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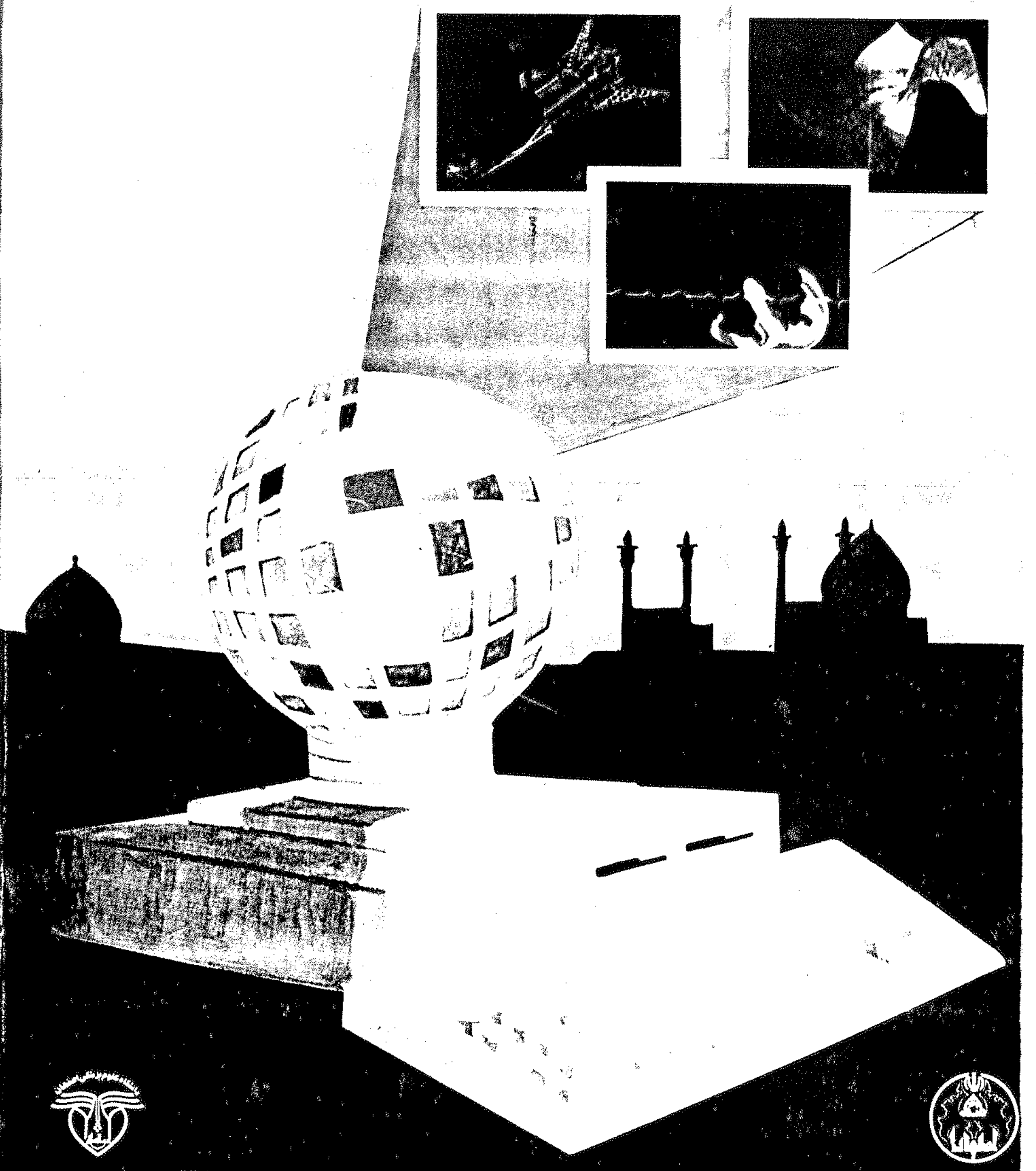

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ON BLOCKING DISTRIBUTION OF NONHIERARCHICAL NETWORKS

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ABSTRACT

It is shown in [19] that different types of routing schemes for circuit-switched traffic in a non-hierarchical network with uniform traffic intensity lead to different performance trade-offs. It is also assumed in [19] that the network is symmetric and the redirected traffic on the links is Poisson. It is, however, not clear how the trade-offs would be effected by using different traffic intensity on links, i.e., if nodes generate calls with different rates. This paper analyzes the traffic behavior without assuming that the redirected traffic is necessarily Poisson or that the network is symmetric. We examine a statistical model to obtain the ***exact limiting distribution*** and the ***end-to-end blocking*** probability of a single link, a group of links and the entire network.

Keywords: circuit-switching, limiting distribution, networks, non-hierarchical, routing.

I. INTRODUCTION

Alternate routing schemes have been widely used in hierarchical and non-hierarchical circuit-switched networks [1], [19], [8]. For nonhierarchical networks, it is known by Yum, T.G. and Schwartz, M.[18] that there is a cut-off point for the traffic intensity (ρ) before which alternative routing gives lower blocking probability than the direct routing for a given *source-destination* pair in a symmetric network [18],[19]. Their analysis is based on the assumption that calls are generated with the same arrival rate (λ) and the same traffic intensity for each link. The call arrival process were assumed to be Poisson and call holding time to be exponentially distributed with the same parameters. Krupp[8] developed a mathematical model for symmetric and uniformly loaded networks with an alternate routing scheme. In his model, the blocking probability is considered as a function of the external traffic load and a bistable blocking behavior was observed during overloads. Later, Akinpelu [1] extended the analysis to nonsymmetric networks and similar instability results were observed. Yum and Schwartz [18] compared different types of routing procedures for fully connected networks. Their results showed that alternate routing performs better than nonalternate (or direct) routing, if the network traffic is light. For heavy traffic, the direct routing performs better as far as the end-to-end blocking probability of a particular link is concerned.

In this paper, we attempt to present an analytical model for the use of direct and alternate routing schemes to compute the exact limiting distribution under various traffic load conditions for both symmetric and asymmetric, fully and non-fully connected networks. We shall show that there exist a cut-off surface for the blocking probability between direct and alternate routing. A few natural questions arise when investigating direct and alternate routing for general networks.

- Do such trade-offs always exist ?
- How sensitive is this surface, when the traffic intensity is not the same for all nodes, i.e., nonuniform traffic generated by different nodes ?
- One would suspect the cut-off surface, when it exists, to be a more complicated function of traffic intensities, especially when the network is not fully connected or asymmetric. Also, one might suspect that, for different topologies, one could observe trade-off results which are different from those described in [19].
- What is the blocking probability and the cut-off surface for two specific nodes, and more generally, what is the behavior of the *joint blocking probability* of a group of nodes or the entire network ?
- What does the cut-off surface look like when the links have different capacities ?

II. NETWORK MODEL AND STATISTICAL ASSUMPTIONS

First, we study the same network as in [19] but with a different approach. This approach is based on the *limiting joint* distribution of the underlying Markov process. We compared the direct and alternate routing in terms of blocking probability for different types of traffic. The main assumption is that the arrival distributions are independent Poisson processes and holding time is exponential.

For large number of nodes, the problem may become intractable when each link has many channels. Even for a small network, the number of possible states could be extremely large, depending upon the topology of the network. We developed a set of algorithms to compute the set of all possible valid states a network could attain, the state transitions, and the balance equations. Then we used LINPACK software to solve these equations to compute the limiting blocking probability over the specified group of nodes under different traffic patterns for both direct and alternate routing.

More precisely, let $n_1, \cdots, n_k$ denote the switching nodes and ℓ_{ij} be the link which directly connects the nodes n_i and n_j. If the system is fully connected then there would be $k(k-1)/2$ links in the system. Otherwise, some of the links may be missing.

The main assumptions for the traffic is that the arrivals at each node and in each direction are independent Poisson processes and the call holding times (service times) are independent and identically distributed exponential with parameter μ. These are standard assumptions in telephone traffic modeling. We will assume that the redirected traffic takes negligible amount of time to find an alternate connection, or to declare that the call is lost due to lack of availability of an end-to-end connection.

Direct routing scheme:

We assume that we have independent arrivals for the use of link ℓ_{ij} where, $i < j, i, j = 1, 2, ..., k$. At any time t, if a call arrives (either at node n_i, or n_j to use the link ℓ_{ij}) then the request will

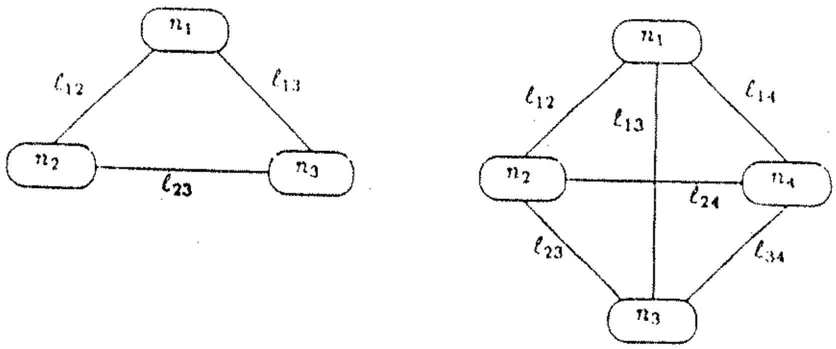

Figure 1: Configuration of fully connected networks.

be granted if there is a free channel on the link. If there is no free channel, the call is declared lost. Thus the nodes in this protocol perform no redirection work. If the call is granted a channel, it holds the channel for a random duration which is exponentially distributed with parameter μ. We assume that the arrival rate for the Poisson process to use the link ℓ_{ij} is λ_{ij}. Clearly, the limiting distribution of the number of busy channels at each link must exist. Let $P_{ij}(c)$ denote the probability that in the limit the link ℓ_{ij} has c channels busy. Thus, $P_{ij}(n)$ is the probability that the link is full. Since the arrival processes are mutually independent, the joint limiting distribution is the product of the marginal limiting distributions of each link. Thus, by the usual Erlang formula, the limiting probability, that the link ℓ_{ij} has c_{ij} channels busy, is:

$$\prod_{i,j=1,i<j}^{k} \frac{\rho_{ij}^{c_{ij}}}{c_{ij}!(\sum_{l=0}^{n} \rho_{ij}^{l}/l!)}$$

where, $c_{ij} \in \{0, 1, ..., n\}$ and $\rho_{ij} = \lambda_{ij}/\mu$. We will compare this distribution with the limiting distribution of the alternate routing scheme.

Alternate routing scheme:

The independence of the link status is no longer valid due to the alternately routed traffic. The situation is easily explained by considering the 3-node network. Further simplify the situation by assuming $n = 1$. The state space of the system can be represented by the corners of a unit cube. Here the x-axis is the state of the link ℓ_{12}, the y-axis is the state of link L_{13} and the z-axis is the state of link ℓ_{23}. These states are numbered as depicted in Figure 2. The corners represent the original eight states and the circles represent extra states representing the redirected traffic. This is is a Markov process[15] as described in [11]. The comparison of these schemes is presented in the next section.

III. COMPARISON OF ROUTING SCHEMES

To avoid triviality, we assume that the network has no isolated node. The node in which the call is originated is called the source node and the node in which the call is connected to is called the destination node. If the direct link connecting the source node to the destination is available then the source node will try to send the information through the direct link. However, if the direct link

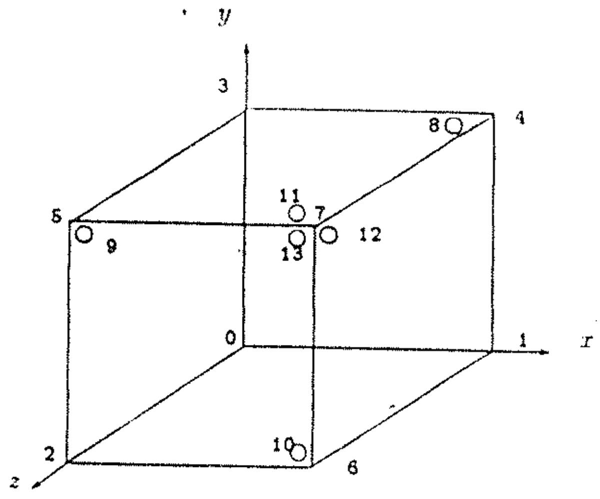

Figure 2: Illustration of states and their numbering.

is not available (either due to congestion or failure) then the source node may or may not reject the request depending upon its routing scheme. In direct routing, the local routing algorithm rejects any call once the direct route is not available. In alternate routing, and in the absence of direct link availability, the local routing algorithm tries to find the next available path leading to the destination with minimum cost. By having this option, the system may utilize available resources better with less overall blocking probability. However, the trade-off of this apparent gain is that the longer the alternate route, the more system resources it will use, and Hence, if there are too many requests, the alternate traffic may occupy sufficient bandwidth to detract significantly from the direct traffic opportunities. The direct routing scheme is very easy to study and the results are well known [11]. In the following sections, we provide a method to compute the trde-off of using alternate routing scheme and then compare the results with the direct routing.

This model provides a mechanism to compute the end-to-end blocking probability for a given source-destination pair, a set of source-destination pairs, and the whole network. We study the performance of the routing schemes for networks with small numbers of channels per link. One of the encouraging observations is that the performance pattern of the two schemes did not change when more channels were added to each link. This gives analytic evidence that one could study a small prototype network of the same topology, rather than the actual network, having a large number of channels per link. The prototype network would have a smaller number of channels per link, and hence a smaller number of balance equations. This approach is fairly general and the algorithms developed here can, in principle, be used for any network.

For the sake of clarity of exposition, we present the analysis for a 3-node network. Therefore, most of the discussion will be restricted to this network. However, we should emphasize that the method is general and can be applied to larger networks. The Kolmogorov equations of this net work are given below.

The system of equations can be solved by hand with some effort and the solution was presented in [11]. In this paper we look at the case where the links have two channels per link. Now it can be shown that there are eighty five different states. It is not easy to solve these equations by hand especially when the traffic rates ρ_{12}, ρ_{13} and ρ_{23} are different. We solved the system symbolically using the MACSYMA symbolic algebra program developed at M.I.T., for the case $\rho_{12} = \rho_{13} = \rho_{23} = \rho$.

$$
\begin{aligned}
(\lambda_{12}+\lambda_{13}+\lambda_{23})P(0) &= \mu(P(1)+P(2)+P(3)+P(8)+P(9)+P(10)) \\
(\lambda_{12}+\lambda_{13}+\lambda_{23}+\mu)P(1) &= \mu(P(4)+P(6)+P(12))+\lambda_{12}P(0) \\
(\lambda_{12}+\lambda_{13}+\lambda_{23}+\mu)P(2) &= \mu(P(5)+P(6)+P(13))+\lambda_{13}P(0) \\
(\lambda_{12}+\lambda_{13}+\lambda_{23}+\mu)P(3) &= \mu(P(4)+P(5)+P(11))+\lambda_{23}P(0) \\
(\lambda_{13}+2\mu)P(4) &= \mu P(7)+\lambda_{23}P(1)+\lambda_{12}P(3) \\
(\lambda_{12}+2\mu)P(5) &= \mu P(7)+\lambda_{23}P(2)+\lambda_{13}P(3) \\
(\lambda_{23}+2\mu)P(6) &= \mu P(7)+\lambda_{13}P(1)+\lambda_{12}P(2) \\
3\mu P(7) &= \lambda_{13}P(4)+\lambda_{12}P(5)+\lambda_{23}P(6) \\
(\lambda_{13}+\mu)P(8) &= \mu P(13) \\
(\lambda_{12}+\mu)P(9) &= \mu P(12) \\
(\lambda_{23}+\mu)P(10) &= \mu P(11) \\
2\mu P(11) &= \lambda_{23}(P(10)+P(3)) \\
2\mu P(12) &= \lambda_{12}(P(9)+P(1)) \\
2\mu P(13) &= \lambda_{13}(P(8)+P(2)) \\
\textstyle\sum_{i=0}^{13} P(i) &= 1.
\end{aligned}
$$

We obtain the following blocking probability, for link ℓ_{12}, using the alternative routing scheme, When each link has only one channel

$$P_{alt}(\ell_{12} \text{ traffic is blocked}) = \frac{\rho^4+7\rho^3+9\rho^2}{\rho^4+8\rho^3+15\rho^2+8\rho+2}.$$

It is trivial to get the corresponding probability for the direct routing scheme :

$$P_{dir}(\ell_{12} \text{ traffic is blocked}) = \frac{\rho}{1+\rho}.$$

The inequality $P_{alt} \le P_{dir}$ is equivalent to solving $\rho^2+\rho-2 \le 0$ which gives that the alternate routing scheme performs better than the direct routing scheme if and only if $\rho \le 1$. In fact, we can find the cut-off surface in the more general case when the input traffic rates are different as well. In this case we obtain the surface given in Figure 3. The values of ρ_{12} , ρ_{13}, and ρ_{23} which lie below the surface are the ones for which the alternate routing scheme performs better (i.e., has smaller blocking probability) than the direct routing scheme and the situation is reversed above the surface. For purposes of comparison with the more complicated case considered later, we also include a contour plot of this surface in Figure 4.

Similarly, if we consider the probability that both links ℓ_{12}, ℓ_{13} traffic is being blocked, then for the equal traffic rates we get

$$P_{alt}(\ell_{12} \text{ and } \ell_{13} \text{ traffic is blocked}) = \frac{\rho^4+6\rho^3+6\rho^2}{\rho^4+8\rho^3+15\rho^2+8\rho+2}.$$

It is again trivial to get the corresponding probability for the direct routing scheme:

$$P_{dir}(\ell_{12} \text{ and } \ell_{13} \text{ traffic is blocked}) = \left(\frac{\rho}{1+\rho}\right)^2.$$

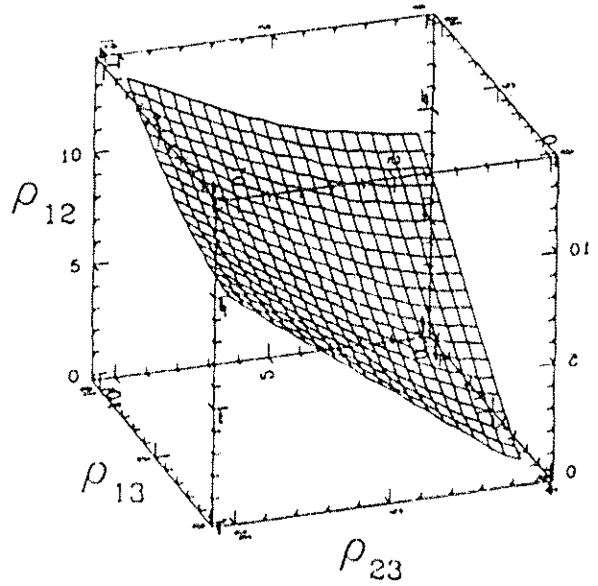

Figure 3: Cutoff surface for 3-node network, with 1 channel per link

For this case we can easily show that for all values of ρ we have $P_{\text{dir}} \leq P_{\text{alt}}$. This should not be surprising since we intuitively suspect that the alternative routing scheme will use up the empty links quickly by allowing indirect traffic to utilize them and hence will provide poorer availability for the direct traffic on the two links ℓ_{12} and ℓ_{13}. In fact, this result remains true even when the traffic input rates are different.

If we consider the situation that no call is getting through anywhere (i.e., each link in the network is blocking traffic) then for the equal arrival rates we have

$$P_{\text{alt}}(\text{all traffic is blocked}) = \frac{\rho^4 + 5\rho^3 + 3\rho^2}{\rho^4 + 8\rho^3 + 15\rho^2 + 8\rho + 2}.$$

It is again trivial to get the corresponding probability for the direct routing scheme to be

$$P_{\text{dir}}(\text{all traffic is blocked}) = \left(\frac{\rho}{1+\rho}\right)^3.$$

Again we have $P_{\text{dir}} \leq P_{\text{alt}}$ for all values of ρ as one would suspect. We can also show that this phenomenon remains true even for the case when the input traffic rates are not necessarily equal.

For the event that some traffic is being blocked somewhere in the network we have the probabilities

$$P_{\text{alt}}(\text{some traffic is blocked}) = \frac{\rho^4 + 8\rho^3 + 12\rho^2}{\rho^4 + 8\rho^3 + 15\rho^2 + 8\rho + 2}.$$

It is again trivial to get the corresponding probability for the direct routing scheme to be

$$P_{\text{dir}}(\text{some traffic is blocked}) = 1 - \left(\frac{1}{1+\rho}\right)^3.$$

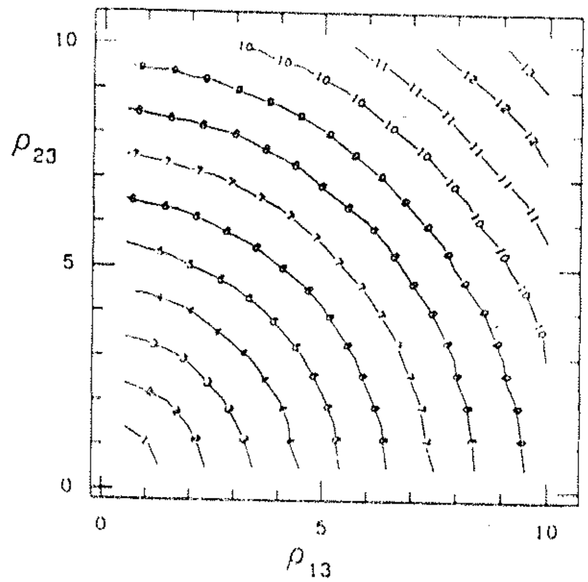

Figure 4: Contour plot of cutoff surface for 3-node network, with 1 channel per link

Again, one would suspect that in this case the alternate routing scheme should outperform the direct routing scheme. This is indeed the case as one can easily verify that $P_{\text{alt}} \le P_{\text{dir}}$ for all values of ρ. In fact, this remains true for all input traffic rates ρ_{12}, ρ_{13}, and ρ_{23}.

Now we examine the 3-node network when each link has two channels. As stated earlier, there are eighty five different states the system can be in at any particular time. We again get simple closed form formulae for the above probabilities in the case of equal input traffic rates. We list these probabilities below.

$$
\begin{aligned}
P_{\text{alt}}(\ell_{12}\text{ traffic is blocked}) = {} & (4374\rho^{21} + 196101\rho^{20} + 3888648\rho^{19} + 45788490\rho^{18} + 361018998\rho^{17} \\
& +2031625506\rho^{16} + 8475935664\rho^{15} + 26845417248\rho^{14} + 65514779952\rho^{13} \\
& +124249225840\rho^{12} + 183692060688\rho^{11} + 211174905184\rho^{10} \\
& +187013165248\rho^{9} + 125196776576\rho^{8} + 61277812992\rho^{7} \\
& +20671133184\rho^{6} + 4286960640\rho^{5} + 410572800\rho^{4})/denom,
\end{aligned}
$$

where,

$$
\begin{aligned}
denom = {} & 4374\rho^{21} + 204849\rho^{20} + 4272102\rho^{19} + 53331048\rho^{18} + 450282294\rho^{17} + 2748261126\rho^{16} \\
& +12640038840\rho^{15} + 45070169844\rho^{14} + 127214108984\rho^{13} + 288890718880\rho^{12} \\
& +534772537264\rho^{11} + 815500339936\rho^{10} + 1032697868416\rho^{9} + 1091325434240\rho^{8} \\
& +963194629504\rho^{7} + 706529780224\rho^{6} + 425317519872\rho^{5} + 205296832512\rho^{4} \\
& +76459769856\rho^{3} + 20610588672\rho^{2} + 3573227520\rho + 298598400.
\end{aligned}
$$

and

$$P_{\text{dir}}(\ell_{12} \text{ traffic is blocked}) = \frac{\rho^2}{2+2\rho+\rho^2}.$$

$$\begin{aligned} P_{\text{alt}}(\ell_{12} \text{ and } \ell_{13} \text{ traffic is blocked}) = \; & (4374\rho^{21} + 187353\rho^{20} + 3557682\rho^{19} + 40196736\rho^{18} \\ & +304618050\rho^{17} + 1649974458\rho^{16} + 6634270536\rho^{15} \\ & +20276793780\rho^{14} + 47812703936\rho^{13} + 87720931600\rho^{12} \\ & +125594399840\rho^{11} + 139939871648\rho^{10} + 120165926656\rho^{9} \\ & +78003245440\rho^{8} + 37001445120\rho^{7} + 12084585984\rho^{6} \\ & +2422794240\rho^{5} + 223948800\rho^{4})/denom, \end{aligned}$$

and

$$P_{\text{dir}}(\ell_{12} \text{ and } \ell_{13} \text{ traffic is blocked}) = \left(\frac{\rho^2}{2+2\rho+\rho^2}\right)^2.$$

$$\begin{aligned} P_{\text{alt}}(\text{all traffic is blocked}) = \; & (4374\rho^{21} + 178605\rho^{20} + 3226716\rho^{19} + 34604982\rho^{18} + 248217102\rho^{17} \\ & +1268323410\rho^{16} + 4792605408\rho^{15} + 13708170312\rho^{14} + 30110627920\rho^{13} \\ & +51192637360\rho^{12} + 67496738992\rho^{11} + 68704838112\rho^{10} \\ & +53318688064\rho^{9} + 30809714304\rho^{8} + 12725077248\rho^{7} \\ & +3498038784\rho^{6} + 558627840\rho^{5} + 37324800\rho^{4})/denom, \end{aligned}$$

and

$$P_{\text{dir}}(\text{all traffic is blocked}) = \left(\frac{\rho^2}{2+2\rho+\rho^2}\right)^3$$

$$\begin{aligned} P_{\text{alt}}(\text{some traffic is blocked}) = \; & (4374\rho^{21} + 204849\rho^{20} + 4219614\rho^{19} + 51380244\rho^{18} \\ & +417419946\rho^{17} + 2413276554\rho^{16} + 10317600792\rho^{15} \\ & +3414040716\rho^{14} + 83216855968\rho^{13} + 160777520080\rho^{12} \\ & +241789721536\rho^{11} + 282409938720\rho^{10} + 253860403840\rho^{9} \\ & +172390307712\rho^{8} + 85554180864\rho^{7} + 29257680384\rho^{6} \\ & +6161127040\rho^{5} + 597196800\rho^{4})/denom, \end{aligned}$$

and

$$P_{\text{dir}}(\text{some traffic is blocked}) = 1 - \left(\frac{2+2\rho}{2+2\rho+\rho^2}\right)^3$$

For the case when the traffic arrival rates are different we can present the cut-off surface and contour plot in Figures 5 and 6 respectively. These figures show that the direct routing scheme does not perform as well as the alternate routing scheme for all those arrival rates which lie below the surface and converse holds above the surface (as far as the blocking is concerned for a particular node).

One should note that all the conclusions drawn from the network having one channel per link remain valid for the network having two channels per link. The differences are only in terms of order of magnitude. In fact, as can be seen from the contour plots for the two cases, they are

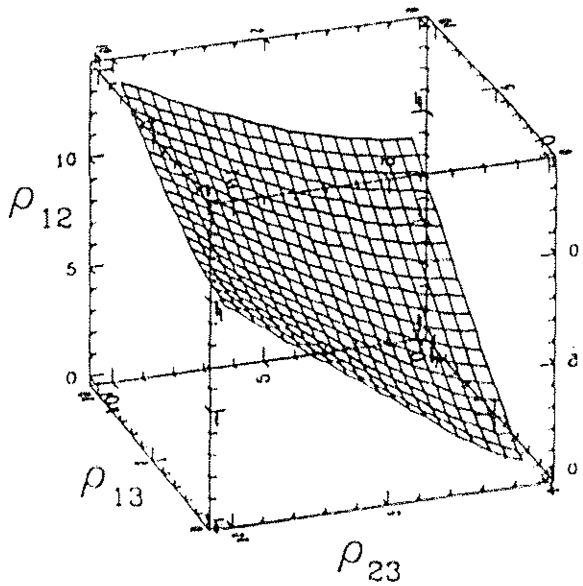

Figure 5: Cutoff surface for 3-node network, with 2 channels per link

almost visually indistinguishable. Therefore, we do not present the results for the larger network in detail. This is a useful observation which gives some credence to assuming that when a network has a large number of channels per link, then we can reduce this number for the analysis purposes and the conclusions could be projected to the original network.

The second observation is that the findings, for a network with small number of channels per link, should also be visible in the same topology with larger number of channels. This is due to the fact that as soon as the extra channels are occupied then the two networks behave identically for a short time period. It could be argued that the larger network may have more complicated performance attributes which may not be achievable, for the same network with smaller channel capacities. We leave the exhaustive analysis of topic for further research.

IV. CONCLUSIONS

In this paper, we have introduced a new technique to compute the exact end-to-end blocking probability of nonhierarchical networks. We have analyzed the traffic behavior without assuming that the redirected traffic is necessarily Poisson or the network is symmetric. In conclusion we surmise that

- One can obtain the exact limiting distribution (for the alternate routing scheme) of the underlying Markov process for networks with sparse connectivity, provided one has enough computing facilities. When the network has a large number of links one may have to resort to numerical methods.

- The trade-off surface between the two schemes may not always exist if one wants to study the

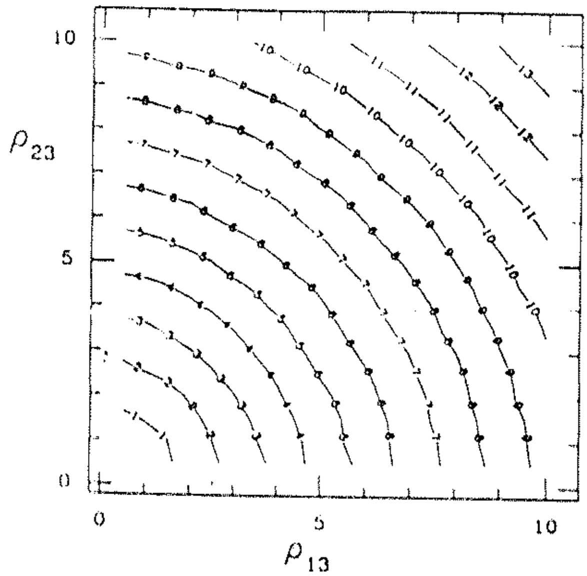

Figure 6: Contour plot of cutoff surface for 3-node network, with 2 channels per link

blocking probability behavior over a group of links. For instance, for the network studied in this paper, the direct routing scheme performs better (has lower blocking probability) than the alternate routing scheme when we are interested in the traffic of both links ℓ_{12} and ℓ_{13} simultaneously. The same conclusion holds when we want to see the traffic flow over all three links. Only when we restrict our attention to one link does there exist a trade-off surface.

- When such a trade-off surface exists, it is a function of the traffic intensities at different links. For the examples presented here, this surface does not seem to be a very complicated function. We studied some networks which are not fully connected as well. these, results will be presented elsewhere.

- The channel capacity of links does not seem to play a major role in the analysis of network performance as far as the blocking probabilities are concerned (at least for the networks we studied). It seems as if one can keep the topology of the network and reduce the channel capacity and study the smaller network with little loss in generality.

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