TRAFFIC FLOW CONTROL

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Real-World Problem

- Consider an intersection without traffic lights
- The problem is to install traffic lights at an intersection in such a way that traffic flows safely and efficiently at the intersection
What are compatible traffic streams?

- We say that the traffic stream \textit{a} and \textit{b} are compatible with each other if they can be moving at the same time without dangerous consequences.
- The decision about the compatibility is made a head of time by traffic engineers.
Formulation of the Problem

- Look at the intersection in Figure 1
- Streams $x, v$ are compatible with each other, whereas $w, t$ are incompatible
- We can build a graph whose vertices are the traffic streams and edges are comparability relations among them, "compatibility graph"
Translate the compatibility information to a Graph

- Vertices in G represent traffic streams
- Two vertices of G adjacent if and only if the corresponding streams are compatible

Figure 1
Compatibility graph G
The Problem in Depth

We want to assign a period of time to each stream during which it receives a **green** light, whereas other streams are **red**.

We may think of a large clock, and the time during which a given traffic stream gets a **green** light corresponds to an arc on the circumference of the clock circle.

There is a cycle of green and red lights, and then after the cycle is finished, it begins again, over and over.

Lead us directly to **Circular-Arc Graphs**.
Circular-Arc Graph and its special characteristics

- Is the intersection graph of a set of arcs on the circle.
- Every vertex is represented by an arc, such that two vertices are adjacent if and only if the corresponding arcs overlap.
- Circular-arc graphs are not always perfect, as the odd chordless cycles $C_5$, $C_7$, etc., are circular-arc graphs.
- If a circular-arc representation fails to cover some point $p$ on the circle, we can cut the circle at $p$ and straighten it out to a line, the arcs becoming intervals.
- Every interval graph is a circular-arc graph. However, the converse is false.
- **Application:** they can be used to model objects of a circular or a repetitive nature.
Interval Graph

- Is the intersection graph of a family of intervals on the real line.
- G is an interval graph if its vertices can be put into one-to-one correspondence with a set of intervals of a linearly ordered set.
- It is a perfect graph
- The four graph problems can be solved efficiently in linear time on Interval graphs
Construct a Circular-Arc Graph from the Problem

- We take a circle and assume that its perimeter corresponds to the total cycle period.
- Assign an arc of the circle to each traffic stream.
- Two arcs of the circle can overlap only if the corresponding streams are compatible.
- It turns out that always there is a point *p on the circle has no arc*.
- Any feasible representation of circular-arc graph of the problem is always interval graph.

![Compatibility graph G and Circular-arc graph for G](image)
Why is it Hard on General Graphs?

- Solution might not be feasible
- Repetitive nature of the problem
- Providing efficiency (Finding maximal cliques)
Make The Solution more Efficient

- We want to assign a green light for streams in such a way that only compatible traffic streams can get green light at the same time.
- We can do that by finding Maximal Cliques
- **Maximal Clique is a clique that cannot be extended by including one more adjacent vertex**
- Assign a green light for maximal clique Ki at a time.

Figure 1
Special properties for solving the problem Efficiently

- Since the intersection graph of the phasing traffic light problem is an interval graph
- Finding maximal cliques can be done efficiently in linear time
- $K_1 = \{t,v,y\}$  $K_2 = \{x,v,y\}$  $K_3 = \{v,z\}$  $K_4 = \{z,w\}$
- Each clique $K_i$ corresponds to a phase during which all streams in that clique get green lights

Intersection graph representation of Circular-arc graph
Interpret the graph problem’s solution to the R-W problem

- Each clique $K_i$ corresponds to a phase during which all streams in that clique get green lights.
- Start a given traffic stream off with green during the first phase in which it appears, and keep it green until the last phase appears.

$K_1 = \{t,v,y\}$  $K_2 = \{x,v,y\}$  $K_3 = \{v,z\}$  $K_4 = \{z,w\}$

Circular-arc graph for $G$

Figure 1
Resources

- [1978] Graph Theory and its Applications to Problems of Society, Roberts, Fred S.
- Wiki, Circular-Arc Graph, Interval Graph
- [2004] Algorithmic Graph Theory and Perfect Graphs, Golumbic, Martin Charles
- Traffic Flow and Circular-arc Graph, PPT, Abdulhakeem Mohammed
- [2007] Recognition of Circular-Arc Graphs and Some Subclasses, Yahav Nussbaum
Thank you...

Questions are Welcome!