Lecture #4

Plan

- **Each graph is the intersection graph of some family of objects.**
- **Interval graphs as the intersection graphs of a family of subpaths of a path.**
- **What are the intersection graphs of subtrees of a tree? Are they special?**
- **Triangulated graphs (alias chordal graphs).**
- **Simplicial vertices and perfect elimination orderings (P.E.O.s).**
- **Basic properties of chordal graphs**
  - **Existence of at least two simplicial vertices.**
  - **Minimal separators.**
- First characterization: three equivalent statements
  - G is chordal ←→
  - G has a P.E.O. ←→
  - Every minimal separator is a clique.
- How to find a P.E.O.?
  - Naïve (polynomial)
  - LexBFS (linear)
  - MCS (linear)
- Linear time recognition algorithm.
- Chordal graphs as the intersection graphs of a family of subtrees of a tree.
  - A family of subtrees of a tree has the Helly property.
  - The intersection graph of a family of subtrees of a tree is chordal.
- Second characterization: three equivalent statements
  - G is chordal ←→
  - G is the intersection graph of a family of subtrees of a tree ←→
  - There is tree T(G) whose nodes are the maximal cliques of G and for every vertex v of G the cliques containing it form a subtree in T(G).
- Solving 4 classical problems on chordal graphs in linear time.
  - Equalities between corresponding numbers.
  - Chordal graphs are perfect.
- p-Centers and r-domination in trees.
  - Efficient solutions via chordal graph.
- Other applications of chordal graphs.

References:

1. Chapters 4 of M. Golumbic's book plus related papers/material
2. http://www.fi.muni.cz/~hlineny/Vyuka/GT/Grafy-lect-eng-9.pdf
3. http://en.wikipedia.org/wiki/Intersection_graph
4. http://www.cse.iitd.ac.in/~naveen/courses/CSL851/uwaterloo.pdf