Addressing, Distances and Routing in Triangular Systems with Application in Cellular and Sensor Networks

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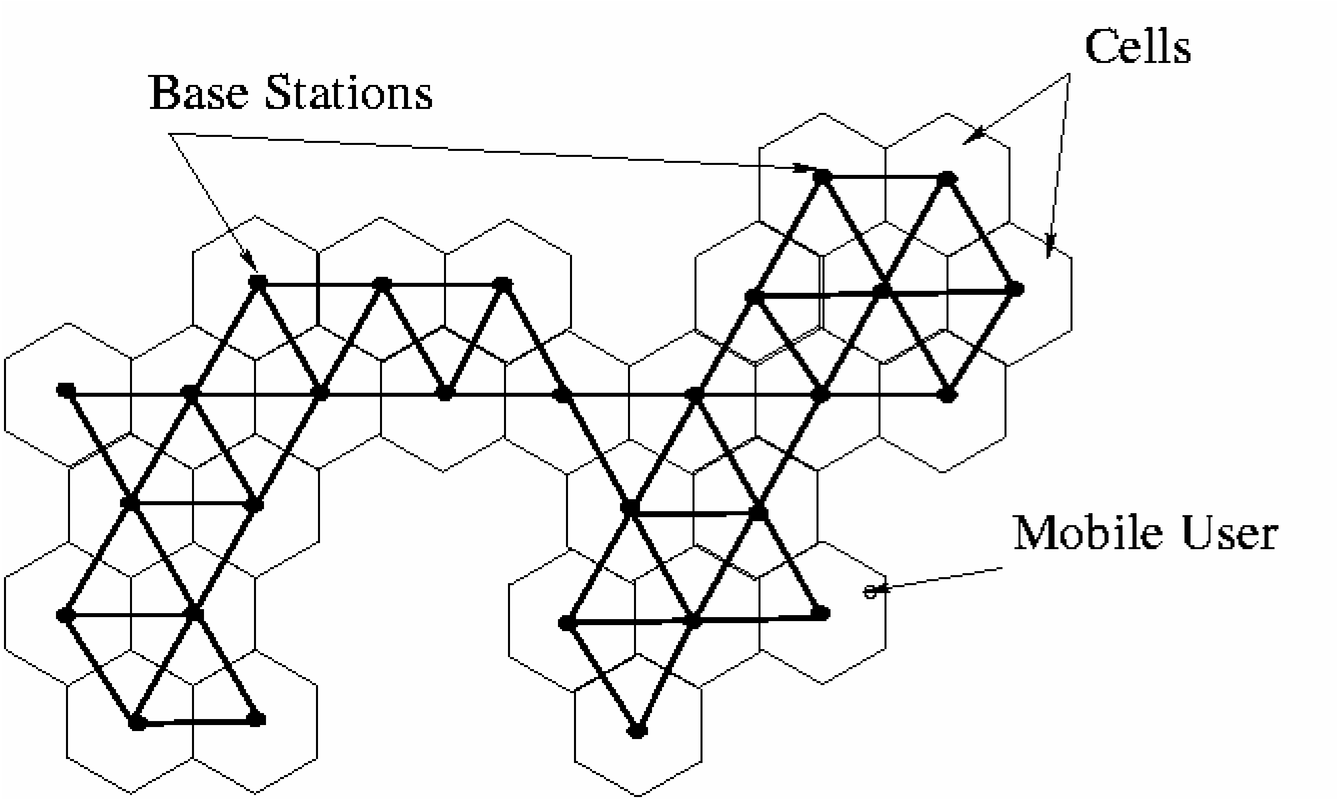
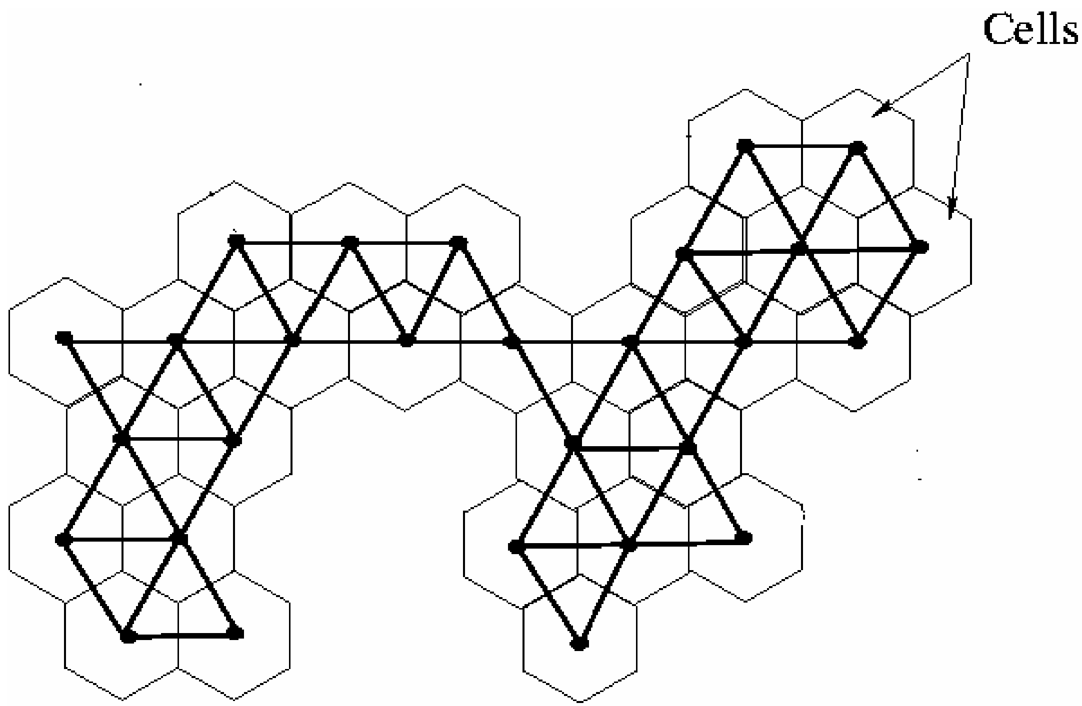
Cellular Network



Benzenoid and Triangular Systems

- Benzenoid Systems: is a simple circuit of the hexagonal grid and the region bounded by this circuit.



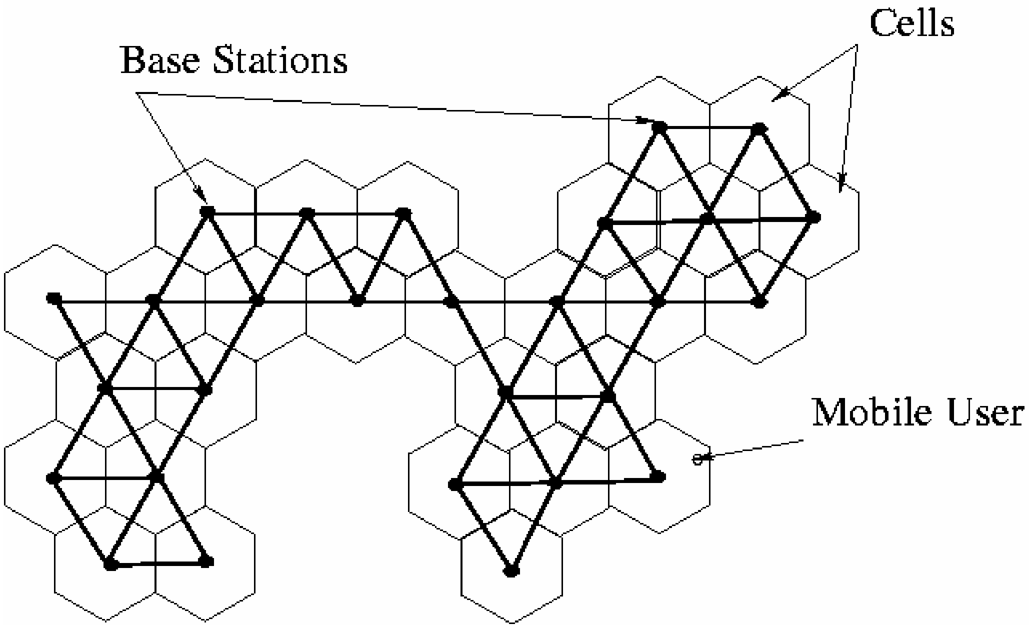
- The Duals to Benzenoid Systems are Triangular Systems

Addressing, Distances and Routing in Triangular Systems: Motivation

Applications in cellular networks

- Identification code (CIC) for tracking mobile users
- Dynamic location update (or registration) scheme
  - time based
  - movement based
  - distance based

(cell-distance based is best, according to [Bar-Noy&Kessler&Sidi'94])



- ➔Distances
- Routing protocol

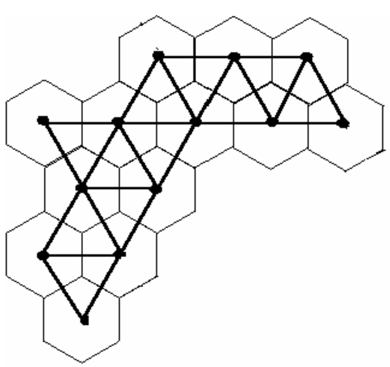
Current situation

- Current cellular networks do not provide information that can be used to derive cell distances
  - It is hard to compute the distances between cells (claim from [Bar-Noy&Kessler&Sidi'94])
  - It requires a lot of storage to maintain the distance information among cells (claim from [Akyildiz&Ho&Lin'96] and [Li&Kameda&Li'00])

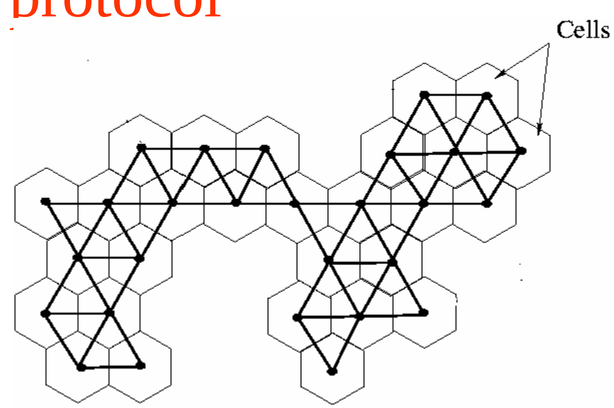
Recent results

[Nocetti&Stojmenovic&Zhang'02] recently considered isometric subgraphs of the regular triangular grid and give among others

- A new cell addressing scheme using only three small integers, one of them being zero
- A very simple method to compute the distance between two sells
- A short and elegant routing protocol



isometric



not isometric

Recent results

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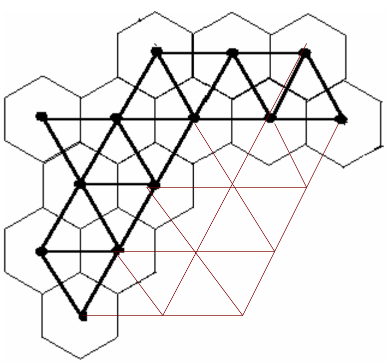
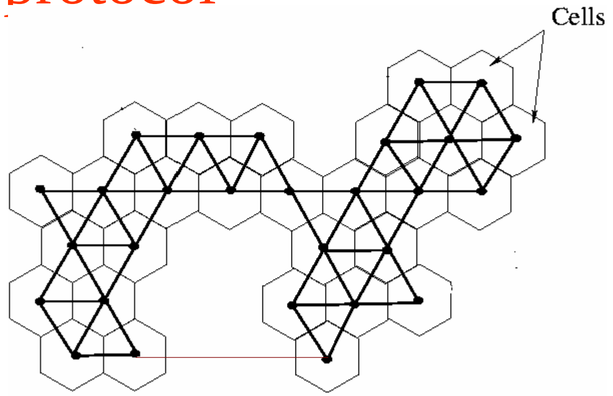


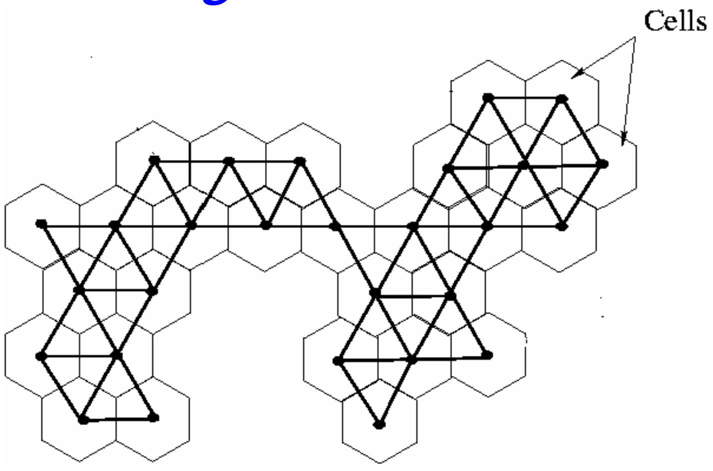
isometric



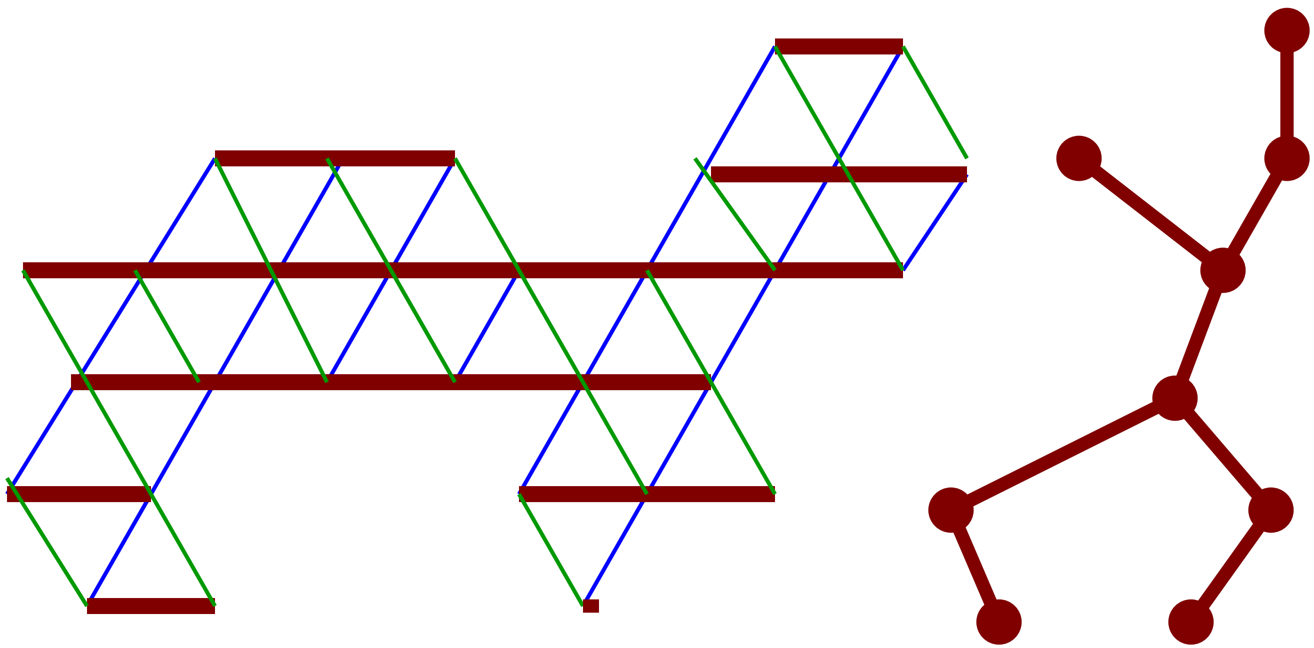
not isometric

Our results for triangular systems

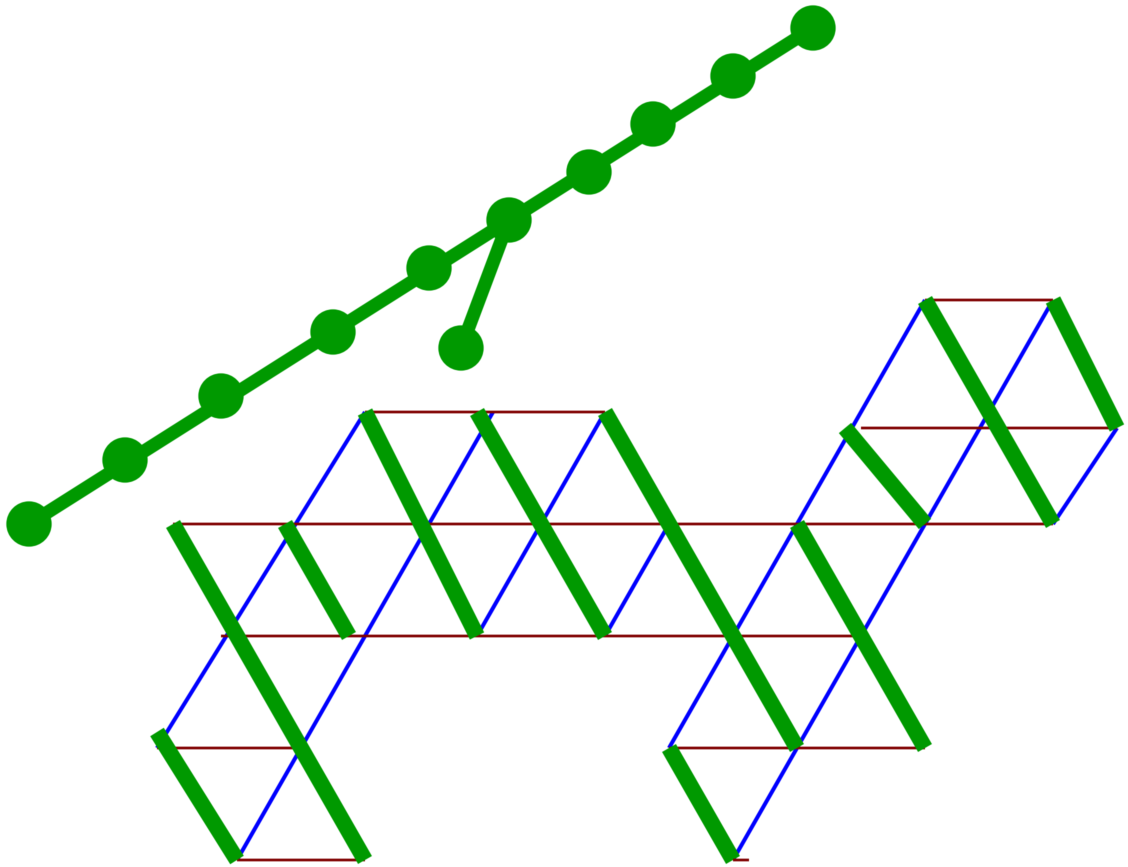
- Scale 2 isometric embedding into Cartesian product of 3 trees
  - ➔ cell addressing scheme using only three small integers
  - ➔ distance labeling scheme with labels of size $O(\log^2 n)$ -*bits per node and constant time distance decoder*
  - ➔ routing labeling scheme with labels of size $O(\log n)$*-bits per node and constant time routing decision.*



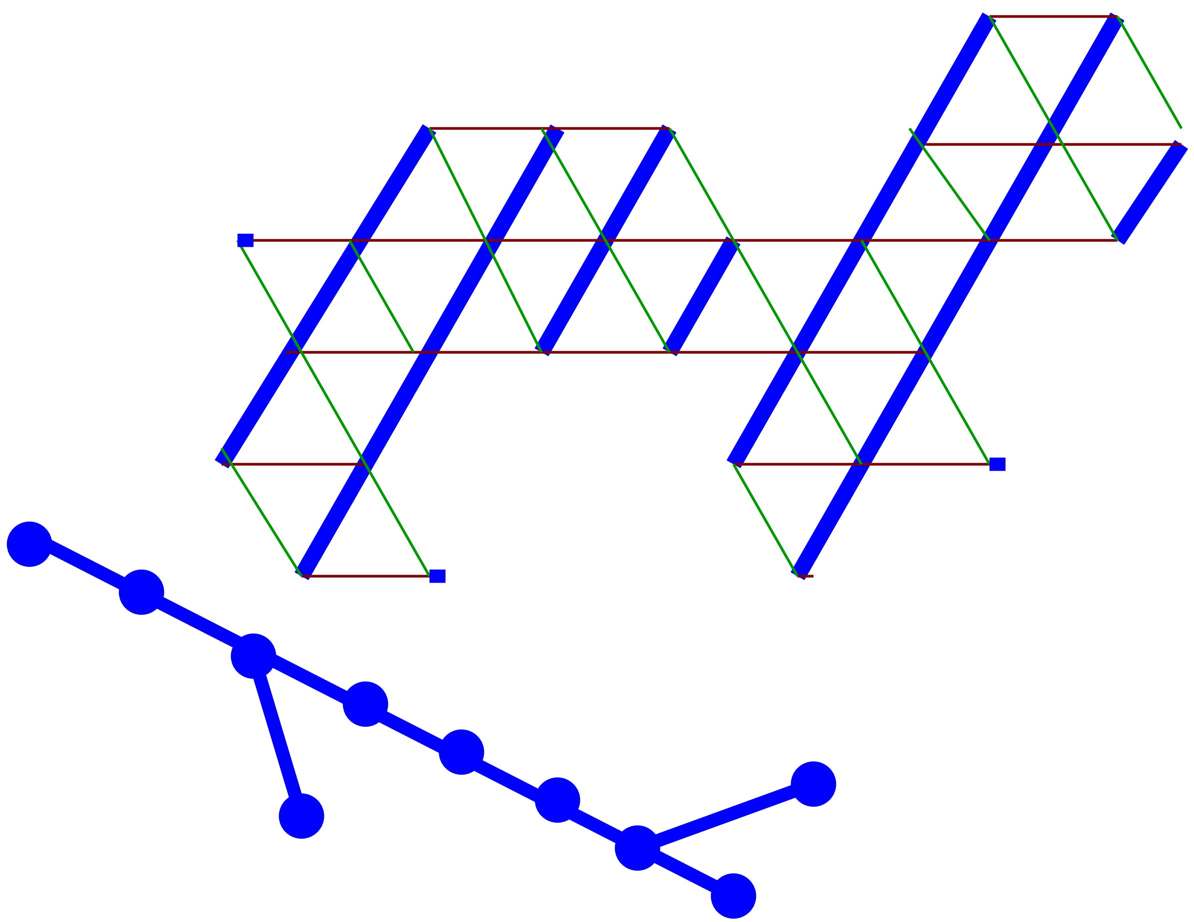
Three edge directions➔three trees



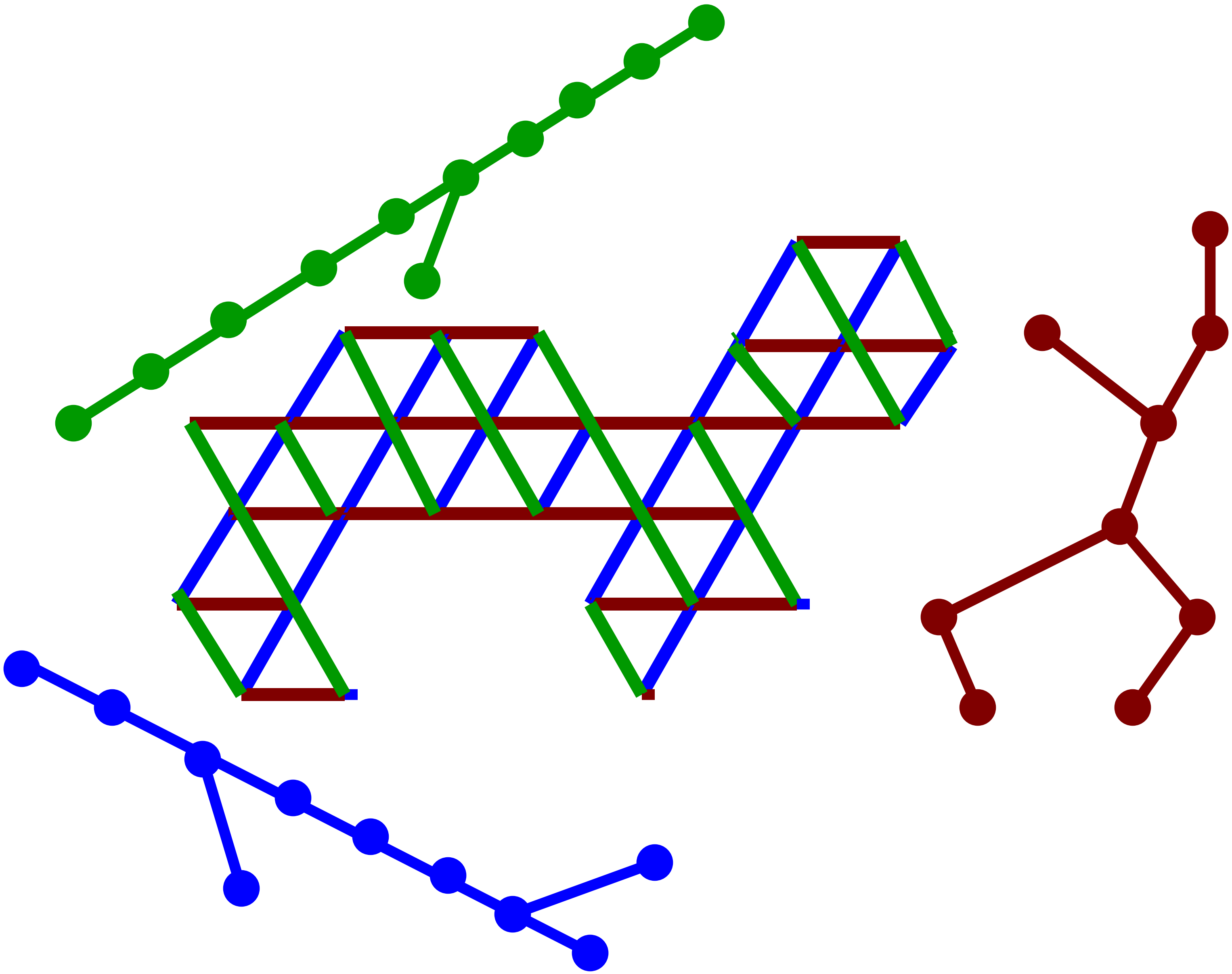
Three edge directions→three trees



Three edge directions→three trees

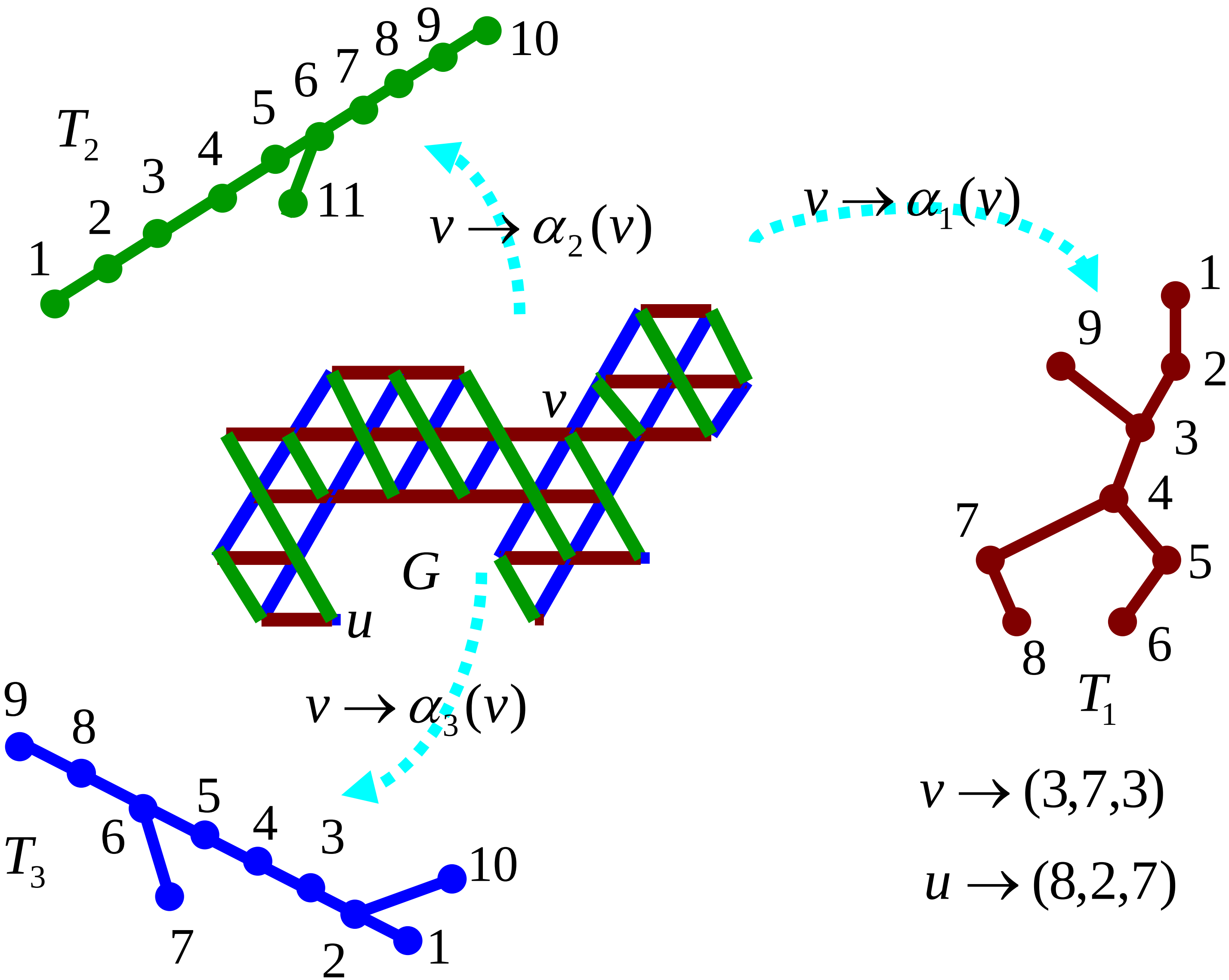


Three edge directions➔three trees



Addressing

$$v \to (\alpha_1(v), \alpha_2(v), \alpha_3(v))$$



$$v \to (3,7,3)$$

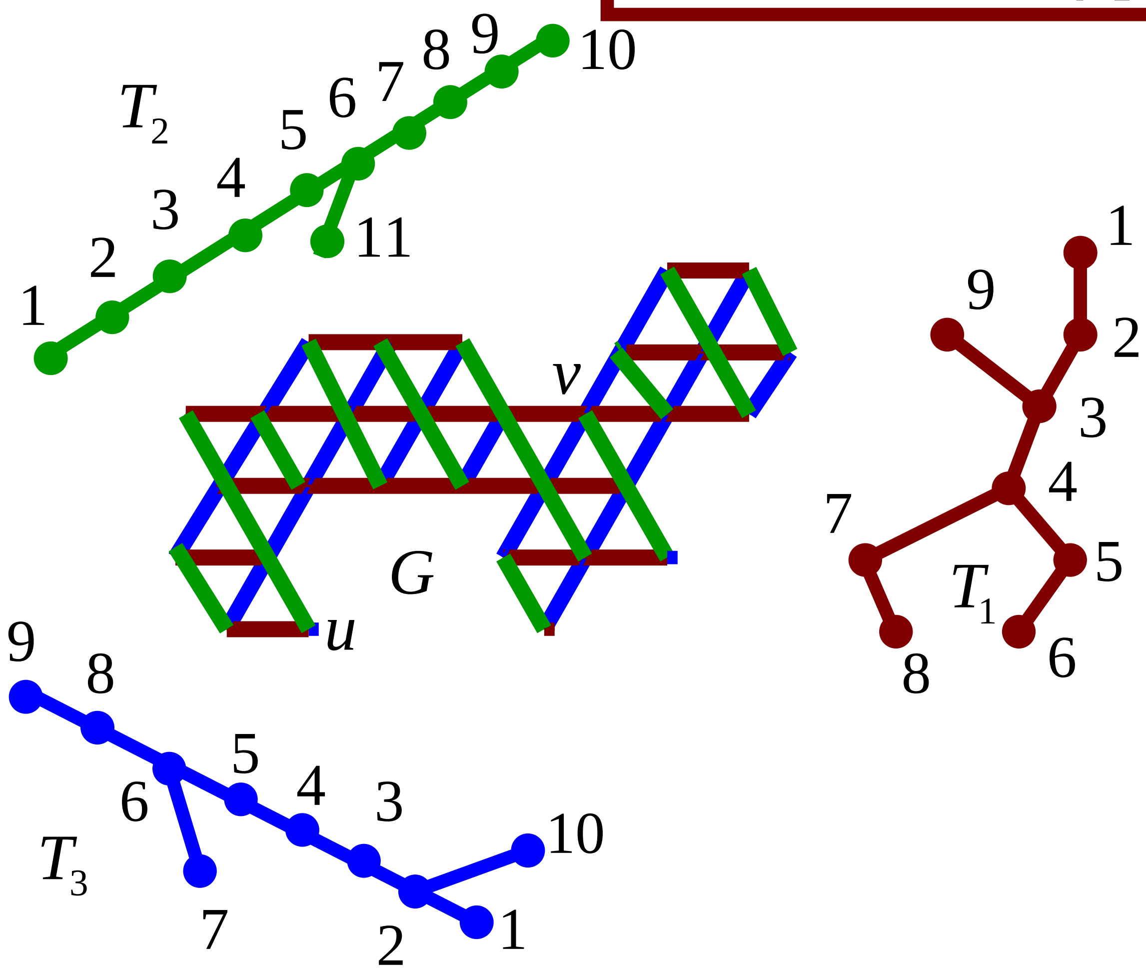
$$u \to (8,2,7)$$

Scale 2 embedding into 3 trees

$$v \to (\alpha_1(v), \alpha_2(v), \alpha_3(v))$$

$$u \to (\alpha_1(u), \alpha_2(u), \alpha_3(u))$$

$$2dist_G(v,u) = \sum_{i=1}^{3} dist_{T_i}(\alpha_i(v), \alpha_i(u))$$



$$v \to (3,7,3)$$

$$u \to (8,2,7)$$

$$dist_{T_1}(\alpha_1(v), \alpha_1(u)) +$$
$$dist_{T_2}(\alpha_2(v), \alpha_2(u)) +$$
$$dist_{T_3}(\alpha_3(v), \alpha_3(u)) =$$
$$3 + 5 + 4 = 12 =$$
$$2dist_G(v,u)$$

Distance Labeling Scheme

Goal: Short labels that encode distances and distance decoder, an algorithm for inferring the distance between two nodes only from their labels (in time polynomial in the label length)

- **Labeling:** *v* → *Label(v)*

  (for trees, $O(\log^2 n)$ *bits per node* [Peleg'99]*)*

- **Distance decoder:** *D(Label(v), Label(u))* → *dist(u,v)*

  (for trees, constant decision time)

Distance labeling scheme for triangular systems

- Given *G*, find three corresponding trees T_1, T_2, T_3 and addressing $v \rightarrow (\alpha_1(v), \alpha_2(v), \alpha_3(v))$ (*O(n) time*)

- Construct distance labeling scheme for each tree

  $\alpha_i(v) \rightarrow Label(\alpha_i(v))$ (*O(n*log*n) time*)

- Then, set $Label(v) = (Label(\alpha_1(v)), Label(\alpha_2(v)), Label(\alpha_3(v)))$

- $O(\log^2 n)$ -bit labels and constructible in total time $O(n \log n)$

Distance decoder for triangular systems

Given *Label(u)* and *Label(v)*

Function distance_decoder_triang_syst(Label(u),Label(v))

- Output ***½(distance_decoder_trees(*** $Label(\alpha_1(v)), Label(\alpha_1(u))$ ***)***

  +(distance_decoder_trees($Label(\alpha_2(v)), Label(\alpha_2(u))$ ***)***

  +(distance_decoder_trees($Label(\alpha_3(v)), Label(\alpha_3(u))$ ***))***

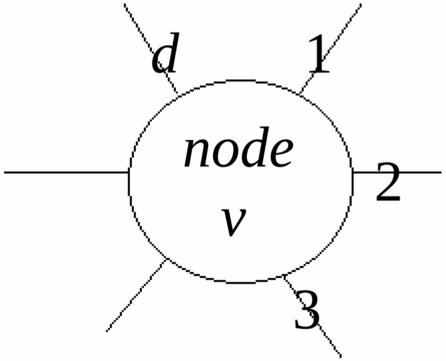
Thm: The family of *n*-node triangular systems enjoys a distance labeling scheme with $O(\log^2 n)$ -bit labels and a constant time distance decoder.

Routing Labeling Scheme

Goal: Short labels that encode the routing information and routing protocol, an algorithm for inferring port number of the first edge on a shortest path from source to destination, giving only labels and nothing else

- **Labeling:** *v* → *Label(v)*
- **Distance decoder:** *R(Label(v), Label(u))* → *port(v,u)*

(for trees, $O(\log n)$ *bits per node and constant time decision* [Thorup&Zwick'01]*)*



Routing labeling scheme for triangular systems

- Given *G*, find three corresponding trees T_1, T_2, T_3 and addressing $v \to (\alpha_1(v), \alpha_2(v), \alpha_3(v))$

- Construct routing labeling scheme for each tree using Thorup&Zwick method (*log n* bit labels)

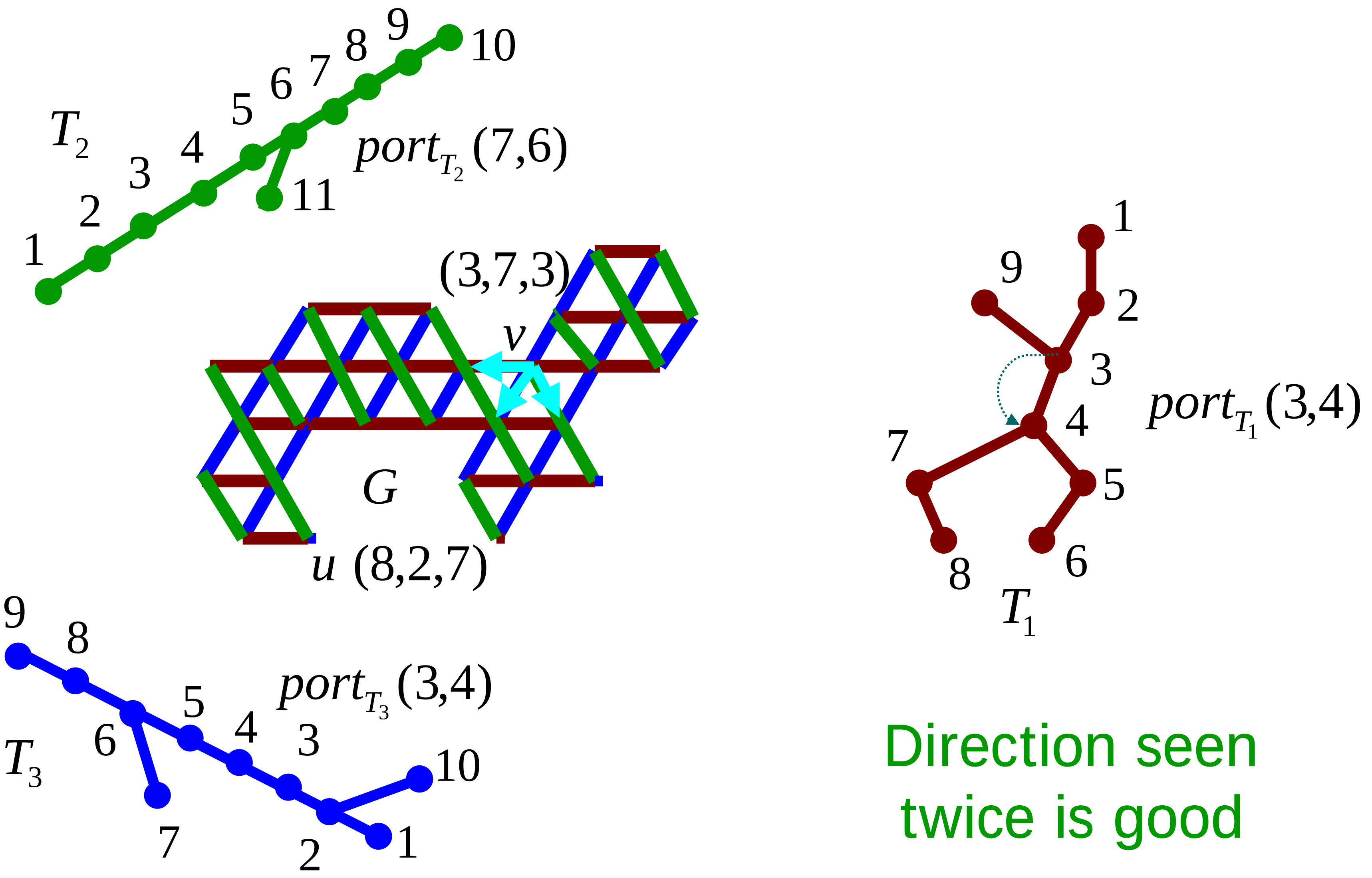
$$\alpha_i(v) \to Label(\alpha_i(v))$$

- Then, set

$$Label(v) = (Label(\alpha_1(v)), Label(\alpha_2(v)), Label(\alpha_3(v)), \ldots.)$$

Something more

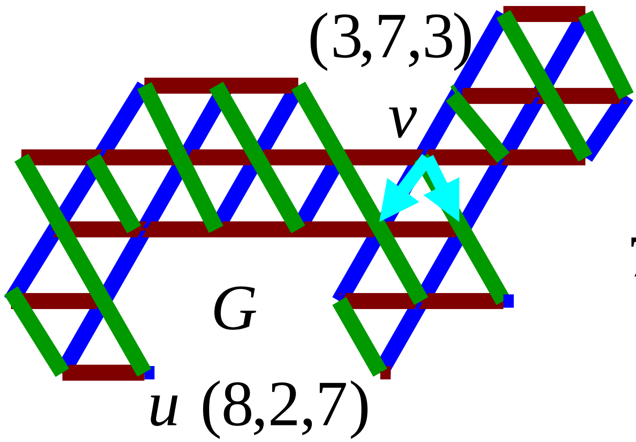
Choosing direction to go from v

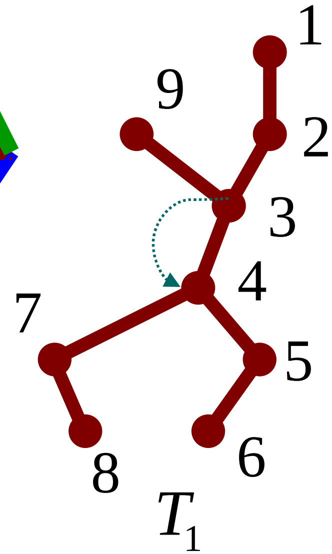


Direction seen
twice is good

Mapping tree ports to graph ports

$$\Delta_i(v) = (\alpha_i^1(v),\ldots,\alpha_i^j(v)) = (2,4) \quad (j \le 4)$$





$$O_i(v) = ((port_{T_i}(\alpha_i(v),\alpha_i^1(v)), Q_i^1(v)),\ldots,(port_{T_i}(\alpha_i(v),\alpha_i^j(v)), Q_i^j(v)) =$$

$$((port_{T_1}(3,2), Q_1^1(v)),\ldots,(port_{T_1}(3,4), Q_1^2(v))$$

$$Q_i^j(v) = (port_G^1, port_G^2)$$

Then, $Label(v) = (Label(\alpha_1(v)), Label(\alpha_2(v)), Label(\alpha_3(v)),\ldots.)$

(i.e., *3xlog n+3x4x3xlogn* bit labels)

Routing Decision for triangular systems

Given *Label(u)* and *Label(v)*

```
function routing_decision_triang_syst(L(x), H(y))

if (α_1(x), α_2(x), α_3(x)) = (α_1(y), α_2(y), α_3(y)) then return "packet reached its destination";
set A ← 0;
for each i ∈ {1,2,3} do
   p ← routing_decision_trees(L_{T_i}(α_i(x)), H_{T_i}(α_i(y)));
   for each j ∈ {1,...,|O_i(x)|} do
      if p = O_i(x)[j] then
         for each entry port_G of the array Q_i^j(x) do
            A[port_G] ← A[port_G] + 1;
            if A[port_G] = 2 then
               return port_G.
```

Thm: The family of n-node triangular systems enjoys a routing labeling scheme with $O(\log n)$-bit labels and a constant time routing decision.

We showed for triangular systems

- Scale 2 isometric embedding into Cartesian product of 3 trees
  - ➔ cell addressing scheme using only three small integers
  - ➔ cell-distance labeling scheme with labels of size $O(\log^2 n)$-*bits per node and constant time distance decoder*
  - ➔ routing labeling scheme with labels of size $O(\log n)$-*bits per node and constant time routing decision.*

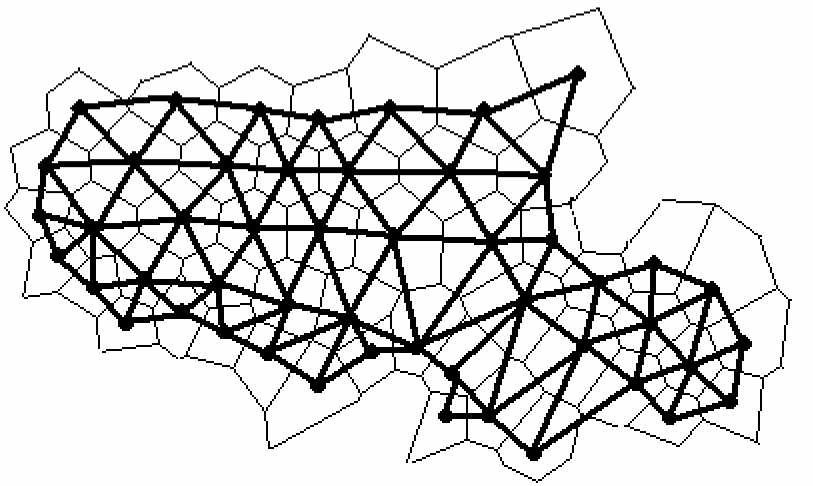
Extensions, Open Problems

- BS at arbitrary positions

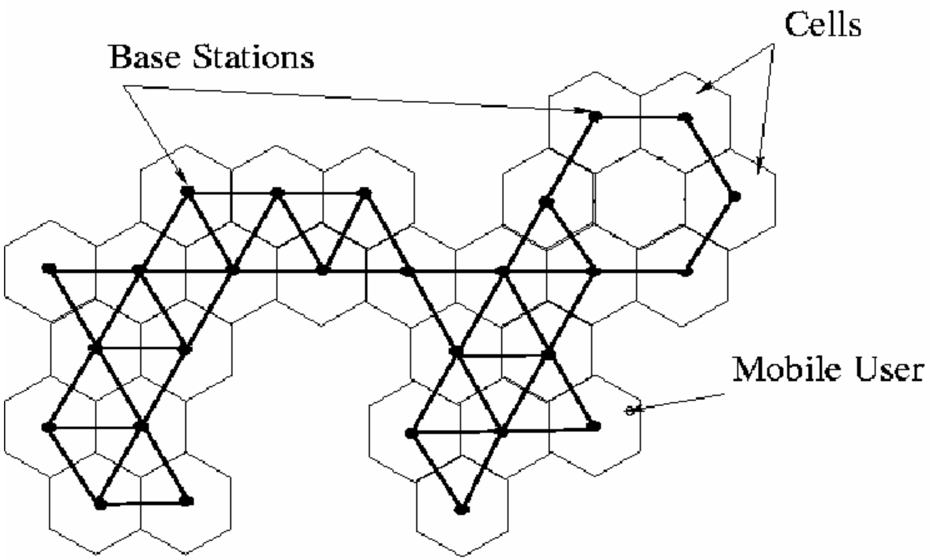
➔ BS determine

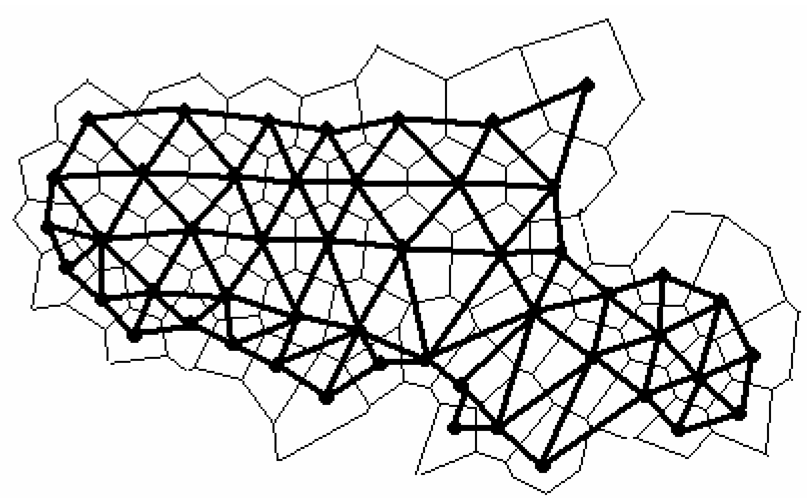
Delaunay triangulation

= dual Voronoi diagram



- Not-Simply Connected Cellular Networks



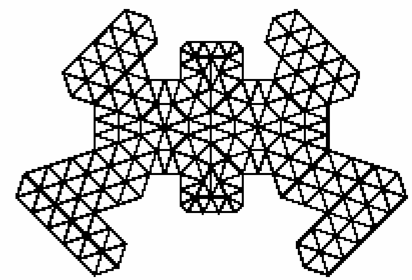


Other Results

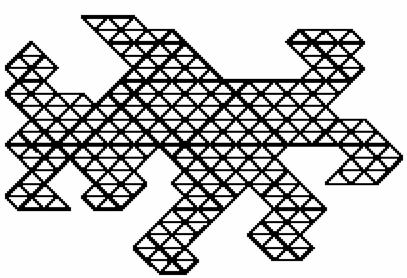
Thm: The families of n-node (6,3)-,(4,4)-,(3,6)-planar graphs enjoy distance and routing labeling schemes with $O(\log^2 n)$ -bit labels and constant time distance decoder and routing decision.

(p,q)-planar graphs:

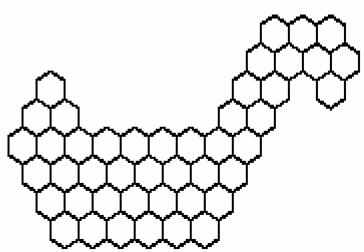
- inner faces of length at least p
- inner vertices of degree at least q



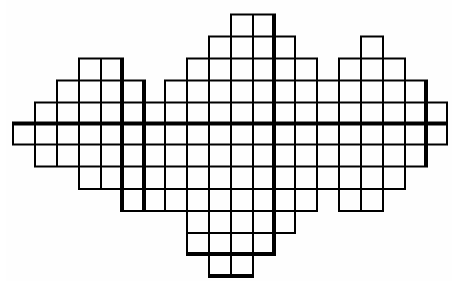
trigraph



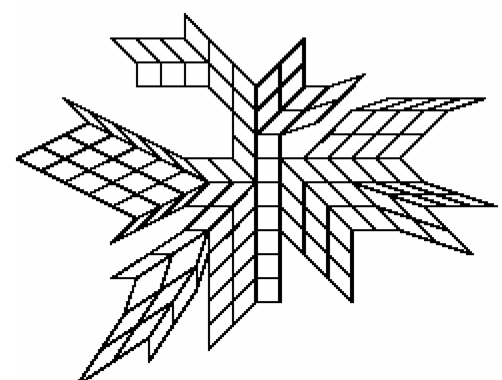
triangular system



hexagonal system



square system



squaregraph

Thank you