Scheduling Bus routes with graph colouring

Presentation by

santosh

outline

1. **Problem statement**
2. **Bipartite graph**
3. **Graph colouring**
4. **Explanation**

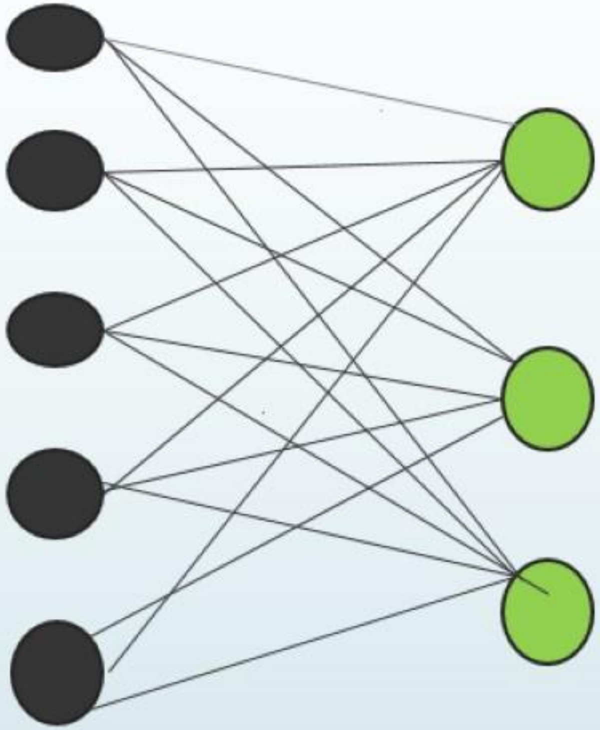
Problem statement

In a particular city they need to allocate buses in various routes such that every route must be travelled by at least one bus and the condition is to take minimum number of buses and there should be no conflicts.

Bipartite graph

1) A bipartite graph is a graph whose vertices can be divided into two disjoint sets. We can also say that there is no edge that connects vertices of same set.
2) Every edge connects a vertex in one set to another.
3) Vertex set are usually called the parts of the graph.
4) A bipartite graph is a graph that does not contain any odd-length cycles

- Below mentioned is an example of complete bipartite graph with order m-5 and n-3



Properties of bipartite graph

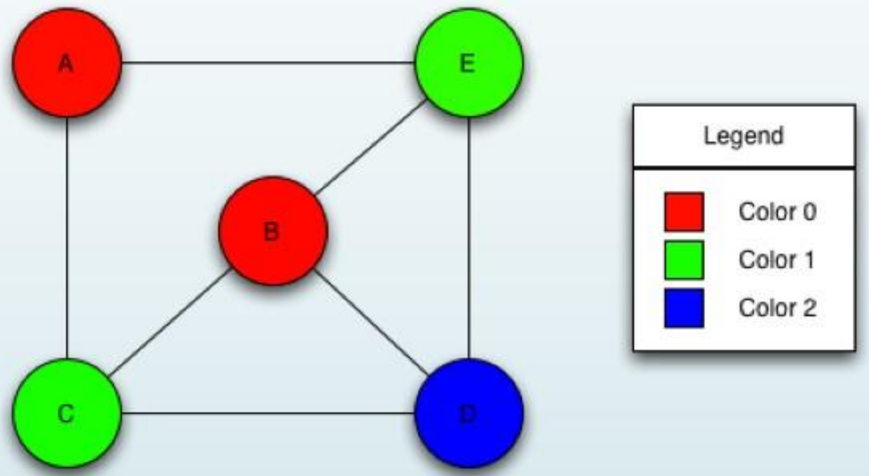
- A graph is said to be bipartite if it does not contain an odd cycle.
- A bipartite graph contains only 2-colors(i.e., its chromatic number is less than or equal to 2)
- Bipartite graphs are used in coding theory, especially to decode code words received from the channels.
- All acyclic graphs are bipartite.

Graph colouring

- **Graph colouring** is a special case of graph labelling; it is an assignment of labels traditionally called "colours" to elements of a graph subject to certain constraints.
- There are different kinds of graph colouring namely vertex colouring, edge colouring etc.

Vertex colouring

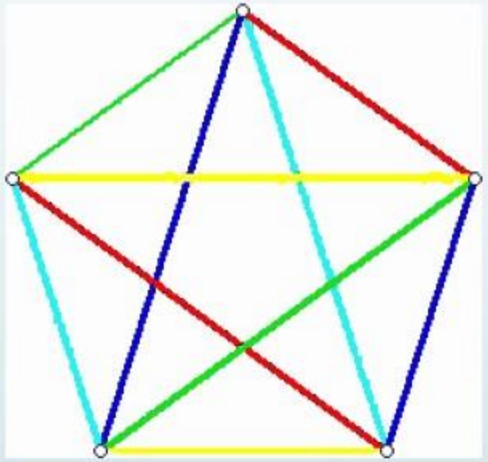
- A vertex colouring is an assignment of labels or colours to each vertex of a graph such that no edge connects two identically coloured vertices. The most common type of vertex colouring seeks to minimize the number of colours for a given graph.



- vertex colouring

Edge colouring

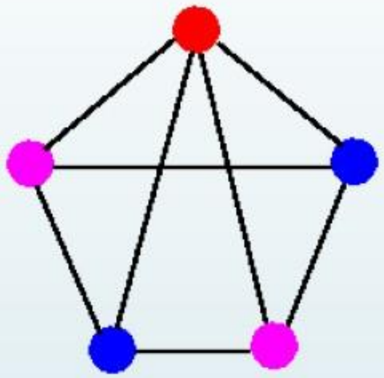
- An **edge colouring** of a graph is an assignment of "colours" to the edges of the graph so that no two adjacent edges have the same colour. Here, two edges are considered to be adjacent when they share a common vertex.



edge colouring

Chromatic number

- The chromatic number of a graph G is the smallest number of colours needed to colour the vertices of G so that no two adjacent vertices share the same colour.

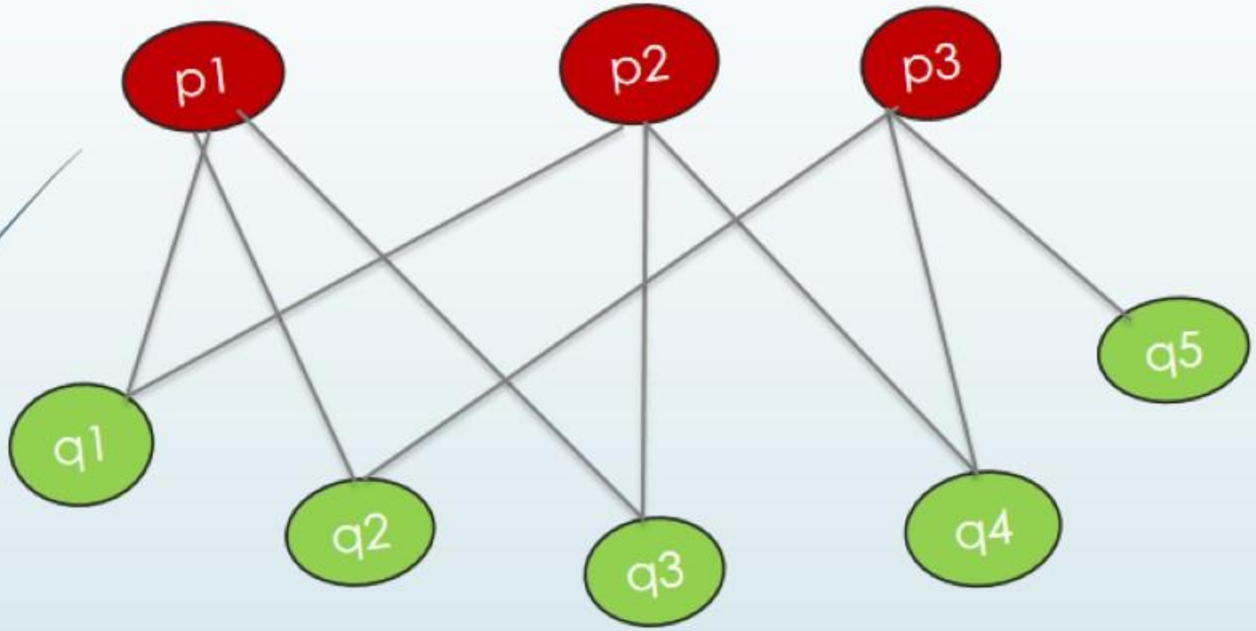


chromatic number 3

Solution:

- Consider a graph with p1,p2,p3 buses and q1,q2,q3,q4,q5 number of routes. The graph obtained is:

- Each bus is connected with at least one route.



Matrix relationship

- The relation between the buses and the routes is considered by taking it in a matrix form.

$$R = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The edge colouring is done for the above problem such that no two edges have same colour.

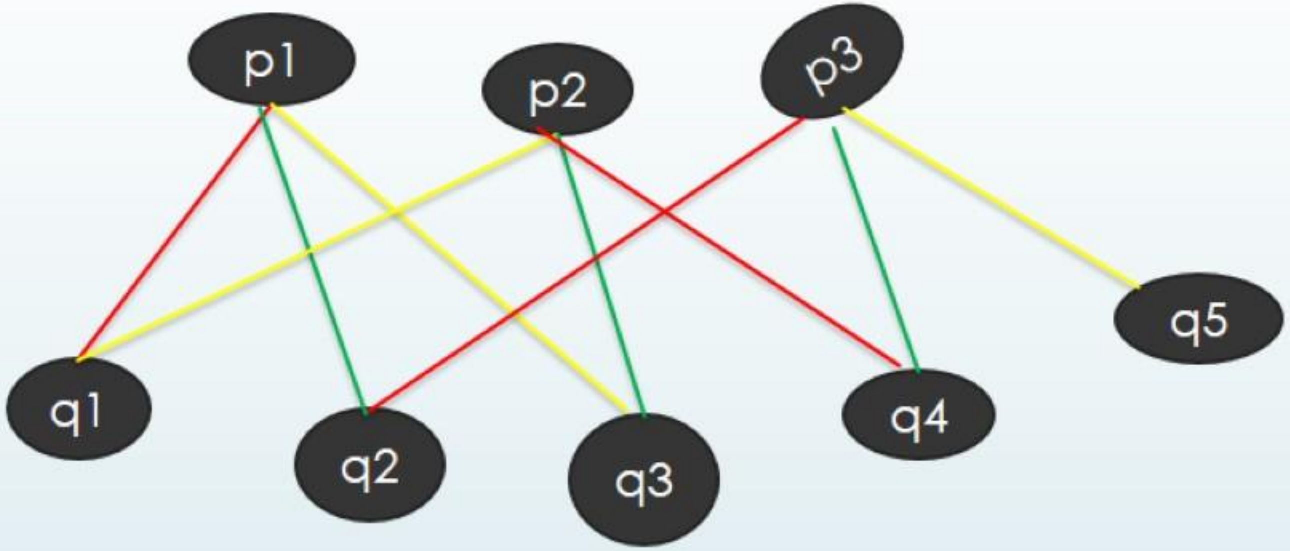


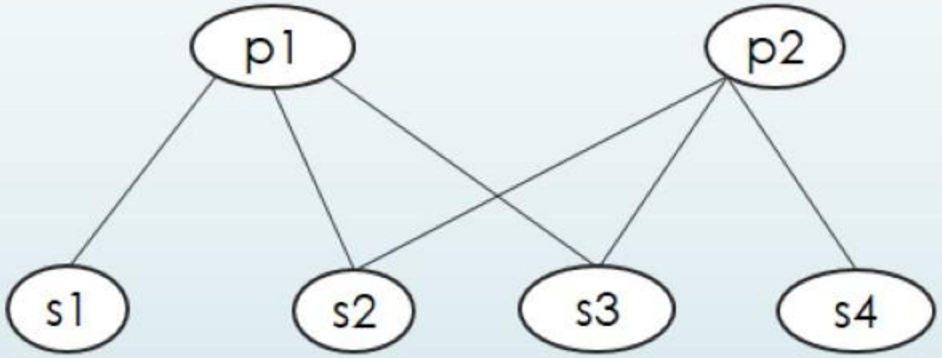
Table formulated

- This is how route are allocated to buses.

	Route1	Route 2	Route 3	Route 4	Route 5
bus1	Q1	Q2	Q3	0	0
bus2	Q1	0	Q3	Q4	0
bus3	0	Q2	0	Q4	Q5

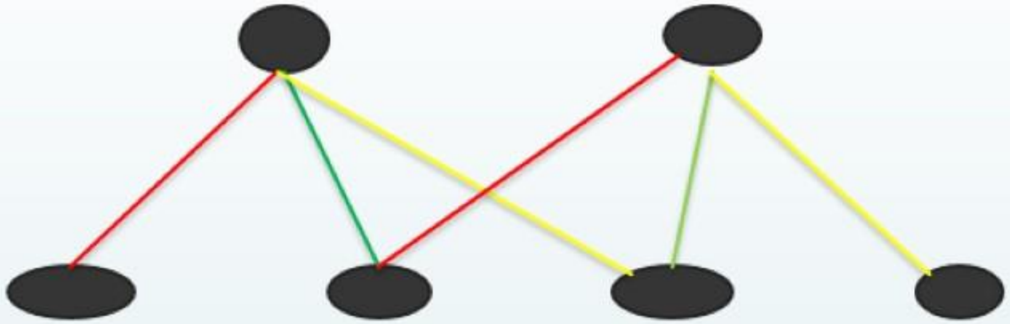
scheduling

- If we consider any two buses and different stops as vertex in particular route.
- Consider buses p1 and p2 with different vertex points as stops.



Graphs colouring

- Edge colouring is done for the above diagram.



- Table is formulated for the diagram.
- By using this table timings are allocated for the buses such that no two buses will intersect and conflict occurs.
- If we consider the table, the starting point of bus1 is stop1 and ending is stop3 where as for bus2 starting point is stop2 and ending is stop4.

	Stop 1	stop 2	stop 3	stop 4
bus1	Q1	Q2	Q3	0
bus2	0	Q2	Q3	Q4