Homework 2

You only need to complete and submit 5 out of the first 9 math problems. You need complete the last (10-th) problem.

There are two deadlines. The first deadline is March 22th. You need to submit 3 out 5 math problems you worked on. The second deadline is April 2nd. You need to submit the rest of problems.

Problem 1: Show that the complement graph $\overline{G_p}$ of a graph G_p is precisely G_q, where $q = 1 - p$.

Problem 2: Let $\vec{G} = (V, \vec{E})$ be a directed graph with m edges and without loops. Use expectation to show that V can be partitioned into sets V_1 and V_2 such that $\vec{G}$ contains more than $m/4$ edges from V_1 and V_2.

Problem 3: Show that a.e. (almost every) graph $G_{n,1/2}$ has maximal degree at least $n/2 + \sqrt{n}$ and minimal degree at most $n/2 - \sqrt{n}$.

Problem 4: Show that a.e. graph $G_{n,1/2}$ has at least $n^{1/3}$ vertices of degree precisely $\lfloor n/2 \rfloor$. (Hint: Compute the expectation and variance of the number of these vertices).

Problem 5: Show that for $\epsilon > 0$, for any graph $G \in G_{n,p}$, a.s. (almost surely), G has at least $\frac{1}{2}(p-\epsilon)n^2$ edges and at most $\frac{1}{2}(p+\epsilon)n^2$ edges.

Problem 6: Show that when $n \to 0$, $\omega(n) \leq logloglog$, $p = (logn + \omega(n))/n$,

$$\sum_{k=1}^{\lfloor n/2 \rfloor} \binom{n}{k} (1-p)^{k(n-k)} \to 0$$

(Hint: considering two cases separately, $1 \leq k \leq n^{3/4}$ and $n^{3/4} \leq k \leq n/2$).

Problem 7: Let $G \in G_{n,p}$, show

1. Find $p(n)$ such that $E[\#K_5] = 1$.
2. Find $p(n)$ such that $E[\#K_{3,3}] = 1$.
3. Find $p(n)$ such that $E[\#Hamitonian\ path] = 1$.

Problem 8: Let $G = (V, E)$ be a graph with n vertices and e edges. Then G contains a bipartite subgraph with at least $e/2$ edges.

Problem 9: Suppose you have a file containing n records. How can you choose m records randomly without knowing n initially? Write a single pass algorithm which sample m records from a large file. Proof your algorithm does generate a random sample.

Problem 10 (Must Do Problem): Run your programs on random graphs and show the threshold phenomenon for

1. Existence of K_3 and K_4.
2. Connectivity
3. Size of the Giant Components.

(Hint: you can draw the diagram to show the effect using matlab.)