**Q1**) Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs? [10 points]

- a) If *n* is a real number such that n > 1 then  $n^2 > 1$ . Suppose that  $n^2 > 1$ . Then n > 1.
- b) The number  $log_23$  is irrational if it is not the ratio of two integers. Therefore. Since  $log_23$  cannot be written in the form a/b where *a* and *b* be are integers, it is irrational.
- c) If *n* is real number with n > 3, then  $n^2 > 9$ . Suppose that  $n^2 \le 9$ . Then  $n \le 3$ .
- d) If *n* is real number with n > 2, then  $n^2 > 4$ . Suppose that  $n \le 2$ . Then  $n^2 \le 4$ .

Q2) What rules of inferences is used in each of these arguments? [10 points]

- *a)* Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
- *b)* It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
- *c)* Linda is an excellent swimmer. If Linda is an excellent swimmer, then she works as a lifeguard. Therefore, Linda can work as a lifeguard.
- *d*) Steve will work at a computer company this summer, therefore, this summer Steve will work at a computer company or he will be a beach bum.
- *e)* If I work all night on this homework, then I can answer all the exercises. If I answer all the exercise, will understand the material. Therefore, if I work a night on this homework, then I will understand the material.

**Q3**) For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises. [20 points]

- a) "If I play hockey, then I am sore the next day." "I use the whirlpool if I am sore." "I did not use the whirepool"
- b) "If I work, it is either sunny or partly sunny." "I worked last Monday or I wroker last Friday." "It was not partly sunny on Friday."
- c) "All insects have six legs." "Dragonflies are insects." "Spiders do not have six legs." "Spiders eat dragonflies."
- d) "Every student has an Internet account." "Homer does not have Internet account.""Maggie has an Internet account."
- e) "All food are healthy to eat do not taste good.""Tofu is healthy to eat." "You only eat what tastes good." "You do not eat tofu.""Cheeseburgers are not healthy to eat."
- f) "I am either dreaming or hallucinating." "I am not dreaming." "If I am hallucinating, I see elephantts running down the road."

Q4) Prove that the square of an even integer is an even integer using: [15 points]

- a) A direct proof b) a proof by contradiction
- **Q5**) prove that if integer k > 0 then  $k^2 + 2k + 1$  is composite. [10 points]
- **Q6**) Prove that if *n* is positive integer, then n is even if and only if 7n+4 is even. [15 points]
- Q7) Prove or disprove that the product of two irrational numbers is irrational. [10points]

**Q8**) Prove or disprove that the sum of any three consecutive integers is divisible by 3. (Two integers are consecutive if, and only if, one is one more than the other.) [10 points]