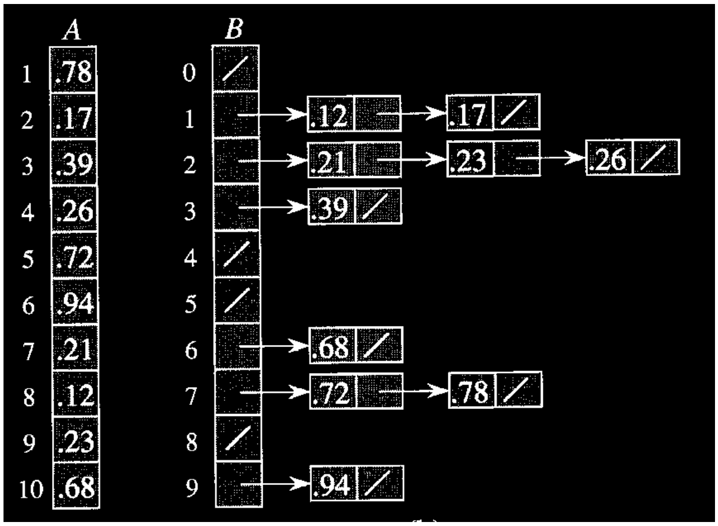
Bucket Sort

Idea

- Counting sort assumes that the input consists of integers in a small range.
- Bucket sort assumes that the inputs are generated by a random process and elements are uniformly distributed over the interval [0,1].

- Algorithm:
  - Throws the numbers in their right buckets.
  - Sort each bucket with regular insertion sort.
  - Concatenate the buckets.

Example



DESIGN & ALALYSIS OF ALGORITHM

Code

```
BUCKET-SORT(A)
n ← length[A]
for i ← 1 to n
     do insert A[i] into list B[⌊nA[i]⌋]
for i ← 0 to n − 1
     do sort list B[i] with insertion sort
concatenate the lists B[0], B[1], . . . , B[n − 1] together in order
```

DESIGN & ALALYSIS OF ALGORITHM

Proof of Correctness

- If two items are in the same bucket then they are in proper relative order.
- If two items are in two different buckets, even then they are in right order.

Complexity

- Each small list is sorted with insertion sort:

$$\sum_{i=0}^{n-1} O(E[n_i^2]) = O\left(\sum_{i=0}^{n-1} E[n_i^2]\right)$$

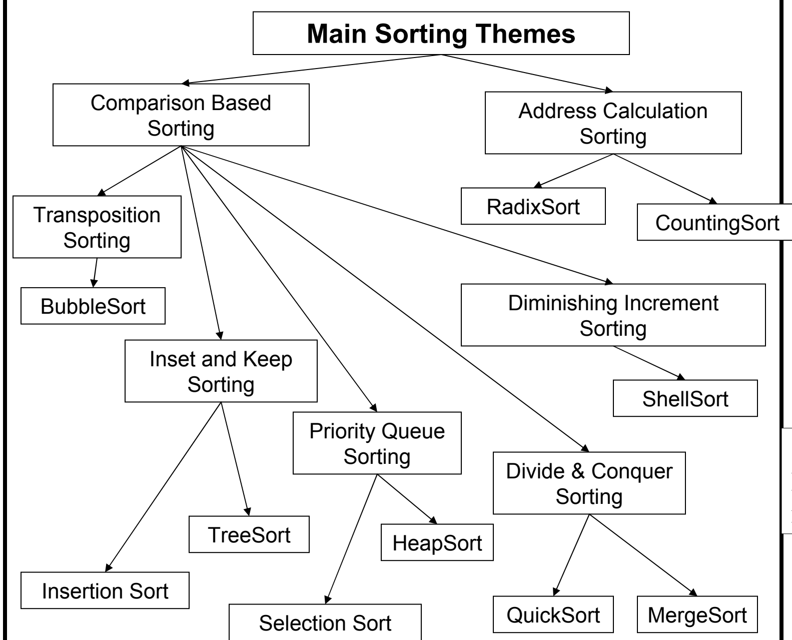
- What is the expected length of the small lists?
  - Probability of a a list to have length 1.. , 2… n?
  - Binomial distribution:

$$p = \frac{1}{n}, E[n_i] = np = 1, Var[n_i] = npq = 1 - \frac{1}{n}$$

$$E[n_i^2] = Var[n_i] + E^2[n_i],$$

$$= (1 - \frac{1}{n}) + 1^2 = 2 - \frac{1}{n} = \Theta(1)$$

$$\sum_{i=0}^{n-1} O(E[n_i^2]) = O\left(\sum_{i=0}^{n-1} E[n_i^2]\right) = O(n)$$



Quiz: Fit BucketSort in this Tree

Class Mechanics

- Discussion about Final Project
- Feedback on Grade
- Quiz

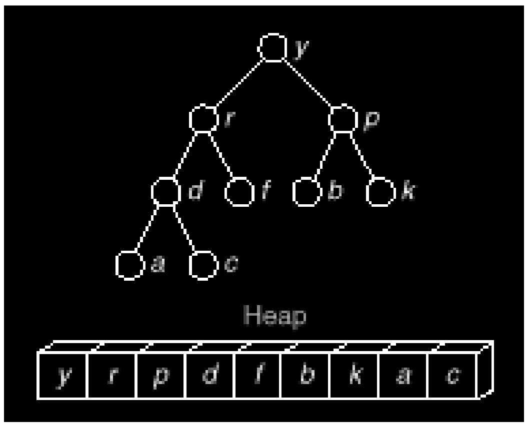
Heap Sort

Motivation

- For Contiguous array, the best is QuickSort. Quicksort has no O(nlogn) worst case bound. HeapSort has worst case bound O(n log n) and sorts in place.

Heap

DEFINITION A ***heap*** is a list in which each entry contains a key, and, for all positions k in the list, the key at position k is at least as large as the keys in positions $2k$ and $2k + 1$, provided these positions exist in the list.



DESIGN &
ALALYSIS OF
ALGORITHM

Idea

- Build the heap,
  - by one by one inserting the keys.
- One by one take the roots out of the tree.
  - Insert it in the sorted list.
  - After each deletion rearrange the heap so that the largest again reaches at the top.

DESIGN &
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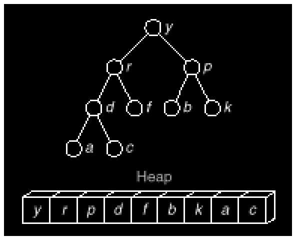
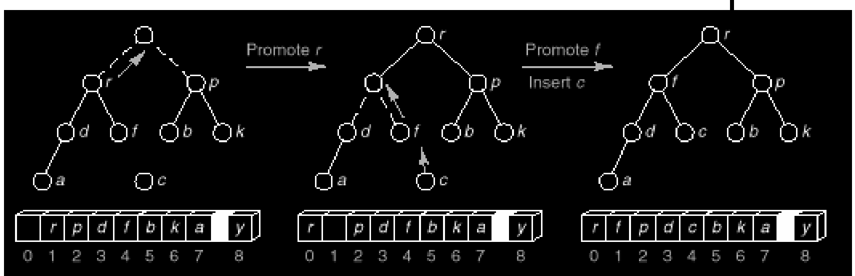
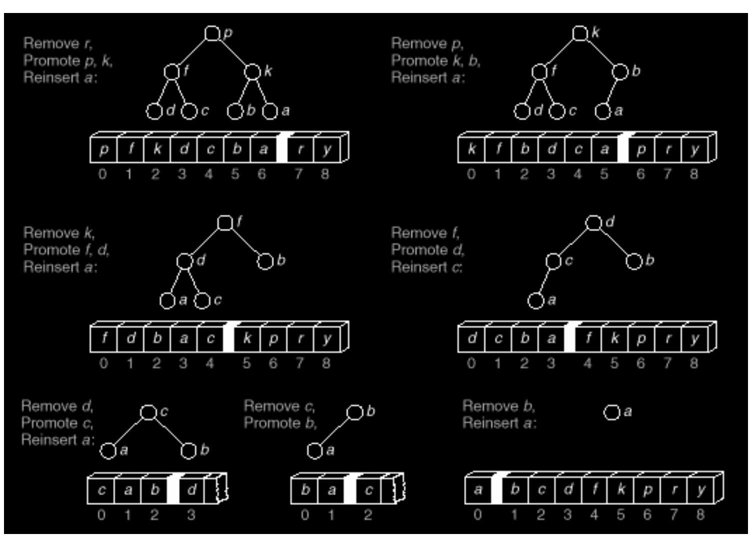
ALGORITHM

Example





Example (contd..)



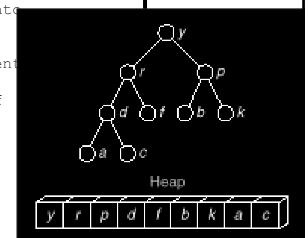
Main & BuildHeap Code

```
void HeapSort(List *list)
{
    Position lu;            /* Entries beyond lu have been
sorted. */
    ListEntry current;      /* holds entry temporarily
removed from list */

    BuildHeap(list);        /* First phase: turn list into
a heap. */
    for (lu = list->count - 1; lu >= 1; lu--) {
        current = list->entry[lu]; /* Extract last element
from list. */
        list->entry[lu] = list->entry[0]; /* Move top of
heap to end of list. */
        InsertHeap(current, 0, lu - 1, list);
    }
}

void BuildHeap(List *list)
{
    Position low;           /* Entries beyond low form a
heap. */

    for (low = list->count / 2 - 1; low >= 0; low--)
        InsertHeap(list->entry[low], low, list->count,
list);
}
```

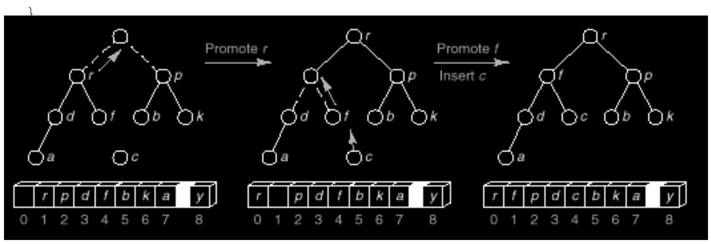


Second half of the ndoes already satisfies the heap condition.

DESIGN & ALALYSIS OF ALGORITHM

InsertHeap

```
void InsertHeap(ListEntry current,
Position low, Position high, List *list)
{
    Position large;
    large = 2 * low + 1;
    while (large <= high) {
        if (large < high && LT(list->entry[large].key,
         list->entry[large + 1].key))
            large++;
         if (GE(current.key, list->entry[large].key))
            break;
        else {
         list->entry[low] = list->entry[large];
            low = large;
            large = 2 * low + 1;
        }
    }
}
```



DESIGN & ALALYSIS OF ALGORITHM

Analysis

- Each insertion may check log n heap nodes.
- Each check has two comparisons and one assignment.
- Let m=n/2, k varies from m-1 to 0. Cost of building is:

$$2\sum_{k=0}^{m-1} \log \frac{n}{k} = 2(m \log n - \log m!) \approx 5m \approx 2.5n$$

$$\log m = \log n - 1$$

$$\log m! = m \log m - 1.5m \approx m \log m - 2.5m$$

- For sorting n elements we need to Insert (or restore the heap) n times. The cost is:

$$2\sum_{k=2}^{n} \log k \leq 2n \log n$$

- The worst case complexity is *2n log n* comparisons
- and *n log n* assignments.

DESIGN &
ALALYSIS OF
ALGORITHM

Heap Sort and Quick Sort

- Worst-case performance of Heap Sort (2nlogn) is poorer than the average-case performance of Quick Sort (1.39nlogn).
- However, the worst-case of Quick Sort is far worse than that of Heap Sort.
- The average-case analysis of Heap Sort is quite complex, however it shows it is almost same as its worst-case.
- On the average, therefore Quick Sort runs almost twice as fast as Heap Sort.