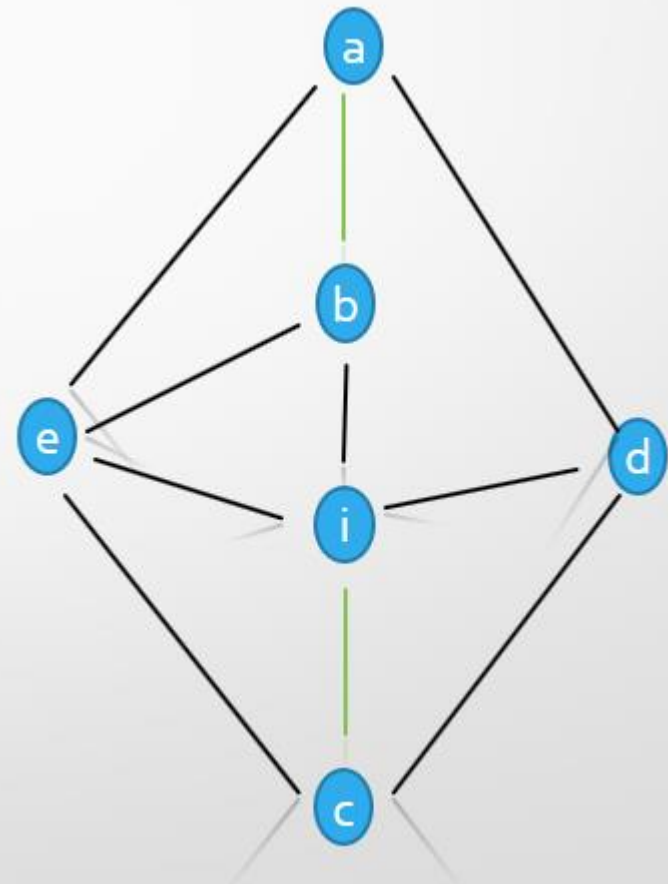
{A, B, I, D}

{A, E, I, D}

{A, E, C, D}

Ida is NOT a liar:

She is missing from the third cycle.



Solving the Mystery

One professor from the chordless 4-cycle must be a liar.

There are three chordless 4-cycles:

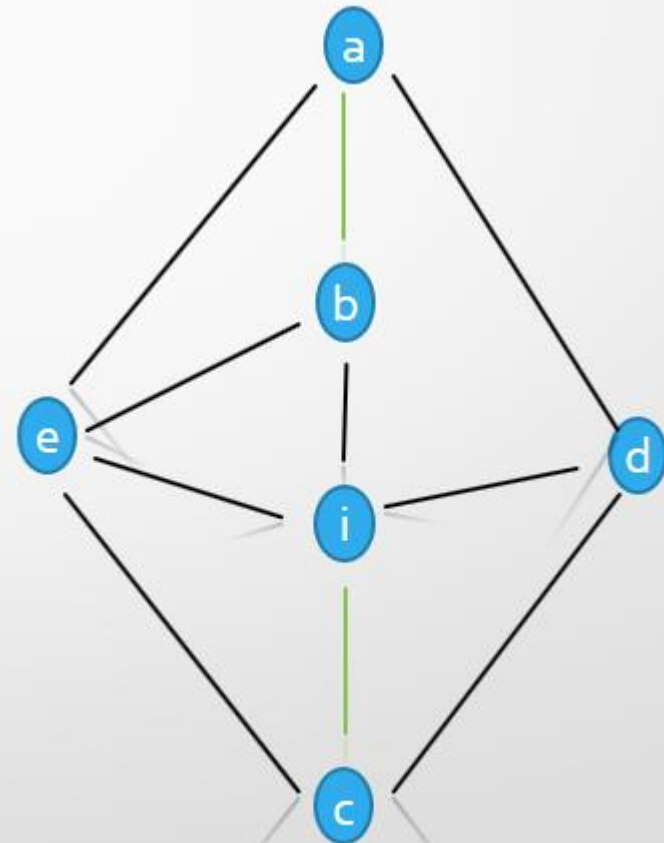
$\{A, B, I, D\}$

$\{A, E, I, D\}$

$\{A, E, C, D\}$

Charlotte is NOT a liar:

She is missing from the second.



Solving the Mystery

One professor from the chordless 4-cycle must be a liar.

There are three chordless 4-cycles:

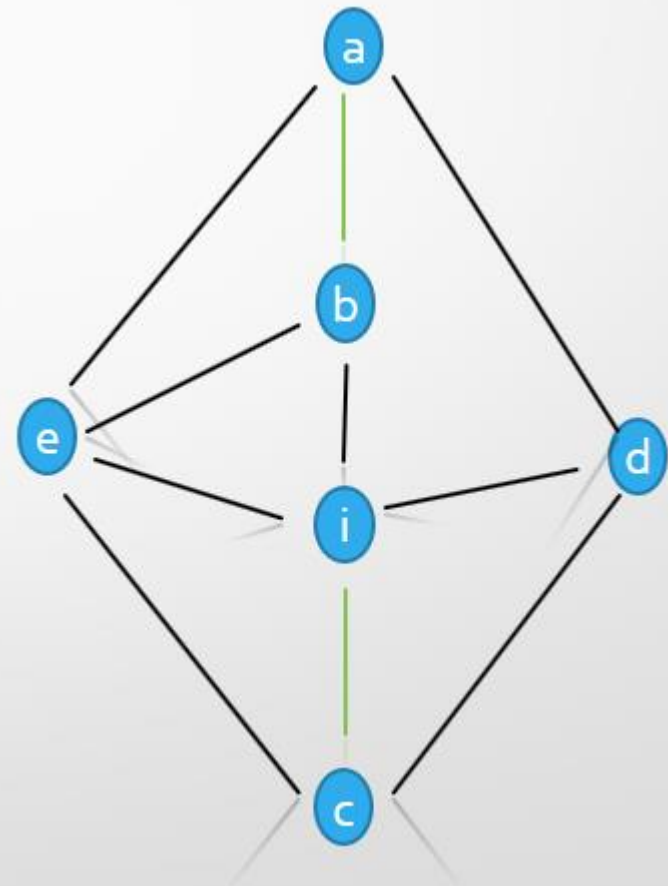
$\{A, B, I, D\}$

$\{A, E, I, D\}$

$\{A, E, C, D\}$

Eddie is NOT a liar:

He is missing from the first cycle.



Solving the Mystery

Burt is NOT a liar: He is missing from the second cycle.

Ida is NOT a liar: She is missing from the third cycle. Charlotte is

NOT a liar: She is missing from the second. Eddie is NOT a liar: He is missing from the first cycle.

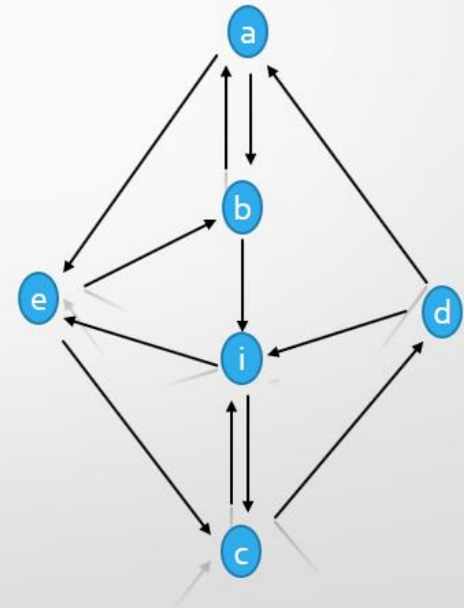
WHO IS THE LIAR? Abe or Desmond ?

Solving the Mystery

WHO IS THE LIAR? Abe or Desmond ?

If Abe were the liar and Desmond truthful,
then {A, B, I, D} would remain a chordless 4-cycle,
since B and I are truthful.

Therefore: **Desmond is the liar.**



References

- An Introduction to Graph Theory
<http://www.cs.haifa.ac.il/~golumbic/courses/algorithmic-graph-theory>
- Wikipedia
https://en.wikipedia.org/wiki/Interval_graph
- Marek Perkowski's Class Notes
http://web.cecs.pdx.edu/~mperkows/CLASS_574/2013/

