

Lecture #6

Plan

- Each graph is the intersection graph of some family of objects.
- Interval graphs as the intersection graphs of a family of subpaths of a path.
- What are the intersection graphs of subtrees of a tree? Are they special?
- Triangulated graphs (alias chordal graphs).
- Simplicial vertices and perfect elimination orderings (P.E.O.s).
- Basic properties of chordal graphs
 - Existence of at least two simplicial vertices.
 - Minimal separators.
- First characterization: three equivalent statements
 - G is chordal \leftrightarrow
 - G has a P.E.O. \leftrightarrow
 - Every minimal separator is a clique.
- How to find a P.E.O.?
 - Naïve (polynomial)
 - LexBFS (linear)
 - MCS (linear)
- Linear time recognition algorithm.
- Chordal graphs as the intersection graphs of a family of subtrees of a tree.
 - A family of subtrees of a tree has the Helly property.
 - The intersection graph of a family of subtrees of a tree is chordal.
- Second characterization: three equivalent statements
 - G is chordal \leftrightarrow
 - G is the intersection graph of a family of subtrees of a tree \leftrightarrow
 - There is tree $T(G)$ whose nodes are the maximal cliques of G and for every vertex v of G the cliques containing it form a subtree in $T(G)$.
- Solving 4 classical problems on chordal graphs in linear time.
 - Equalities between corresponding numbers.
 - Chordal graphs are perfect.
- p -Centers and r -domination in trees.
 - Efficient solutions via chordal graph.
- Other applications of chordal graphs.

References:

1. Chapters 4 of M. Golumbic's book plus related papers/material
2. <http://www.fi.muni.cz/~hlineny/Vyuka/GT/Grafy-lect-eng-9.pdf>
3. http://en.wikipedia.org/wiki/Intersection_graph
4. <http://www.cse.iitd.ac.in/~naveen/courses/CSL851/uwaterloo.pdf>