Distance Approximating Trees in Graphs
Brandstaedt & Chepoi & Dragan,

• A graph $G=(V,E)$
  – Vertex set $V = (v_1, v_2, ..., v_n)$
  – Edges $E \subseteq V \times V$
    • Adjacent
    • Incident
    • Path
    • Connected

• A tree $T=(V,E')$
  – connected and minimum number of edges
Distance Approximating Trees in Graphs

- Shortest path
- Distance in graphs $d_G$
- The problem: approximate $d_G$ by a simpler distance (e.g., by $d_T$)
  - applications in
    - Communication networks
    - Data analysis
    - Motion planning
    - Network design
    - Phylogeny reconstruction
    - Numerical taxonomy
Distance Approximating Trees in Graphs

- Case $E' \subseteq E \rightarrow t$-spanners

  - Multiplicative Tree $t$-Spanners
  \[ d_T(x, y) \leq t \cdot d_G(x, y) \]
  for any $x, y \in V$

  - Additive Tree $r$-Spanners
  \[ d_T(x, y) \leq d_G(x, y) + r \]
  for any $x, y \in V$
Distance Approximating Trees in Graphs

Tree Spanners

• Problem: Given $G$ and integer $t$, decide whether $G$ has a multiplicative tree $t$-Spanner.

• For all graphs (Cai & Corneil)
  • NP-complete for $t > 3$
  • Linear for $t = 1, 2$
  • Open for $t = 3$

• For special graph classes
  – Multiplicative tree 3-spanners in linear time for interval and permutation graphs (Madanlal & Venkatesan & Rangan)
  – Additive tree 2-spanners in linear time for interval and distance hereditary graphs (Prisner)
  – Additive tree 4-spanner for cocomparability graphs (Prisner)

• For chordal graphs
  – For every fixed integer $t$ there is a chordal graph without tree $t$-spanners (additive as well as multiplicative) (McKee)
  – Question: Whether strongly chordal graphs have tree $t$-spanners with small $t$ (Prisner, STACS’97)
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Our result

• Every strongly chordal graph (even every dually chordal graph) has a multiplicative 4-spanner which is also an additive 3-spanner. Such a tree can be constructed in linear time.

  – A graph $G$ is chordal if it does not contain any chordless cycle of length at least four.
  – A chordal graph is strongly chordal if it does not contain any induced sun.