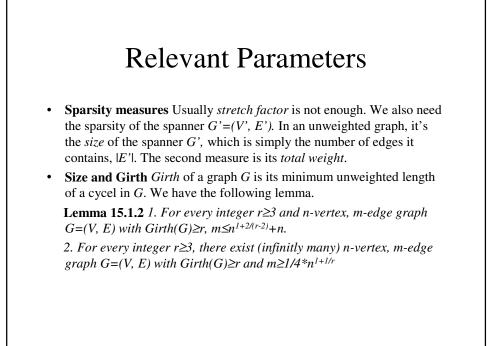
# Spanning Trees, Tree Covers and Spanners

- Consider a weighted connected graph *G*=(*V*, *E*, *w*), where *w*:*E*→*R* assigns a nonnegative weight *w*(*e*) to each edge *e*, representing its length. In the following we only consider unweighted graphs. We omit the weight function *w*. For each edge of the graph, we assume its weight as 1.
- Definition Given a graph G=(V, E, w) and a spanning subgraph G'=(V, E') of G such that E'⊆E, we define the following parameters. For a distinguished root vertex r<sub>0</sub>∈V, define the root-stretch (or simply the stretch) of G' with respect to r<sub>0</sub> as

Stretch (G', r\_0) = 
$$\max_{v \in V} \left\{ \frac{dist_{G'}(r_0, v)}{dist_G(r_0, v)} \right\}$$

The stretch factor of G' is

Stretch (G') = 
$$\max_{u,w\in V} \left\{ \frac{dist \, G'(u,w)}{dist \, G(u,w)} \right\}$$

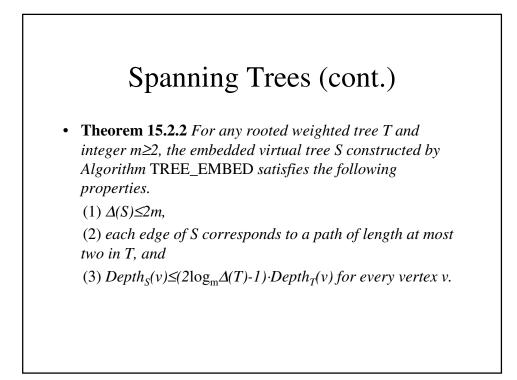


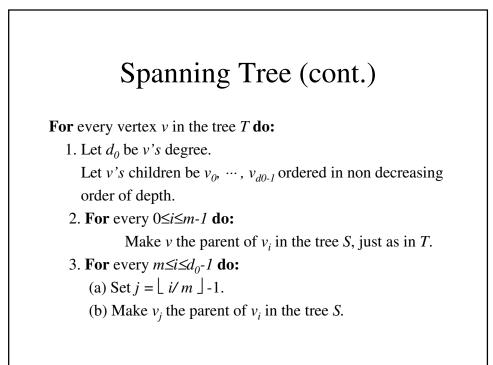
## Spanning Trees

- Definition 15.2.1 The shortest path spanning tree, or SPT, of G with respect to a given root  $r_0$  is a spanning tree  $T_s$ with the property that for every other vertex  $v \neq r_0$ , the path leading from  $r_0$  to v in the tree is the shortest possible, or in other words, Stretch( $T_s$ ,  $r_0$ )=1
- Controlling Tree Degrees Another parameter of revelance to skeletal representations involves vertex degrees. We define  $\{\Delta(G)=max\}$

 $(deg_G(v))$ ,  $\forall x \in V$ . We want this parameter to be as small as possible.

Given a tree T we run TREE\_EMBED algorithm to construct its virtual tree S



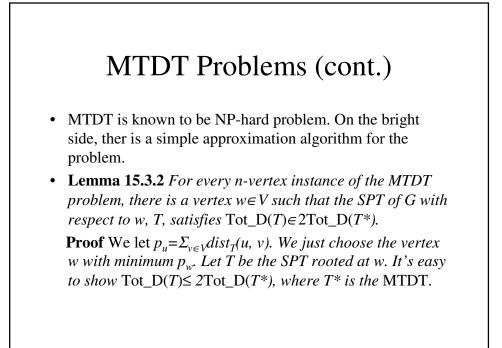


#### Minimum Total Distance Trees

• **Definition 15.3.1** For a subgraph G' spanning the graph G=(V, E), let Tot\_D(G') denote the sum of the distances between any two vertices in G', namely,

$$\operatorname{Fot}_{D}(G') = \sum_{u,v \in V} dist_{G'}(u,v)$$

The minimum total distance tree, or MTDT, of G is a spanning tree  $T_D$  minimizing Tot\_D(T) over all spanning trees T of G.



		_	_	
Good tree sp	anners are us	ually hard to	find.	
	Arbitrary Graph	Planar Graph	Chordal Graph	
t=2	Р	Р	Р	
t=3	?	Р	?	
t≥4	NP	NP	NP	

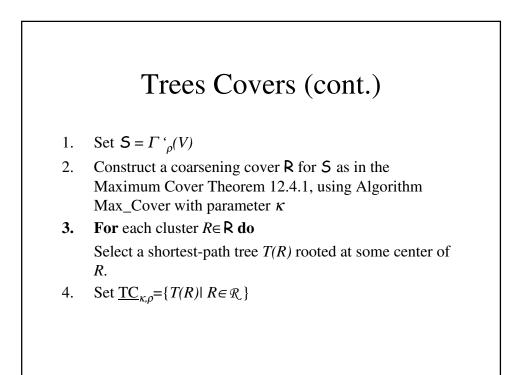
G if  $Stretch(G') \le k$ .

### **Trees Covers**

• **Definition 15.5.1** Given a weighted graph G=(V, E, w), a p-tree cover, or a tree cover for  $\Gamma'_{\rho}(v)$ , is a collection  $\underline{TC}$  of trees in G with the property that for every vertex  $v \in V$ , there is a tree  $T \in \underline{TC}$  that spans its entire p-neighborhood, namely,  $\Gamma_{\rho}(v) \subseteq V(T)$ . The depth of a tree cover  $\underline{TC}$  is

 $Depth(\underline{TC}) = max\{Depth(T)\}, \text{ for all } T \in \underline{TC}$ the maximum degree of  $\underline{TC}$  is

 $\Delta^{TC}(\underline{TC}) = max\{\Delta(T)\}, \text{ for all } T \in \underline{TC}$ and the overlap of  $\underline{TC}$  is the maximum, over all vertices v, of the number of different trees containing v,  $Overlap(\underline{TC}) = max\{|\{T \in \underline{TC}| v \in V(T)\}|\}$ 



# Tree Covers (cont.)

• **Theorem 15.5.2** For every weighted graph G=(V, E, w), |V|=n and integers  $\kappa$ ,  $\rho \ge 1$ , Algorithm Tree\_Cover constructs a  $\rho$ -tree cover  $\underline{TC}=\underline{TC}_{\kappa,\rho}$  for G with  $Depth(\underline{TC}) \le (2\kappa-1)\rho$  and  $Overlap(\underline{TC}) \le [2\kappa n^{1/\kappa}]$ .