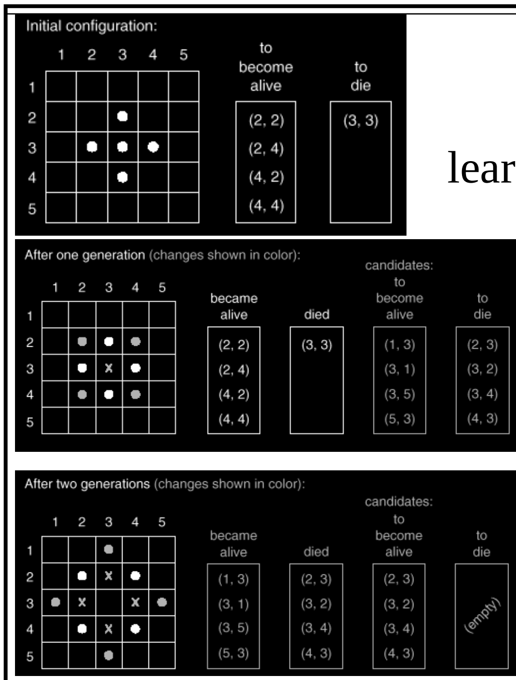
CS 4/56101

Design and Analysis of Algorithms

Kent State University

Dept. of Math & Computer Science

LECT-3



What did we learned in the last class?

Flashback: Two versions of Game of Life..

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Lesson from Last Class

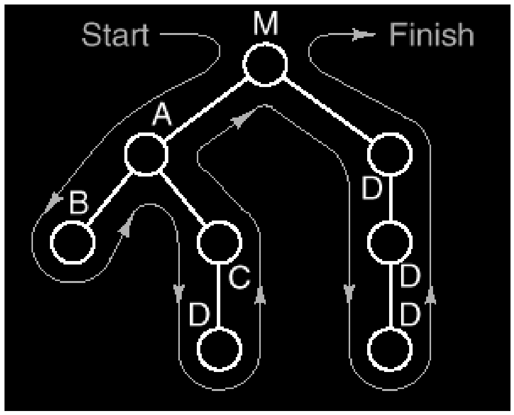
Program Analysis and Program Design are closely interrelated. A good computer engineer must know both.

In this course we will learn a host of new powerful programming techniques. Along with we will learn more formal methods for analyzing their performance.

Technique of Recursion

Concept of Recursion

- Let us consider a set of nested subroutines...



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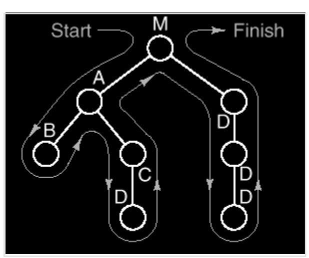
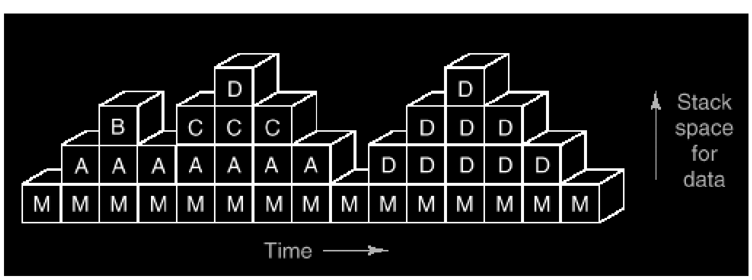
Program Stack



In recursive program, instead of one routine calling a different routine, one routine can repeatedly calls itself.



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Recursion

- Recursion is a powerful tool which can make the solution of many difficult problem astonishingly easy.
- It is a powerful tool to divide and conquer complex problems.
- However, it is also very important to carefully analyze a recursive solution.
- In this class we will see two examples of recursive solutions, and will learn techniques how to analyze recursive programs.

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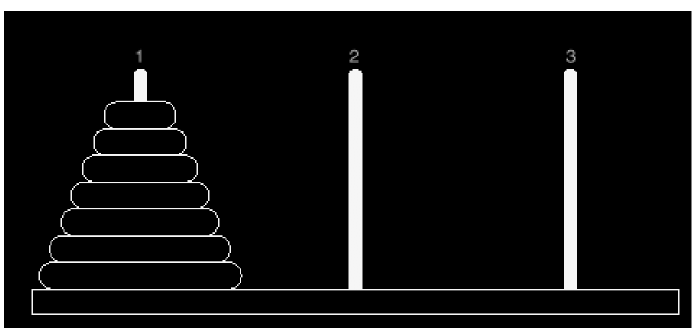
Tower of Hanoi



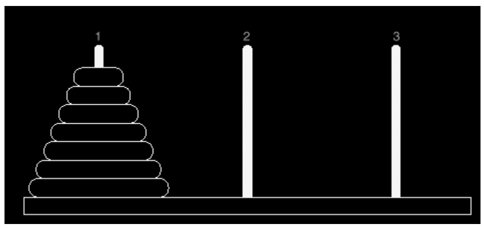
This is task which is underway at the Temple of Brahma. At the creation of the world, the priest were given a brass platform on which were 3 diamond needles. On the first needle were stacked 64 golden disks, each one slightly smaller than the one under it. The priest were assigned the task of moving all the golden disks from the first needle to the third. The end of the task will signify the end of the world.

Solution

- Solution:

Move(64,1,3,2)

- Meaning: Move 64 disks from tower 1 to tower 3 using tower 2 as temporary.



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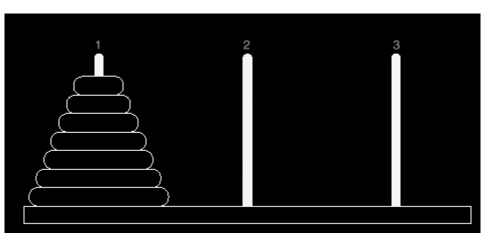
Solution (Divide and Conquer)

- Step 1:
  - Move (63,1,2,3)
  - printf("Move disk #64 from tower 1 to tower 3\n");
  - Move(63,2,3,1)
- Step 2 ?



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Structure of Recursive Program

Every recursive process consists of two parts:

1. A smallest, base case that is processed without recursion; and
2. A general method that reduces a particular case to one or more of the smaller cases, thereby making progress toward eventually reducing the problem all the way to the base case.

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Solution

int Move(int count, int start, int finish, int temp);

Pre: There are at least count disks on the tower start. The top disk (if any) on each of towers temp and finish is larger than any of the top count disks on tower start.

Post: The top count disks on start have been moved to finish; temp (used for temporary storage) has been returned to its starting position.

```
/* Move: moves count disks from start to finish using
temp
for temporary storage. */
void Move(int count, int start, int finish, int temp)
{
    if (count > 0) {
        Move(count-1, start, temp, finish);
        printf("Move a disk from %d to %d.\n", start,
finish);
        Move(count-1, temp, finish, start);
    }
}
```

Demonstration: Tower of Hanoi

Hanoi.exe

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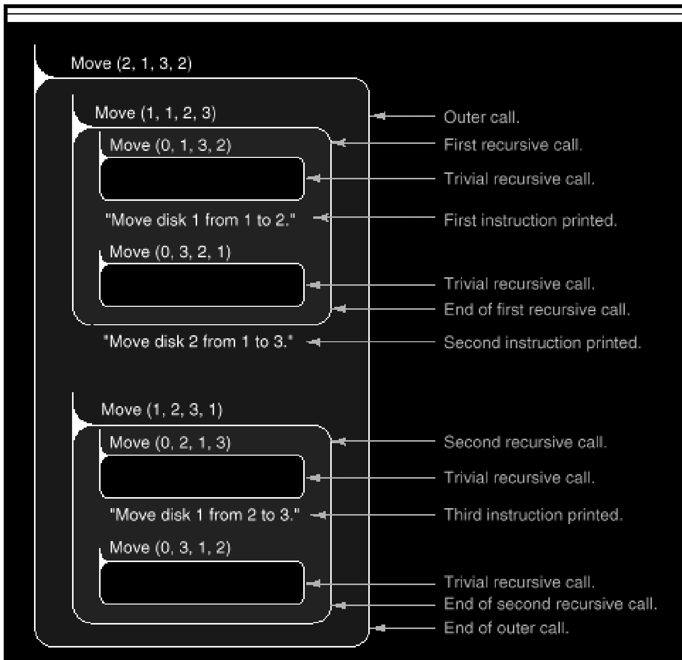

e

Move(2,1,3,2)

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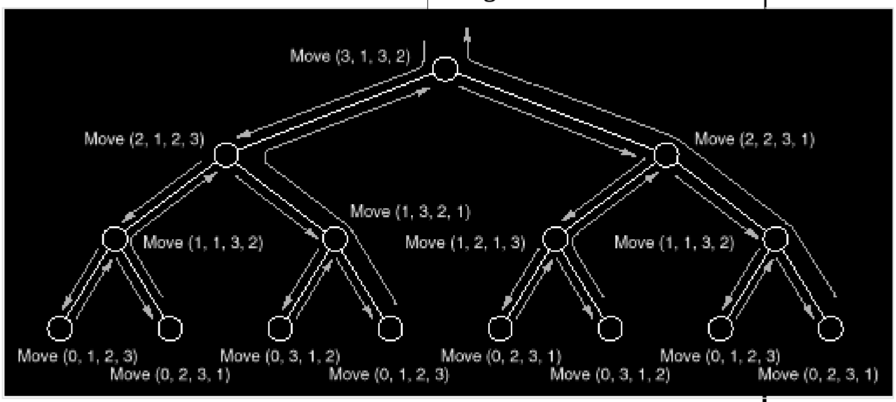
Analysis

- Recursion Tree

Height & Number of Nodes



- Number of Nodes= 1+2+4+…….. $2^{63} = 2^{64} - 1$

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How Large is this number?

- $10^3 \approx 2^{10}$
- Let the priest can perform
  - one move per second then it will take:
  - $2^{64} > 2^4 . 2^{60} = 16 \times 10^{18}$ secs
- There are about:
  - 3.2×10^7 seconds in a year.
  - The life of universe is 20 billion years.
  - It will take 25 times more to complete the task!
- Computers will fail
  - because of time.
  - How much space will be required?

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A Useful Case

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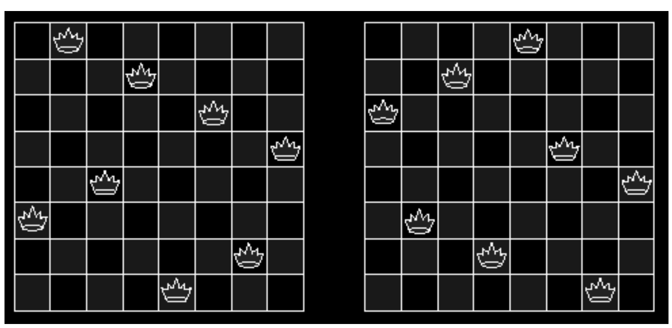
A Fruitful Application of Recursion:

n-queen problem



Apparently an analytically unsolvable problem. Even C. F. Gauss, who attempted this in 1850 was perplexed by this problem. But, solution do exists. See the two shown above.

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Solution Outline

```
void
AddQueen(void)
{
    for (every unguarded position p on the
board) {
        Place a queen in position p;
        n++;
        if (n == 8)
            Print the configuration;
        else
            AddQueen();
        Remove the queen from position p;
        n--;
    }
}
```

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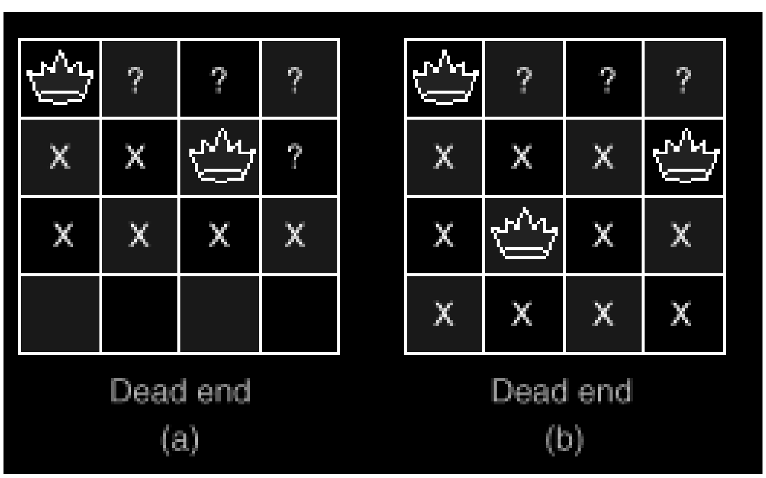
Example 4-Queen



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Example 4-Queen

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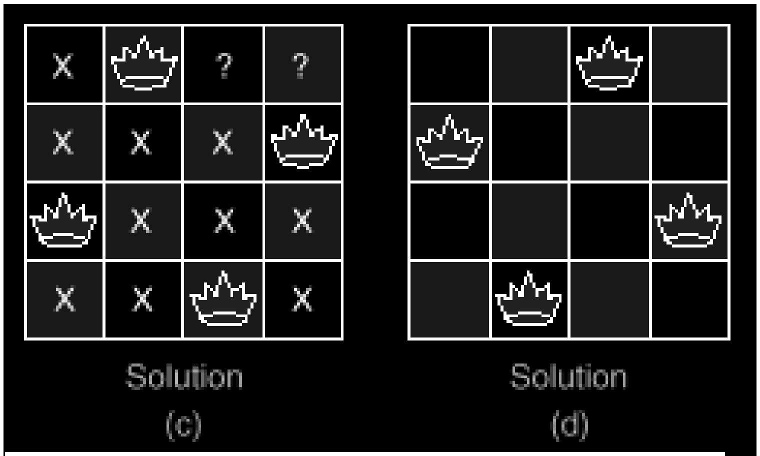

Backtracking: Build correct partial solution and proceed. When an inconsistant state arises, the algorithm backs up to the point of last correct partial solution.

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Choice of Data Structure

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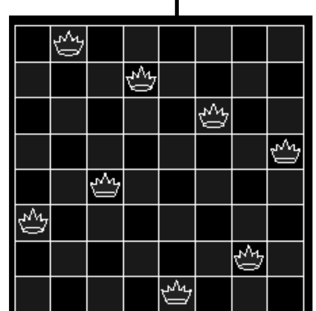
- Boolean array or integer array?
  - Keep a count of check, to help backtracking.
- Search later or mark ahead?
- Pigeon hole principle
  - use one row one queen to reduce search.
- Keep track of free columns
  - int col[8]
- Keep track of free diagonals
  - number of diagonal 2*boardsize -1
  - all downdiagonal (x-y)=constant
  - all updiagonal (x+y)=constant



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```
#include "common.h"
#define BOARDSIZE   8
#define DIAGONAL    (2*BOARDSIZE-1)
#define DOWNOFFSET  7

void WriteBoard(void);
void AddQueen(void);

int queencol[BOARDSIZE];    /* column with the queen    */
Boolean colfree[BOARDSIZE]; /* Is the column free?      */
Boolean upfree[DIAGONAL];   /* Is the upward diagonal free?     */
Boolean downfree[DIAGONAL]; /* Is the downward diagonal free?   */

int queencount = -1,        /* row whose queen is currently placed
*/
    numsol = 0;             /* number of solutions found so far
*/

 int main(void)
{
    int i;

    for (i = 0; i < BOARDSIZE; i++)
        colfree[i] = TRUE;

    for (i = 0; i < DIAGONAL; i++) {
        upfree[i] = TRUE;
        downfree[i] = TRUE;
    }
    AddQueen();

    return 0;
}
```

Main()

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```
void AddQueen(void)
{
    int col;    /* column being tried for the queen */

    queencount++;
    for (col = 0; col < BOARDSIZE; col++)
        if (colfree[col] && upfree[queencount + col] &&
            downfree[queencount - col + DOWNOFFSET]) {
            /* Put a queen in position (queencount, col).   */
            queencol[queencount] = col;
            colfree[col] = FALSE;
            upfree[queencount + col] = FALSE;
            downfree[queencount - col + DOWNOFFSET] = FALSE;
            if (queencount == BOARDSIZE-1)  /* termination condition
*/
                WriteBoard();
            else
                AddQueen();                 /* Proceed recursively.
*/
            colfree[col] = TRUE;    /* Now backtrack by removing the
queen. */
            upfree[queencount + col] = TRUE;
            downfree[queencount - col + DOWNOFFSET] = TRUE;
        }
    queencount--;
}
```

AddQueen()

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Analysis

- Naïve approach:
  - generate a random configuration and test it.

$$\binom{64}{8} = 4,426,165,368$$

- One queen per row

$$8^8 = 16,777,216$$

- One queen per column

$$8! = 40,320$$

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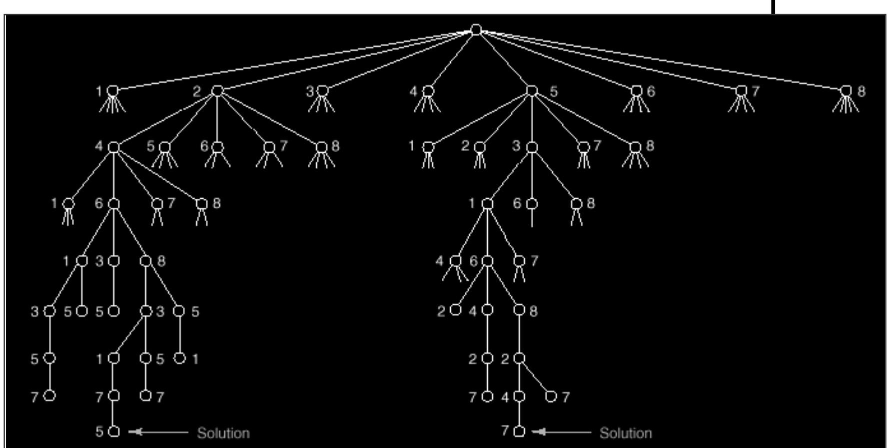
Part of Recursion Tree for 8-queen



Height & Number of Nodes

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