Discrete Structures for Computer Science

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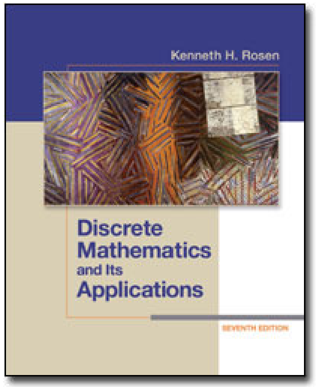
Fall 2012

Textbook

Discrete Mathematics and Its Applications
By Kenneth H. Rosen, McGraw Hill (7th ed.)



Use lecture notes as study guide.

Course Requirements

- Quizzes & Attendance 15%
- Homework 25%
- Midterm Exam 25%
- Extra Credit Problem 2-5%
- Final Exam 35%

Why Discrete Math?

Design efficient computer systems.

- How did Google manage to build a fast search engine?
- What is the foundation of internet security?

algorithms, data structures, database,
parallel computing, distributed systems,
cryptography, computer networks…

Logic, sets/functions, counting, graph theory…

What is discrete mathematics?

Logic: artificial intelligence (AI), database, circuit design

Counting: probability, analysis of algorithm

Graph theory: computer network, data structures

Number theory: cryptography, coding theory

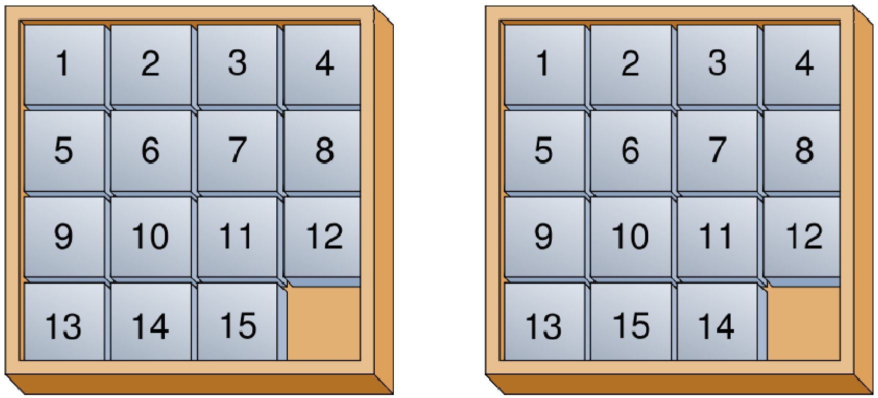
logic, sets, functions, relations, etc

Topic 1: Logic and Proofs

How do computers think?

Logic: propositional logic, first order logic

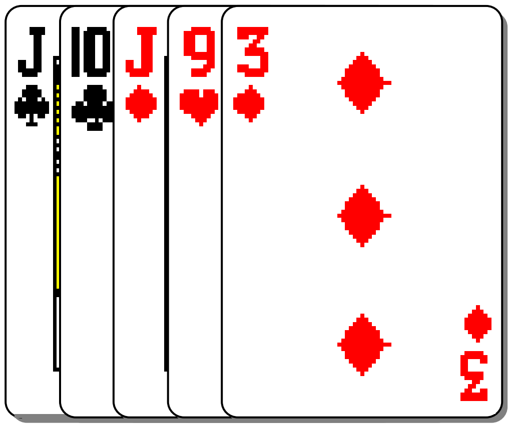
Proof: induction, contradiction



Artificial intelligence, database, circuit, algorithms

Topic 2: Counting

- Sets
- Combinations, Permutations, Binomial theorem
- Functions
- Counting by mapping, pigeonhole principle
- Recursions, generating functions



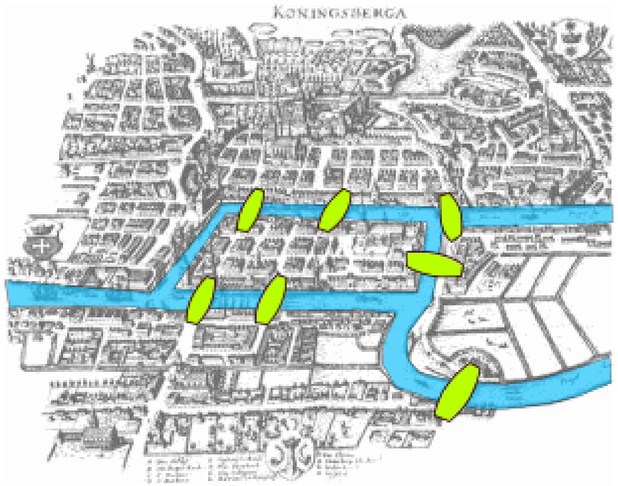
Probability, algorithms, data structures

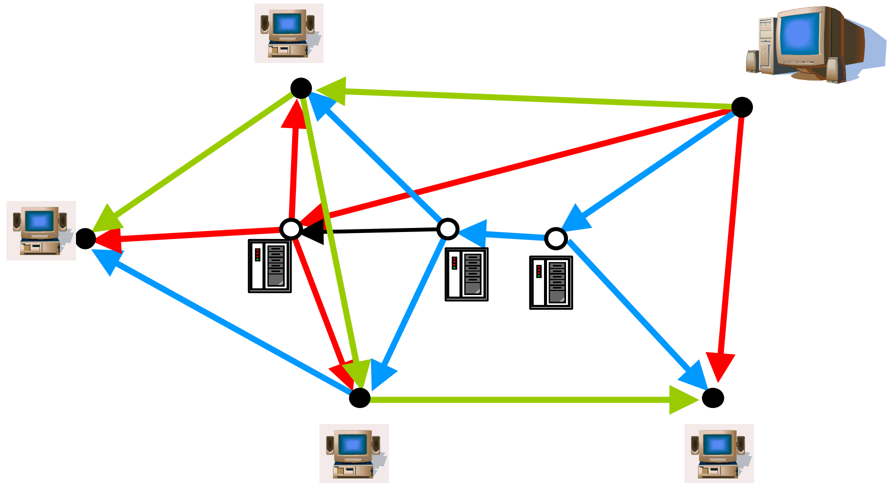
Topic 2: Counting

How many steps are needed to sort n numbers?

Topic 3: Graph Theory

- Relations, graphs
- Degree sequence, isomorphism, Eulerian graphs
- Trees





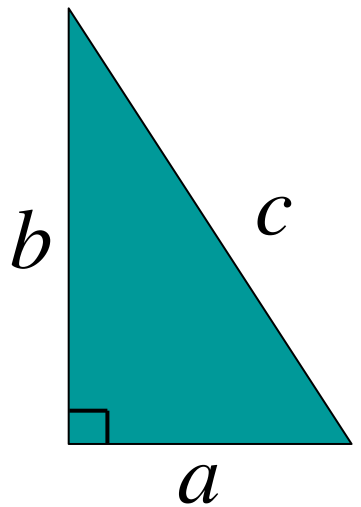
Computer networks, circuit design, data structures

Topic 4: Number Theory

- Number sequence
- Euclidean algorithm
- Prime number
- Modular arithmetic

Cryptography, coding theory, data structures

Pythagorean theorem

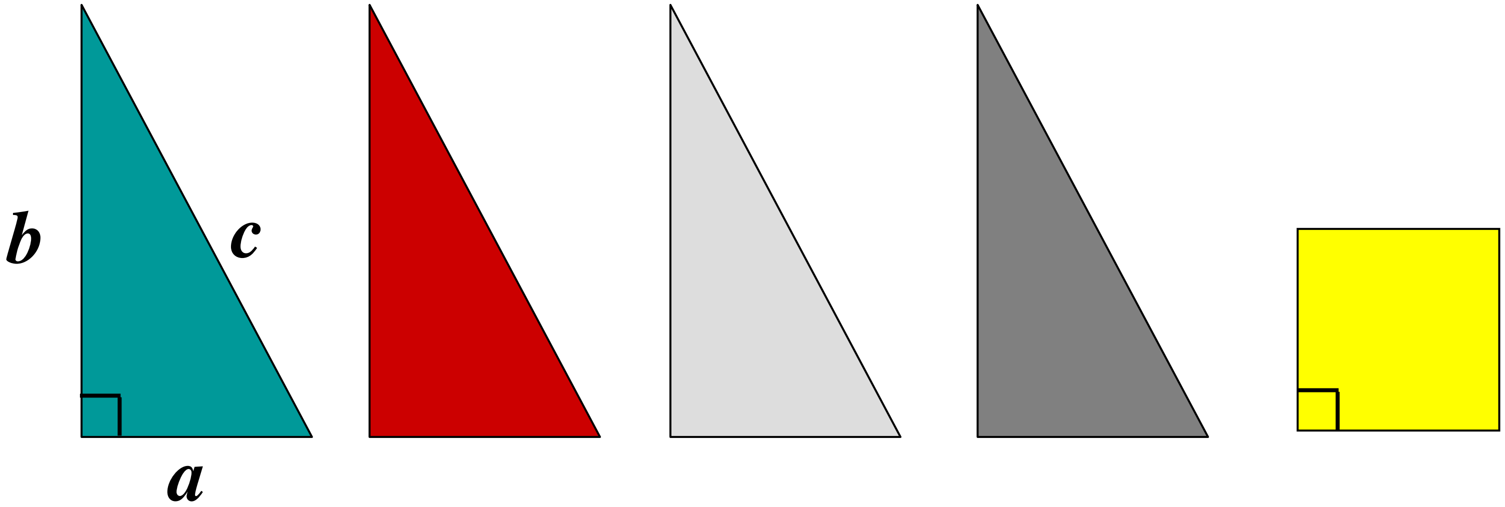


$$a^2 + b^2 = c^2$$

Familiar?

Obvious?

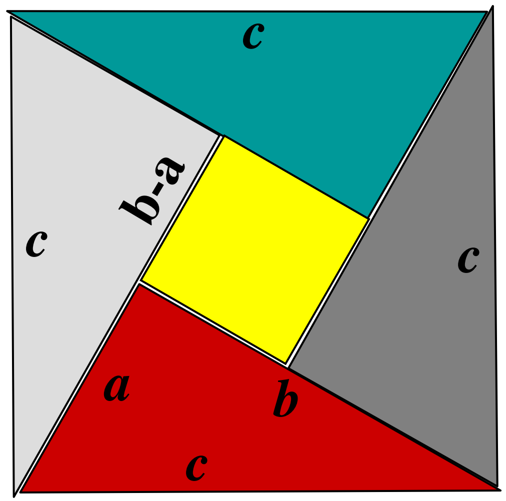
Good Proof



Rearrange into: (i) a c×c square, and then

(ii) an a×a & a b×b square

Good Proof



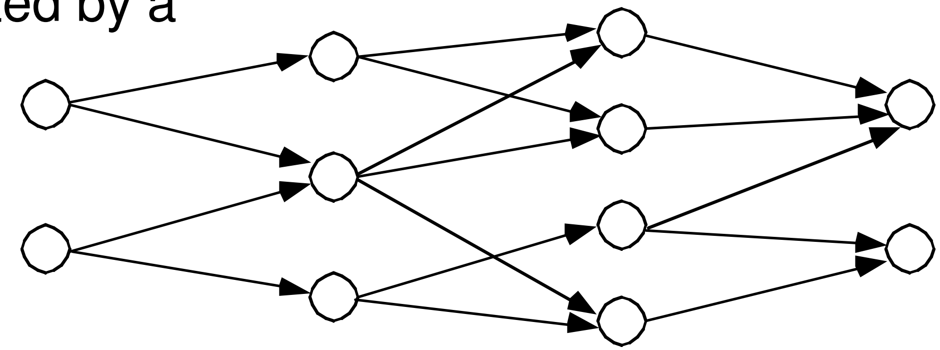
81 proofs in http://www.cut-the-knot.org/pythagoras/index.shtml

Acknowledgement

- Next slides are adapted from ones created by Professor Bart Selman at Cornell University.

Graphs and Networks

•Many problems can be represented by a graphical network representation.



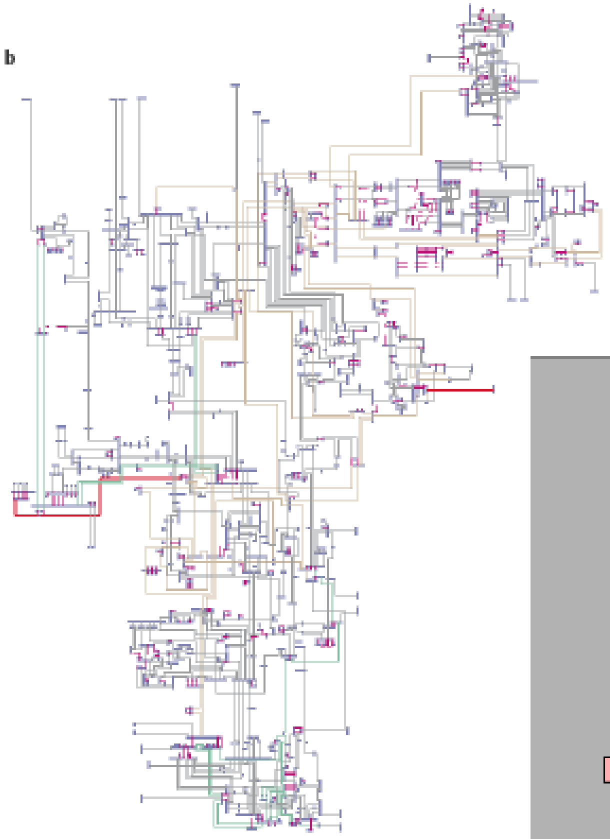
•Examples:

- Distribution problems
- Routing problems
- Maximum flow problems
- Designing computer / phone / road networks
- Equipment replacement
- And of course the Internet

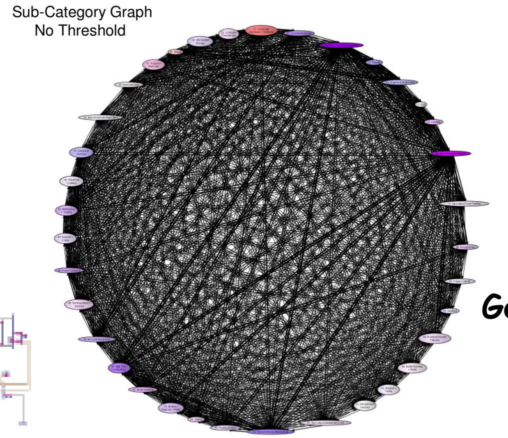
<u>Aside</u>: finding the right problem representation is one of the key issues.

New Science of Networks

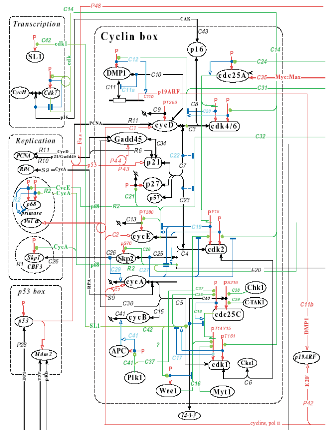
Networks are pervasive



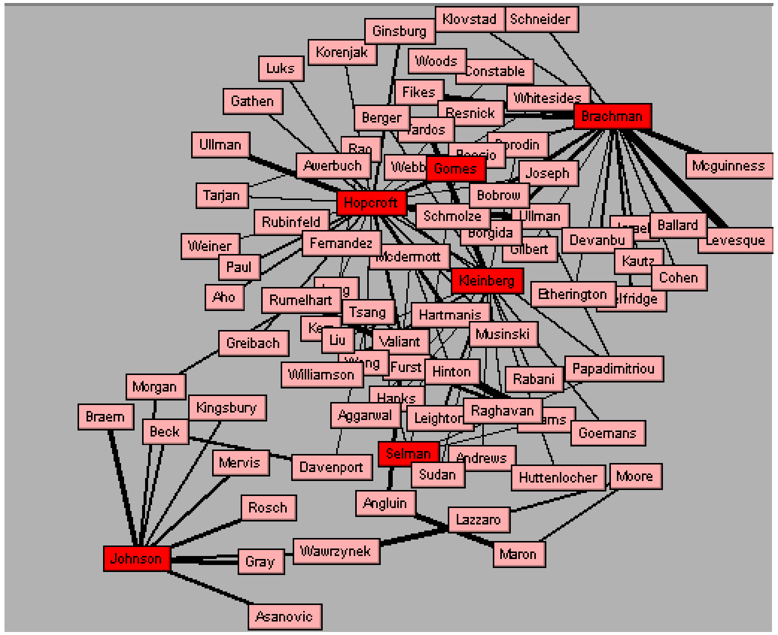
NYS Electric
Power Grid
(Thorp,Strogatz,Watts)



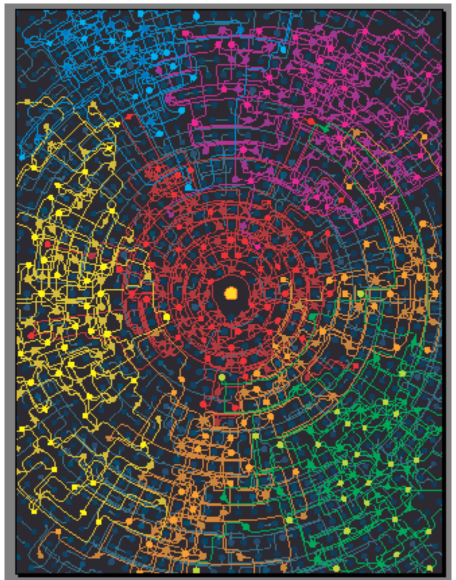
Utility Patent network
1972-1999
(3 Million patents)
Gomes,Hopcroft,Lesser,Selman



Neural network of the nematode worm C- elegans
(Strogatz, Watts)



Network of computer scientists
ReferralWeb System
(Kautz and Selman)

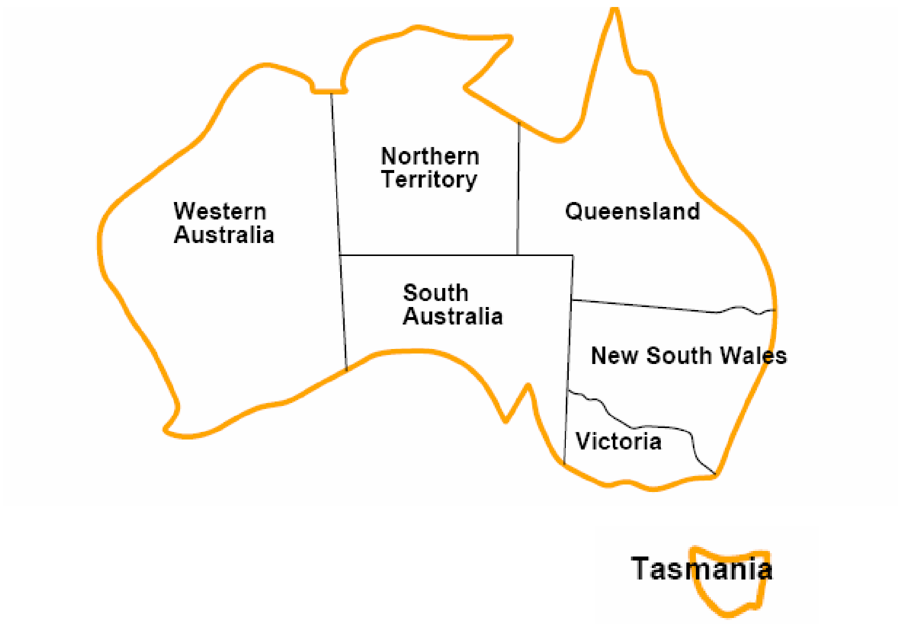


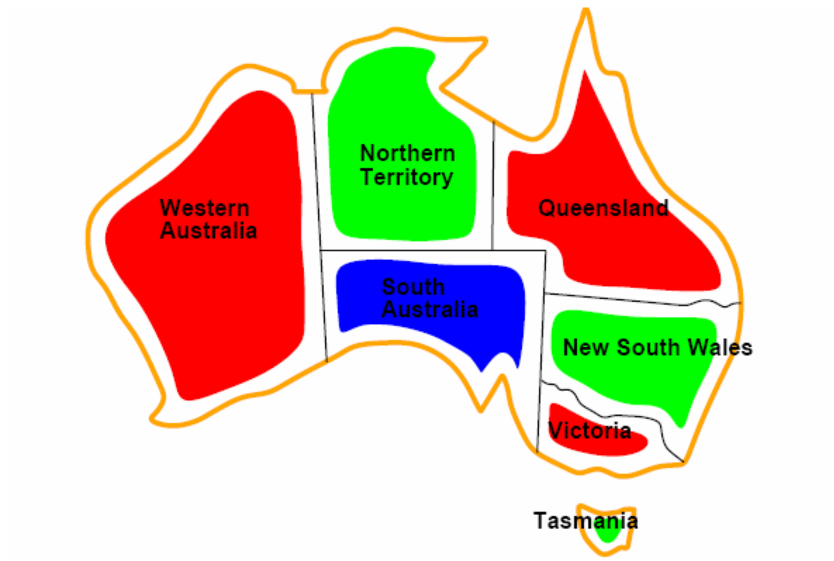
Cybercommunities
(Automatically discovered)
Kleinberg et al

Applications of Networks

Applications	Physical analog of nodes	Physical analog of arcs	Flow
Communication systems	phone exchanges, computers, transmission facilities, satellites	Cables, fiber optic links, microwave relay links	Voice messages, Data, Video transmissions
Hydraulic systems	Pumping stations Reservoirs, Lakes	Pipelines	Water, Gas, Oil, Hydraulic fluids
Integrated computer circuits	Gates, registers, processors	Wires	Electrical current
Mechanical systems	Joints	Rods, Beams, Springs	Heat, Energy
Transportation systems	Intersections, Airports, Rail yards	Highways, Airline routes Railbeds	Passengers, freight, vehicles, operators

Example: Coloring a Map

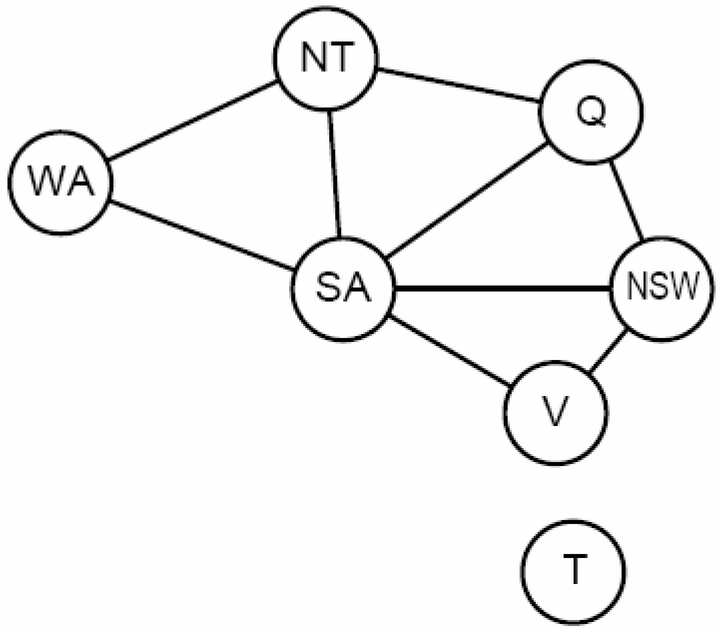




How to color this map so that no two adjacent regions have the same color?

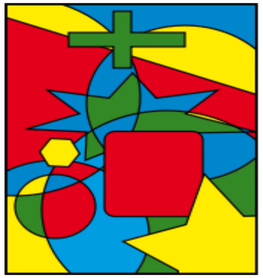
Graph representation

Abstract the essential info:



Coloring the nodes of the graph:
What's the minimum number of colors such that any two nodes connected by an edge have different colors?

Four Color Theorem



Four color map.

- The chromatic number of a graph is the least number of colors that are required to color a graph.

- The Four Color Theorem – ***the chromatic number of a planar graph is no greater than four. (quite surprising!)***

- Proof by Appel and Haken 1976;
- careful case analysis performed by computer;
- Proof reduced the infinitude of possible maps to 1,936 reducible configurations (later reduced to 1,476) which had to be checked one by one by computer.
- The computer program ran for hundreds of hours. The first significant *computer-assisted* mathematical proof. *Write-up was hundreds of pages including code!*

Examples of Applications of Graph Coloring

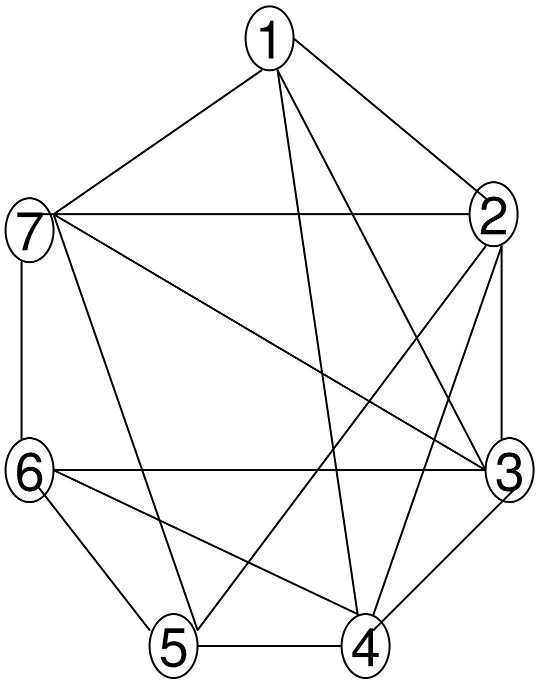
Scheduling of Final Exams

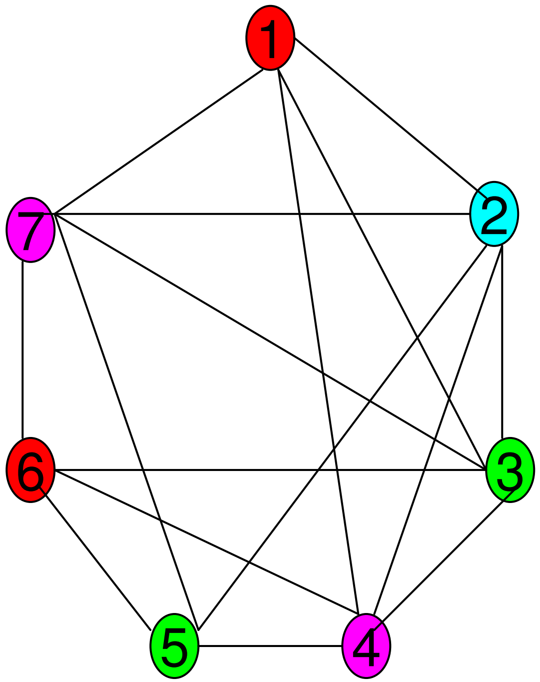
- How can the final exams at Kent State be scheduled so that no student has two exams at the same time? ***(Note not obvious this has anything to do with graphs or graph coloring!)***

Graph:
A vertex correspond to a course.
An edge between two vertices denotes that there is at least one common student in the courses they represent.
Each time slot for a final exam is represented by a different color.

A coloring of the graph corresponds to a valid schedule of the exams.

Scheduling of Final Exams





Time Period	Courses
I	1,6
II	2
III	3,5
IV	4,7

What are the constraints between courses?
Find a valid coloring

Why is mimimum number of colors useful?

Frequency Assignments

- T.V. channels 2 through 13 are assigned to stations in North America so that no two stations within 150 miles can operate on the same channel. How can the assignment of channels be modeled as a graph coloring?

- A vertex corresponds to one station
- There is a edge between two vertices if they are located within 150 miles of each other
- Coloring of graph --- corresponds to a valid assignment of channels; each color represents a different channel.

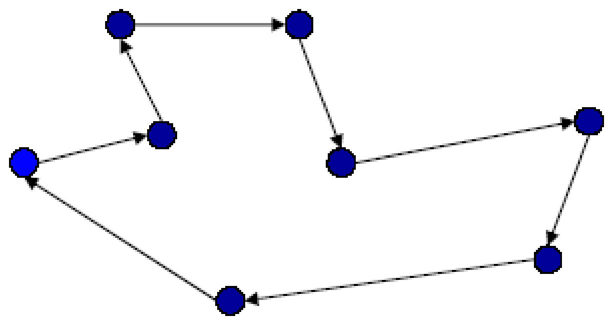
Index Registers

- In efficient compilers the execution of loops can be speeded up by storing frequently used variables temporarily in registers in the central processing unit, instead of the regular memory. For a given loop, how many index registers are needed?

- Each vertex corresponds to a variable in the loop.
- An edge between two vertices denotes the fact that the corresponding variables must be stored in registers at the same time during the execution of the loop.
- Chromatic number of the graph gives the number of index registers needed.

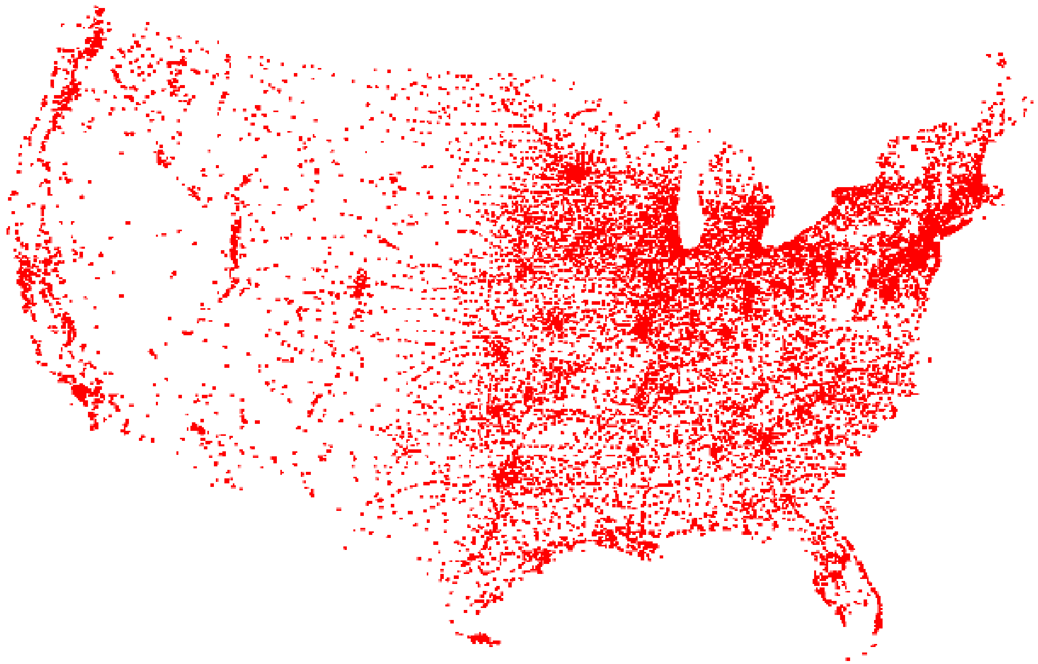
Example 2:
Traveling Salesman

Find a closed tour of minimum length visiting all the cities.



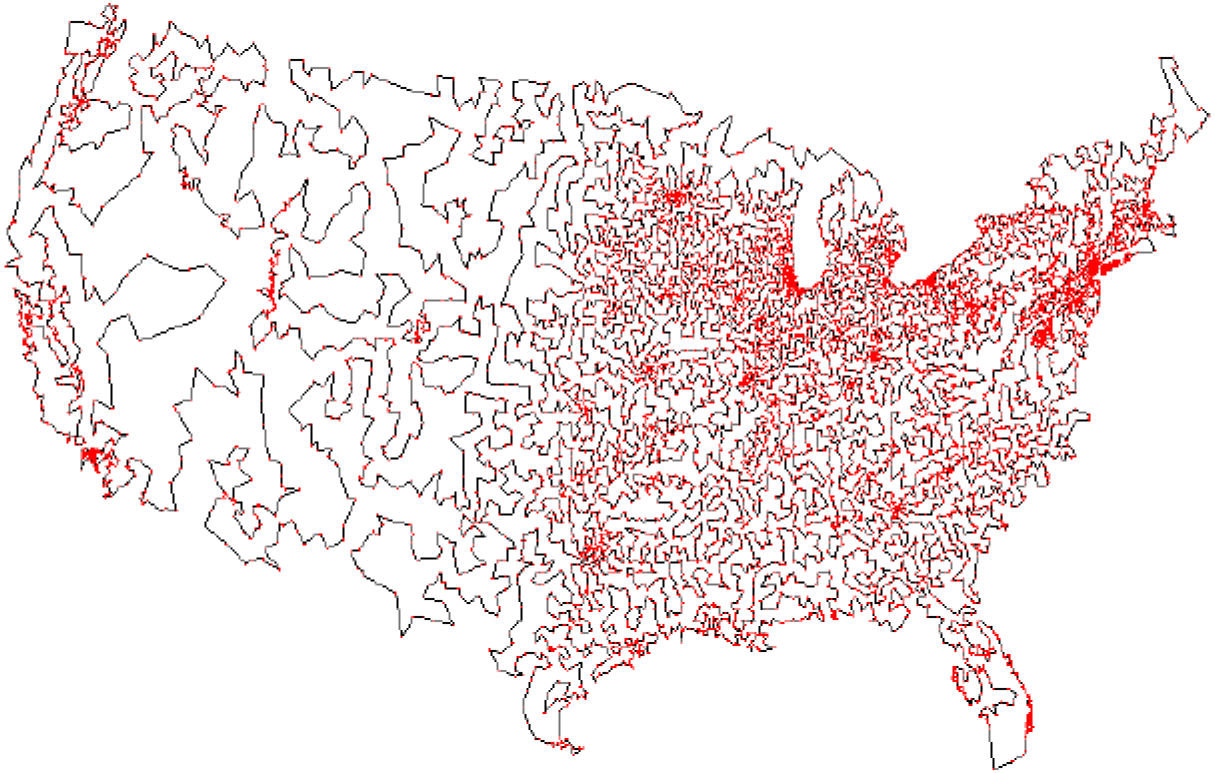
TSP → lots of applications:
Transportation related: scheduling deliveries
Many others: e.g., Scheduling of a machine to drill holes in a circuit board ;
Genome sequencing; etc

13,509 cities in the US



13508!= 1.4759774188460148199751342753208e+49936

13509 cities in the USA



(Applegate, Bixby, Chvatal and Cook, 1998)

The optimal tour!