TRAFFIC FLOW CONTROL

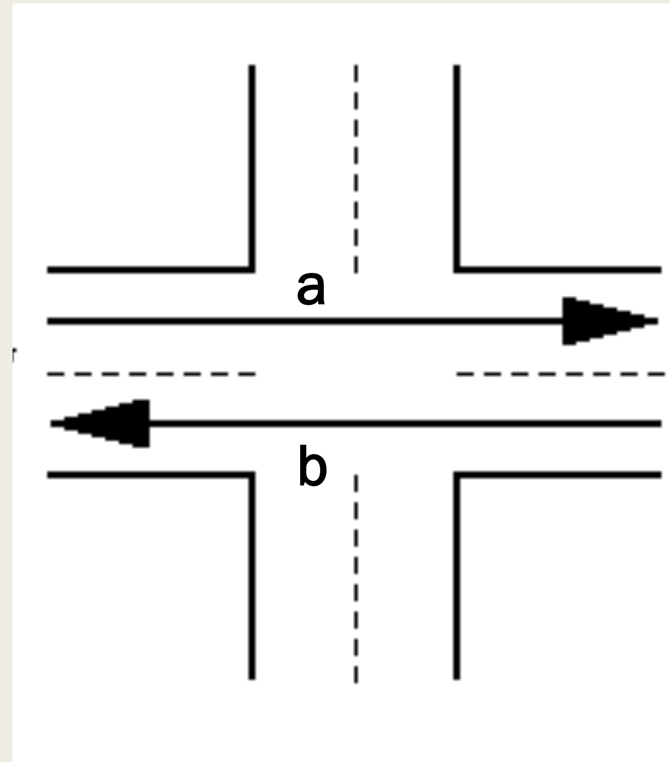
Ahmed Al-Baghdadi

Real-World Problem

- Consider an intersection without traffic lights
- The problem is to install traffic lights at an intersection in such a way that traffic flows safely and efficently at the intersection

What are compatible traffic streams?

- We say that the traffic stream **a** and **b** are compatible with each other if they can be moving at the same time without dangerous consequences
- The decision about the compatibility is made a head of time by traffic engineers



Formulation of the Problem

- Look at the intersection in Figure 1
- streams **x**, **v** are compatible with each other, whereas **w**, **t** are incompatible
- We can build a graph whose vertices are the traffic streams and edges are comparability relations among them, "**compatibility graph**"

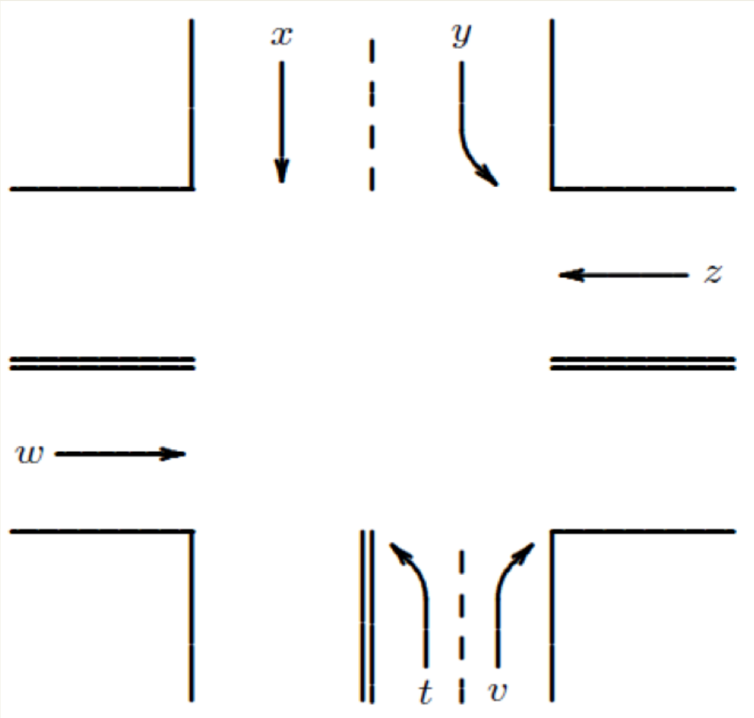


Figure 1

Translate the compatibility information to a Graph

- Vertices in G represent traffic streams
- Two vertices of G adjacent if and only if the corresponding streams are compatible

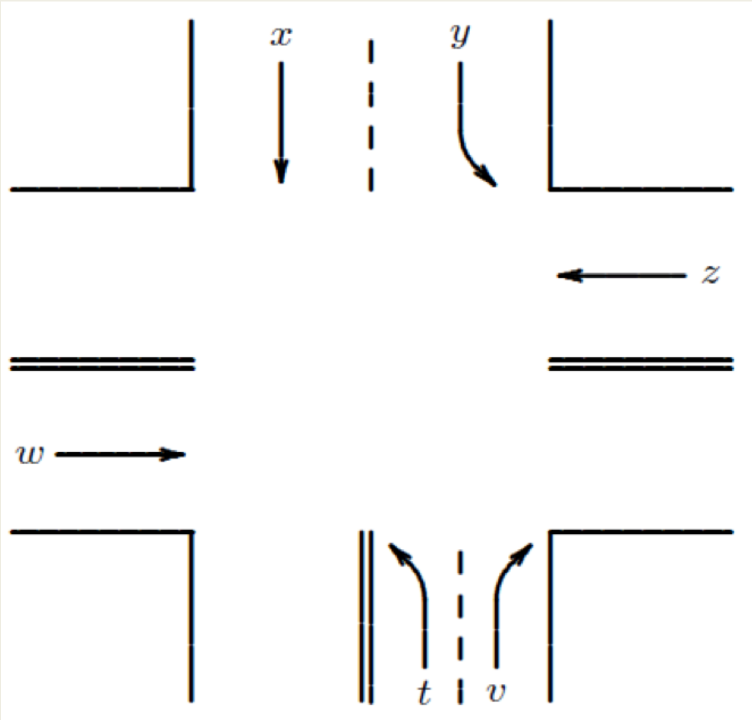
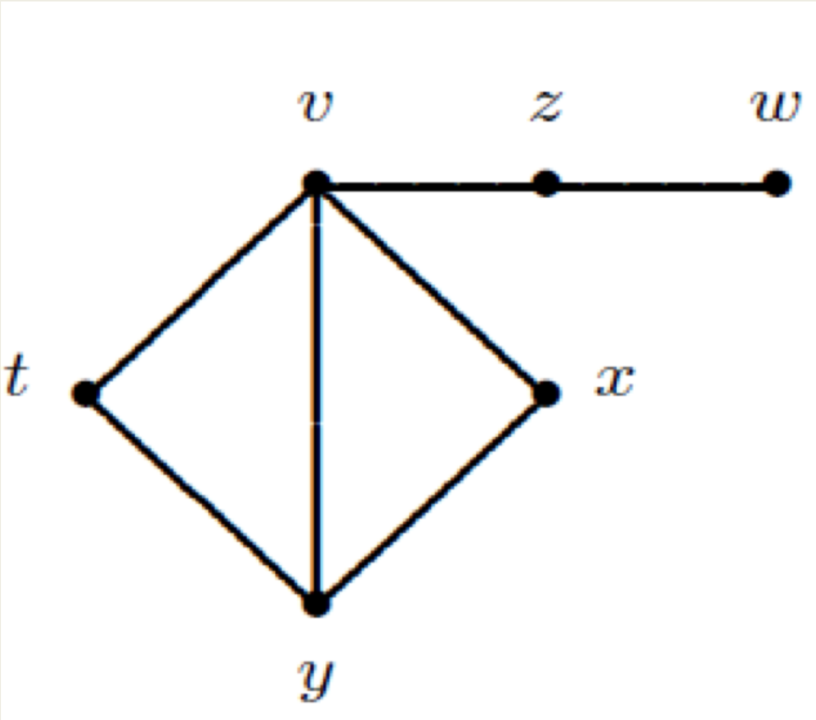


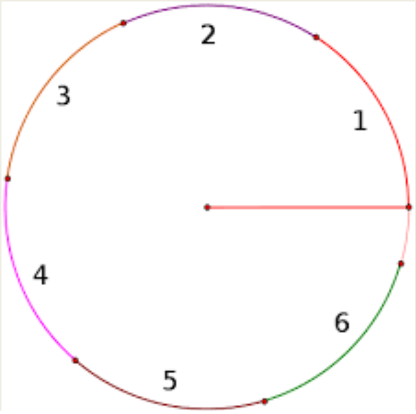
Figure 1



Compatibility graph G

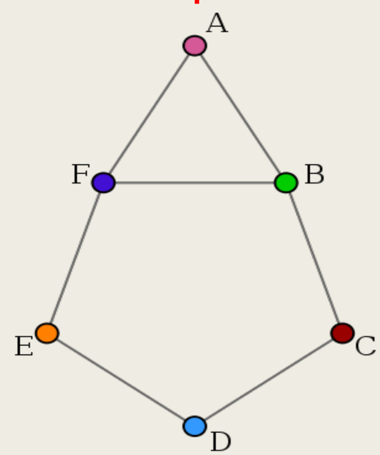
The Problem in Depth

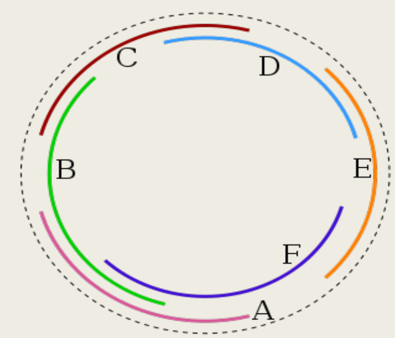
- We want to assign a period of time to each stream during which it receives a green light, whereas other streams are red
- We my think of a large clock, and the time during which a given traffic stream gets a green light corresponds to an arc on the circumference of the clock circle.
- There is a cycle of green and red lights, and then after the cycle is finished, it begins again, over and over
- lead us directly to Circular- Arc Graphs



Circular-Arc Graph and its special characteristics

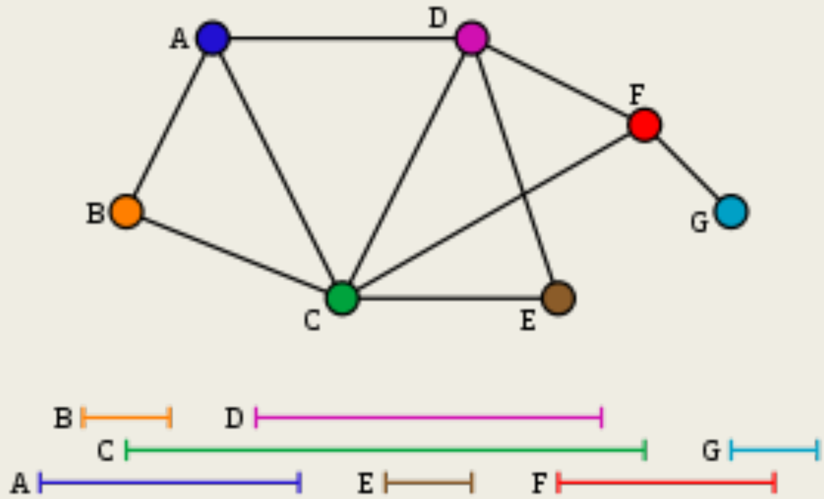
- Is the intersection graph of a set of arcs on the circle.
- Every vertex is represented by an arc, such that two vertices are adjacent if and only if the corresponding arcs overlap.
- Circular-arc graphs are not always perfect, as the odd chordless cycles C_5, C_7, etc., are circular-arc graphs.
- If a circular-arc representation fails to cover some point ***p*** on the circle, we can cut the circle at ***p*** and straighten it out to a line, the arcs becoming intervals.
- Every interval graph is a circular-arc graph. However ,the converse is false.
- **Application: they can be used to model objects of a circular or a repetitive nature.**





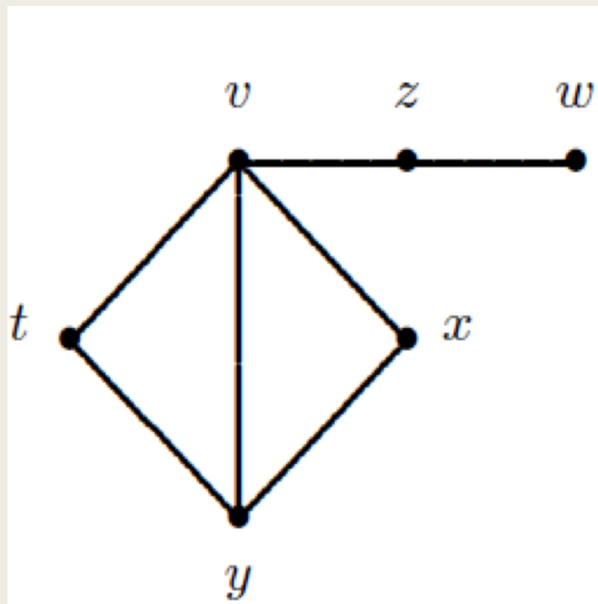
Interval Graph

- Is the intersection graph of a family of intervals on the real line.
- G is an interval graph if its vertices can be put into one-to-one correspondence with a set of intervals of a linearly ordered set.
- It is a perfect graph
- The four graph problems can be solved efficiently in linear time on Interval graphs

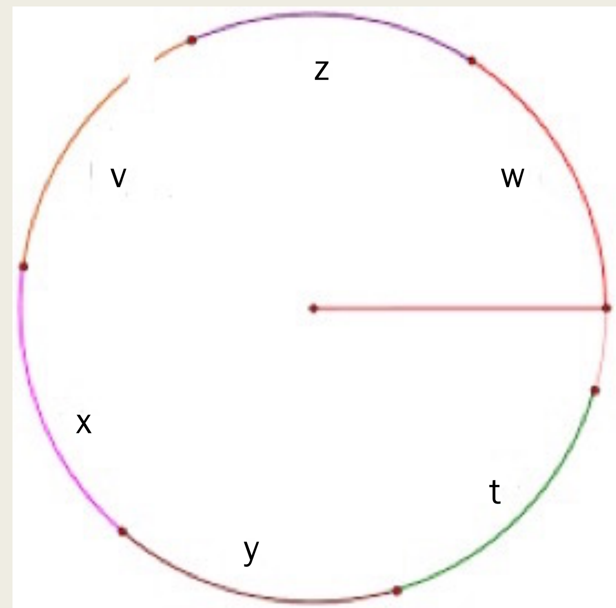


Construct a Circular-Arc Graph from the Problem

- We take a circle and assume that its perimeter corresponds to the total cycle period
- Assign an arc of the circle to each traffic stream
- Two arcs of the circle can overlap only if the corresponding streams are compatible
- It turns out that always there is a point *p on the circle has no arc*
- Any feasible representation of circular-arc graph of the problem is always **interval graph**.



Compatibility graph G



Circular-arc graph for G

Why is it Hard on General Graphs?

- Solution might not be feasible
- Repetitive nature of the problem
- Providing efficiency(Finding maximal cliques)

Make The Solution more Efficient

- We want to assign a green light for streams in such a way that only compatible traffic streams can get green light at the same time.
- We can do that by finding Maximal Cliques
- Maximal Clique is a clique that cannot be extended by including one more adjacent vertex
- Assign a green light for maximal clique Ki at a time.

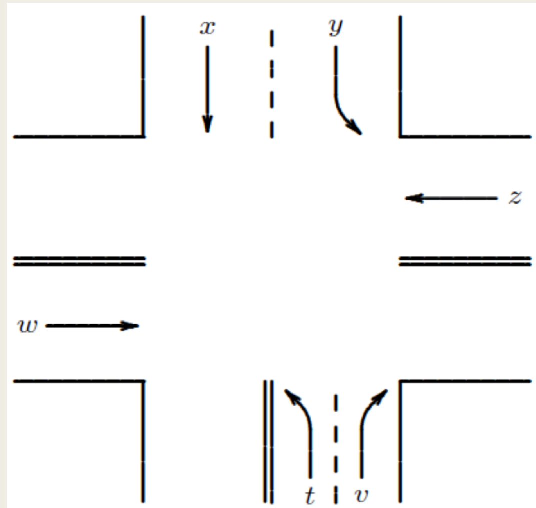
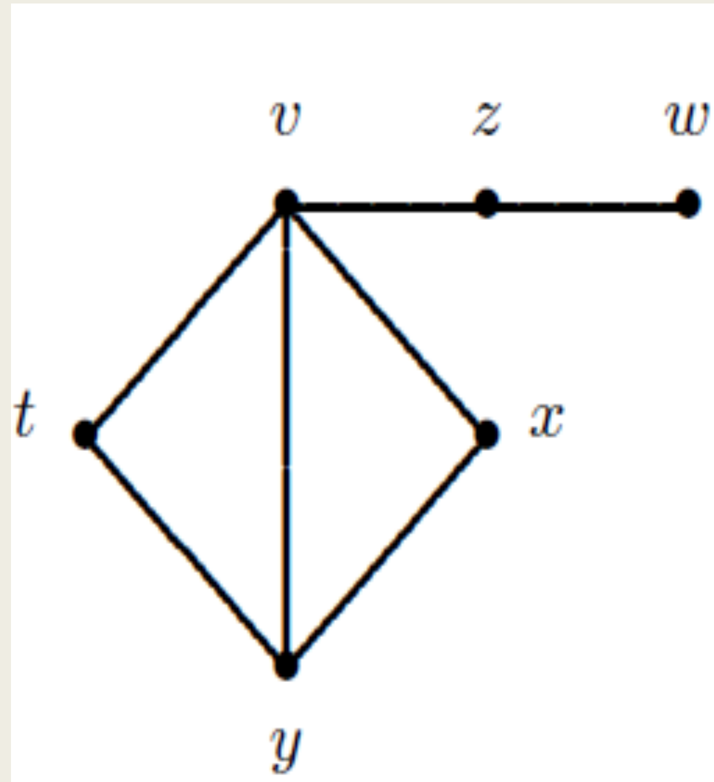


Figure 1

Special prosperities for solving the problem Efficiently

- Since the intersection graph of the phasing traffic light problem is an interval graph
- Finding maximal cliques can be done efficiently in linear time
- K1={t,v,y} K2={x,v,y} K3={v,z} K4={z,w}
- Each clique Ki corresponds to a phase during which all streams in that clique get green lights

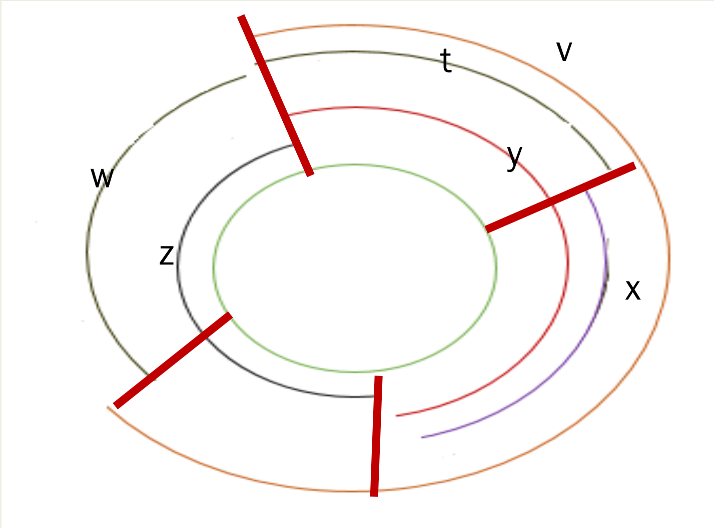


Intersection graph representation of Circular-arc graph

Interpret the graph problem's solution to the R-W problem

- Each clique Ki corresponds to a phase during which all streams in that clique get green lights
- Start a given traffic stream off with green during the first phase in which it appears, and keep it green until the last phase appears.

K1={t,v,y} K2={x,v,y} K3={v,z} K4={z,w}



Circular-arc graph for G

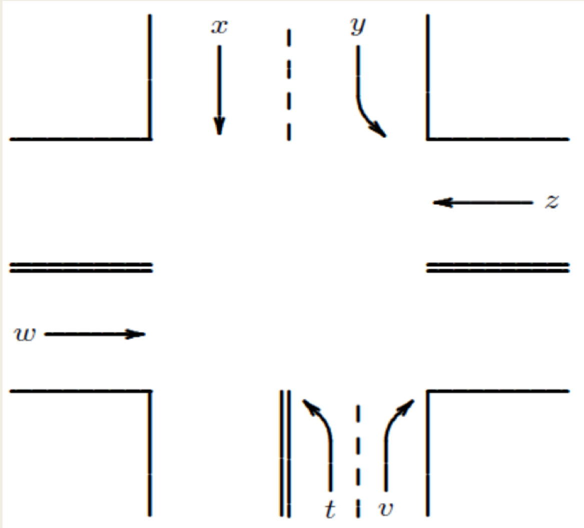


Figure 1

Resources

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Thank you…

Questions are Welcome!