Subgraph Isomorphism and its Application to Layout-Versus-Schematic Verification

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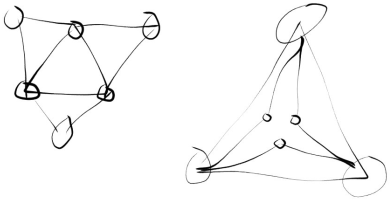
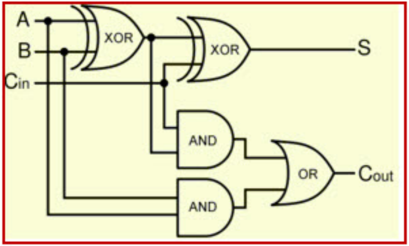
Graph Isomorphism

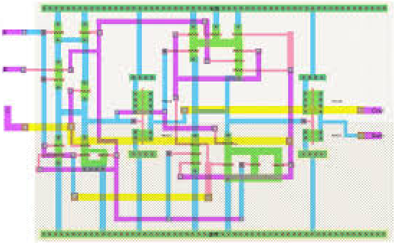
Two graphs $G = (V_1, E_1)$ and $H = (V_2, E_2)$ are called isomorphic iff there exists a bijection $f : G \to H$ that preserves vertex adjacency, that is, for all $u, v \in V_1$
$[f(u), f(v)] \in E_2 \iff [u, v] \in E_1$



Layout v.s. Schematic (LVS) Problem

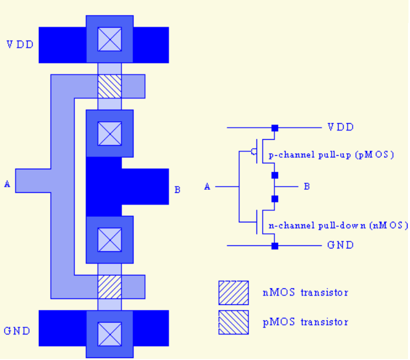
- Schematic (left) shows interconnection of electrical components
- Layout (right) shows geometry of integrated circuit
- Do layout and schematic describe the same circuit?

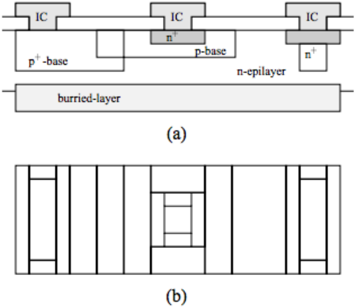




Graph Representation of Layout

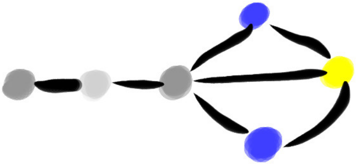
- Layout consists of multiple stacked masks
- Each mask is set of regions colored by material





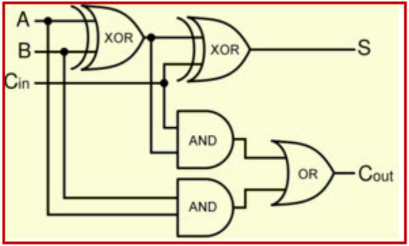
Graph Representation of Layout

- Each region of a given material is represented by a vertex
- Junctions between regions are represented by edges
- Vertices are colored according to material

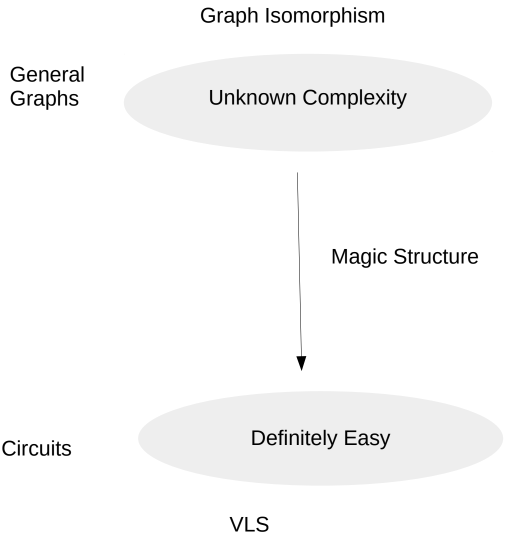


Graph Representation of Schematic

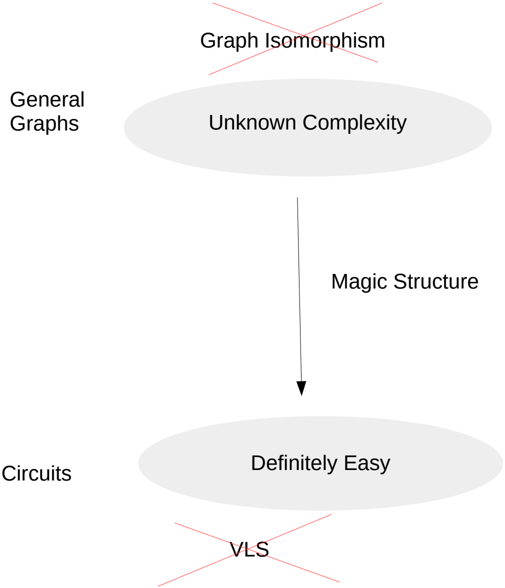
- Each conductor is vertex (called net)
- Each pin is vertex
- (Idea) connect layout graph for components to pins



The Plan


Graph Isomorphism
General
Graphs
Unknown Complexity
Magic Structure
Circuits
Definitely Easy
VLS

Plan Foiled


Graph Isomorphism
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Sub-Circuit Extraction

- Find a small circuit in a larger circuit
- Serves as a subroutine for practical VLS
- Related to subgraph isomorphism problem

Subgraph Isomorphism Problem

Given two graphs G and H, determine whether G contains a subgraph H' which is isomorphic to H.

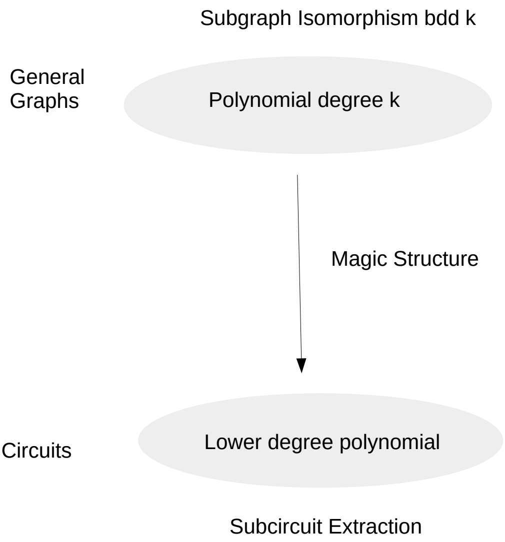
- NP Hard: we can use subgraph isomorphism to search for a clique of size k. Therefore, subgraph isomorphism is at least as hard as max clique.
- If $|H| < k$ then complexity is reduced to $O(|H|^k)$.
- If G has the right structure (in addition to fixing $|H|$), complexity may be reduced further.

Brute Force Algorithm

Check all possible vertex assignments.

- There are $\frac{|E_G|!}{(|E_G|-|E_H|)!}$ possible vertex assignments
- If $|E_H| < k$, this becomes $(|E_G|) * (|E_G| - 1) * ...(|E_G| - k + 1)$ which is $\Theta(|E_G|^k)$
- We can check each vertex assignment for isomorphism with $|E_H|$ edge lookups.

The New Plan


Subgraph Isomorphism bdd k
General
Graphs
Polynomial degree k
Magic Structure
Circuits
Lower degree polynomial
Subcircuit Extraction

Outlook Seems Bright

- Linear algorithm for planar graphs (Eppstein)
- Linear algorithm for graphs of bounded genus (Eppstein)
- "A linear matching time performance of the SCE algorithm is achievable, to the testing circuits that possess EU local distinguishability." (Ling)

Planning Method

- Method from [Ling]
- Edge units in H (pattern graph) are tasks
- Edge units in G (target graph) are resources
- Assign tasks to resources subject to constraints

Constraints

- Pattern vertex must be assigned to host vertex with greater or equal degree.
- Vertices must share labels.
- Edge units that share a vertex in the pattern graph must share the same vertex in the target graph.

Scanning for isomorphic assignments

- In contrast to brute force algorithm, do not check every vertex assignment (or edge assignment).
- Instead, scan the decision tree of valid edge unit assignments, backtracking when no further assignments are possible.

Eliminating starting points

- First, compute center spanning tree of pattern graph
- Choose center as starting vertex in pattern graph
- Eliminate starting vertices in host graph with different label
- Eliminate starting vertices in host graph with eccentricity less than that of the center in the pattern graph.

Criticality and Cruciality

- Criticality: the number of valid host edge vertex assignments to a given pattern edge unit
- Cruciality: given a pairing of edge units (one in patter, one in host) the cruciality is the number of conflicting pairings.

Search Idea: choose the tasks with least criticality and choose the resources with least cruciality.

Performance

- left shows linear performance in size of pattern graph (surprising)
- right shows performance in model graph

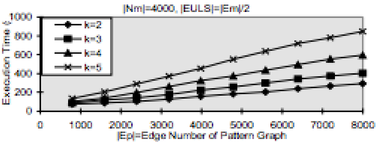


Figure 2. Near-Linear Performance

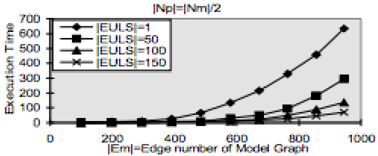


Figure 3. Impact of Label Sets

We didn't quite make it

- Algorithm is heuristic, efficiency claims are experimental
- Algorithm appears to serve general graphs
- What is EU local distinguishability?

References

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