Frequency Assignment For Wireless Networks

Maha A. Allouzi

Outline

- Frequency Assignment for Wireless network
- Unit disk graph *UDG(V)*
- *UDG(V)* properties
- Graph Coloring for *UDG(V)*
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Frequency Assignment for Wireless Network

- *Frequency Assignment Problems* may be modeled as optimization problems having the following form: given a collection of radio transmitters to be assigned operating frequencies, find an assignment that satisfies various constraints and that minimizes the value of a given objective function.

Unit Disk Graph

- ***Unit Disk graphs*** are intersection graphs of equal-sized disks in the plane.

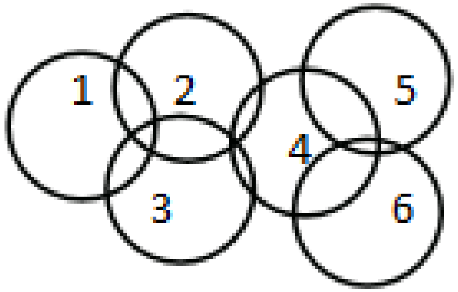


Fig1 Intersection model

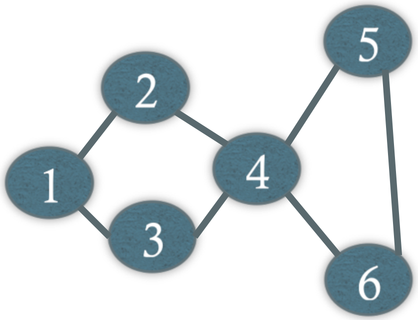
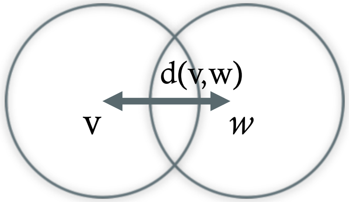


Fig2 Unit disk graph

Unit Disk Graph…

- Unit disk graph also can be described in terms of distance models, which consist of a value $d \geq 0$ and an embedding of the vertices in the plane such that *vw is an edge* iff $d(v,w) \leq d$, where $d(v,w)$ denotes the Euclidean distance of v and w in the specified embedding.



Frequency Assignment in Wireless Network & Coloring Unit Disk Graph

- It has been pointed out by Hale [4] that the problem of assigning different frequencies to nodes which are within communication range from each other can be formalized as a graph coloring problem.

- In this context the vertices of the graph G represent transmitters of the same power in the broadcast network, and two transmitter may interfere if they have a distance of at most d, for some d ≥ 0. In the simplest setting, interfering transmitters should be given different frequencies.

- Since the spectrum available to broadcast services is limited resource, we would also like to keep the number of channels used in a valid channel assignment of a given network as small as possible. Obviously this task can be formulated as a graph coloring problem, on the underlying UD graph.

Unit Disk Graph Properties

- Clark et al [6] proved That coloring problem remains NP-complete on UD graphs.
- In unit disk graph any induced subgraph is also a unit disk graph.
- UD graphs are not necessary perfect, for instance C_5 the cordless cycle with five vertices , is a UD but it not perfect as $\chi(C_5) = 3 > 2 = w(C_5)$.

Unit Disk Graph Properties

Lemma 1 *let C be a circle of radius r and let S be a set circles of radius r such that every circle in S intersects C and no two circles in S intersect each other. Then |S|≤ 5.*

Proof for lemma 1: Suppose |S|≥ 6. let s_i, 1≤ i ≤ 6 , denote the centers of any six circles in S. Let c denote the center of C. Denote the ray c s_i by r_i (1≤ i≤ 6) . Since there are six rays emanating from c, there must at least one pair of rays r_j and r_k such that the angle between them is at most 60°.Now , it can be verified that the distance between sj and s_k is at most 2r which implies that circles centered at s_j and s_k intersect, contradicting our assumption. Thus |S|≤ 5

Unit Disk Graph Properties

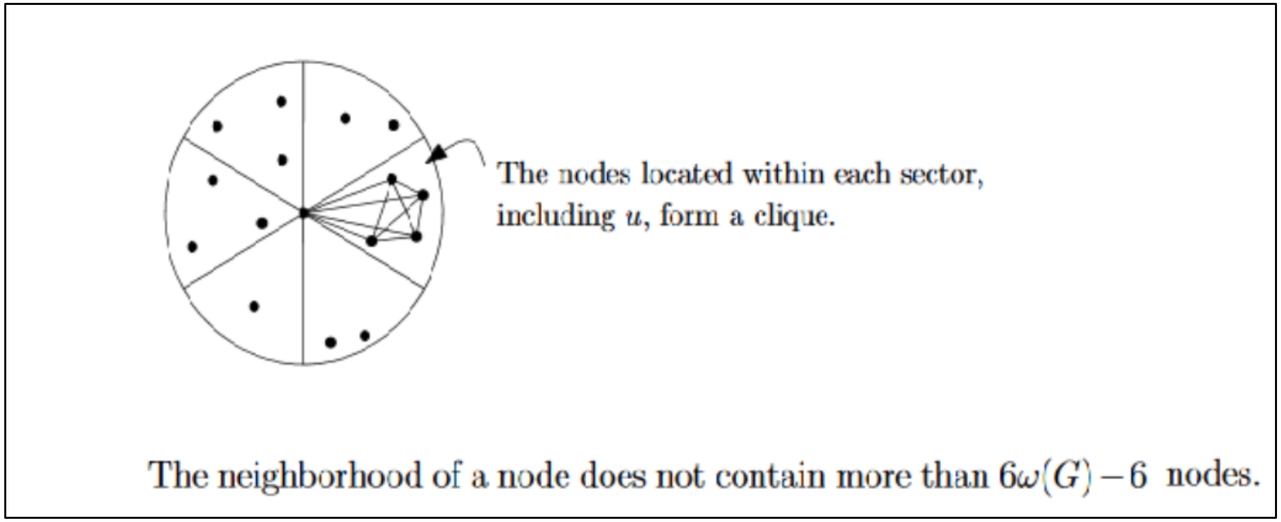
- **Lemma2**: let G(V,E) be a unit disk graph , then G cannot contain an induced subgraph isomorphic to $K_{1,6.}$
- **Lemma3**: let G be a unit disk graph, and let v be a vertex such that the unit disk corresponding to v has the smallest X-coordinate. The size of the independent set in G(N(v)) is at most 3.
- **Lemma 4** a unit disk graph G with maximum node degree Δ contains ***a clique*** of size at least $\Delta/6 + 1$

Graph Coloring

- A standard approach used in the context of coloring graphs is the *sequential coloring algorithm*.
- A sequential coloring algorithm takes a graph as input, computes some ordering on the nodes, and greedily assigns colors to nodes according to that order. Each node is assigned the lowest color that has not been assigned to any of its neighbors.
- We denote the maximum degree of a graph G by $\Delta(G)$, and the size of the largest clique in G (the clique number by $w(G)$, since the number of colors used by a sequential coloring algorithm cannot exceed $\Delta(G)+1$.On the other hand, since no two nodes in a clique can have the same color, we have that $\chi(G) \geq \omega(G)$, we have that $\chi(G) \leq \Delta(G)+1$

Unit Disk Graph Coloring

- It is easy to see that vertex degree for unit disk graph are bounded from above by $\Delta(G) \leq 6w(G) -6$. this implies that all sequential unit disk graph coloring algorithms have a performance ration of at most six.



- Hence G can be colored using at most 6w(G)-6 colors using any variation of the sequential coloring algorithm (consider the vertices in a given order and always assign the “least” color which is allowed at a given vertex).

Unit Disk Graph Coloring Algorithms

- ***Lexicographic ordering***: is the one induces by the(x, y) coordinate of the nodes, nodes with smaller x-coordinate are colored first , with ties broken according to the y-coordinate. Peeters [3] showed that the lexographical ordering archives a performance ratio of three because for every node u, no more than $3w(G) - 3$ neighbors of u will choose their color before u.

Unit Disk Graph Coloring Algorithms

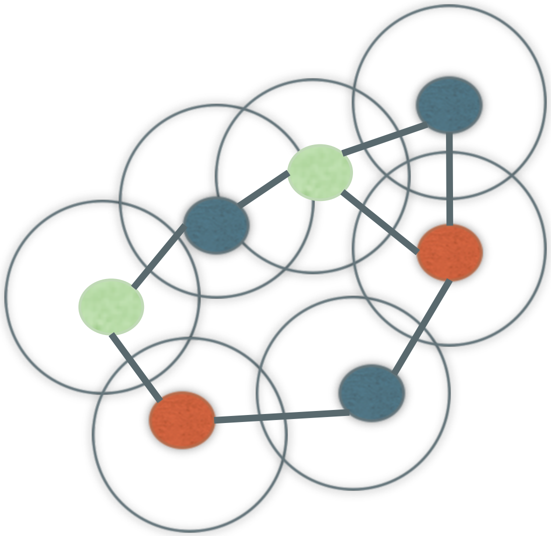
- ***The smallest-last*** coloring algorithm colors the node with the minimum degree last. The rest of the ordering is computed recursively on the graph G/{v}.
- The smallest-last achieves a performance ratio of at most **three** over unit disk graphs. However this algorithm is not distributed.

Unit Disk Graph Coloring Algorithms

	distributed	location oblivious	worst-case perf. ratio
sequential	yes	yes	5
lexicographic	yes	no	3
smallest-last	no	yes	3

Back to Frequency Assignment

- F1 == 
- F2 == 
- F3 == 



References

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Questions!