

Multiway Trees

1

Tree

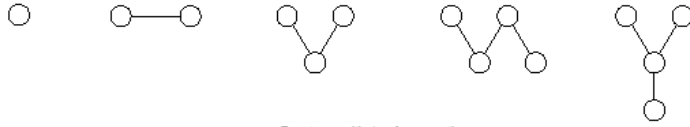
- A *(free) tree* is any set of points (called *vertices*) and any set of pairs of distinct vertices (called *edges* or *branches*) such that (1) there is a sequence of edges (a *path*) from any vertex to any other, and (2) there are no *circuits*, that is, no paths starting from a vertex and returning to the same vertex.
- A *rooted tree* is a tree in which one vertex, called the *root*, is distinguished.
- An *ordered tree* is a rooted tree in which the children of each vertex are assigned an order.
- A *forest* is a set of trees. We usually assume that all trees in a forest are rooted.
- An *orchard* (also called an *ordered forest*) is an ordered set of ordered trees.



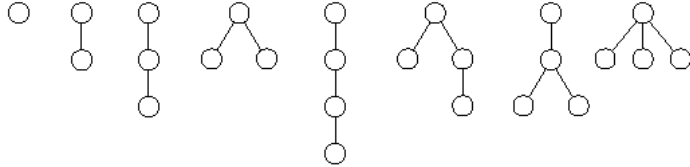
DESIGN &
ANALYSIS OF
ALGORITHM

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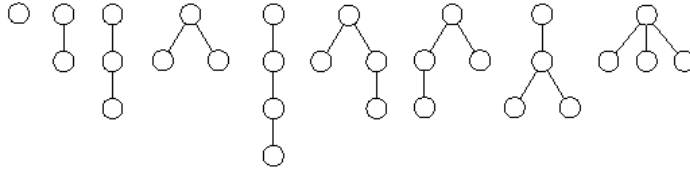
Examples



Free trees with 4 or fewer vertices
(Arrangement of vertices is irrelevant.)



Rooted trees with 4 or fewer vertices



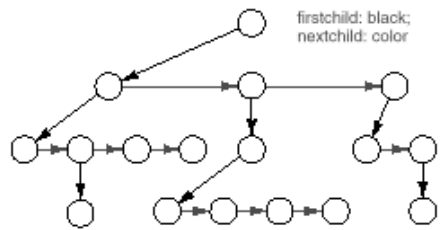
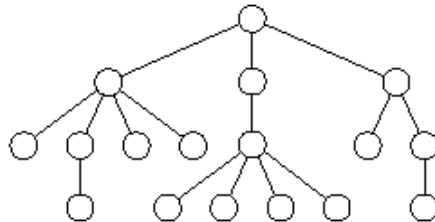
Ordered trees with 4 or fewer vertices



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Implementation

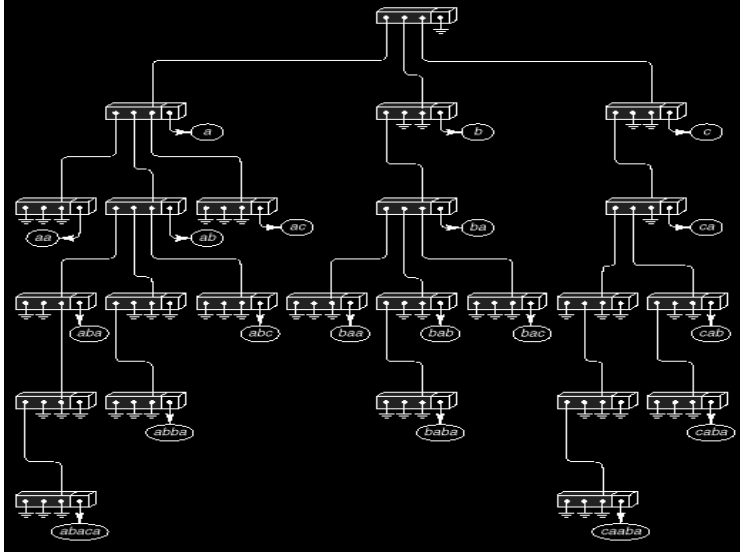


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Tries

Tries



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B-Trees

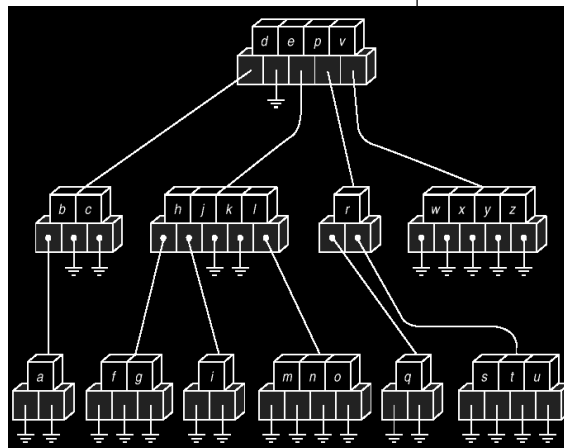
7

B-Trees



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- Idea: In external search disk access is expensive. How can we store more than 2-way information in a block?
- Solution: construct multiway search tree.



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B-Trees

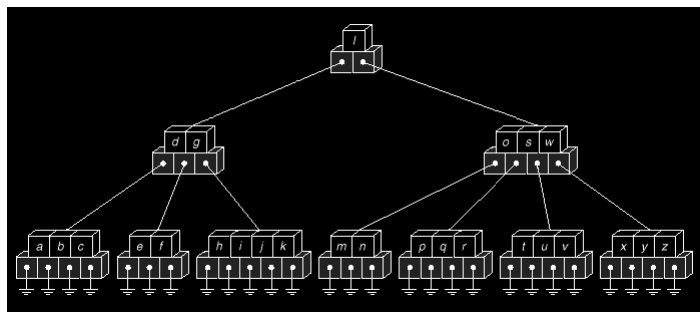
- A *B-tree of order m* is an m -way search tree in which
 - All leaves are on the same level.
 - All internal nodes except the root have at most m non-empty children, and at least $\text{ceiling}(m/2)$ nonempty children.
 - The number of keys in each internal node is one less than the number of its children, and these keys partition the keys in the children in the fashion of a search tree.
 - The root has at most m children, or as few as 2 if it is not a leaf, or none if the tree consists of the root alone.



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Example ($m=4$)



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Insertion

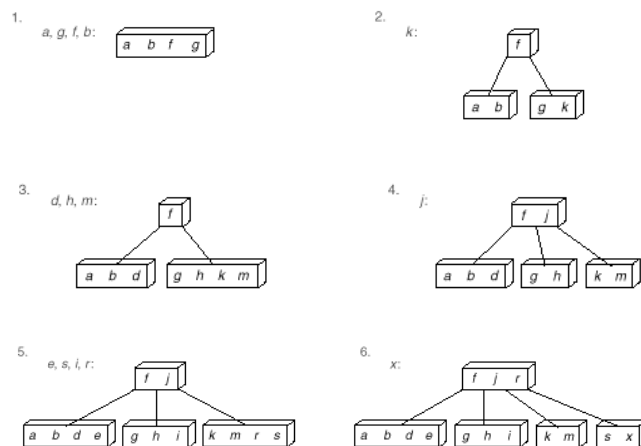
- B-trees grow at the root, not the leaves.
- Find the leaf where the new key belongs and insert it.
- If the leaf now has too many keys, split it into two nodes on the same level, but do not put the median key into either new node.
- Move up one level, insert the median key in this parent node, and repeat the splitting process if necessary.
- If the root node splits, then the resulting median key goes into a new root and the tree grows in height.



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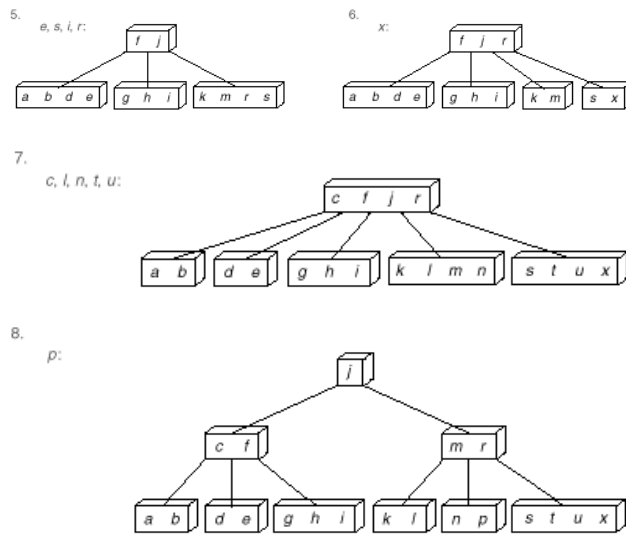
Insertion Example



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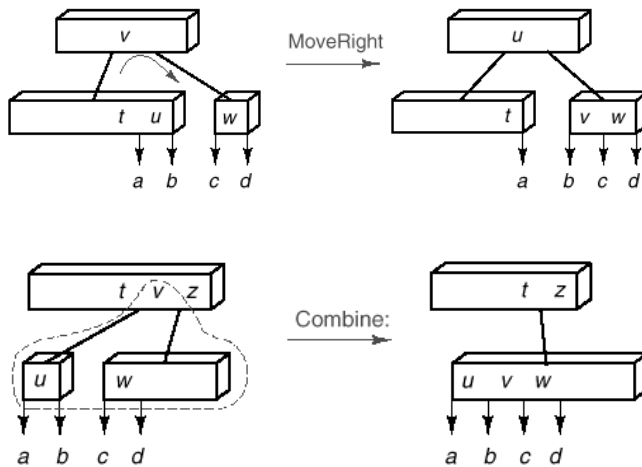
Insertion Example



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Deletion

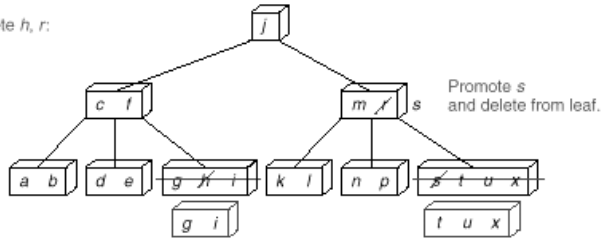


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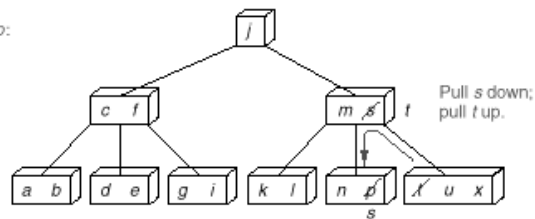
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Deletion Example

1. Delete *h*, *r*:



2. Delete *p*:

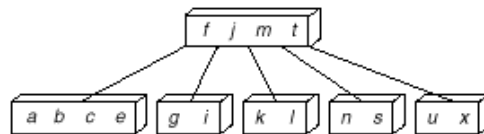
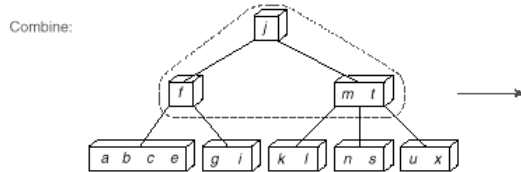
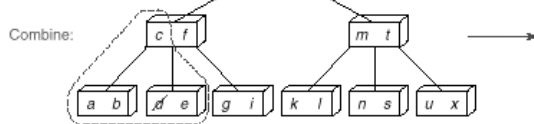


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Deletion Example (cont..)

3. Delete *d*:



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