Q1) Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs? [10 points]
a) If $n$ is a real number such that $n>1$ then $n^{2}>1$. Suppose that $n^{2}>1$. Then $n>1$.
b) The number $\log _{2} 3$ is irrational if it is not the ratio of two integers. Therefore. Since $\log _{2} 3$ cannot be written in the form $a / b$ where $a$ and $b$ be are integers, it is irrational.
c) If $n$ is real number with $n>3$, then $n^{2}>9$. Suppose that $n^{2} \leq 9$. Then $n \leq 3$.
d) If $n$ is real number with $n>2$, then $n^{2}>4$. Suppose that $n \leq 2$. Then $n^{2} \leq 4$.

Q2) What rules of inferences is used in each of these arguments? ${ }_{[10 ~ p o i n t s] ~}^{\text {a }}$
a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she works as a lifeguard. Therefore, Linda can work as a lifeguard.
d) Steve will work at a computer company this summer, therefore, this summer Steve will work at a computer company or he will be a beach bum.
$e)$ If I work all night on this homework, then I can answer all the exercises. If I answer all the exercise, will understand the material. Therefore, if I work a night on this homework, then I will understand the material.

Q3) For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises. [20 points]
a) "If I play hockey, then I am sore the next day." "I use the whirlpool if I am sore." "I did not use the whirepool"
b) "If I work, it is either sunny or partly sunny." "I worked last Monday or I wroker last Friday." "It was not partly sunny on Friday."
c) "All insects have six legs." "Dragonflies are insects." "Spiders do not have six legs." "Spiders eat dragonflies."
d) "Every student has an Internet account." "Homer does not have Internet account." "Maggie has an Internet account."
e) "All food are healthy to eat do not taste good."
"Tofu is healthy to eat." "You only eat what tastes good." "You do not eat tofu." "Cheeseburgers are not healthy to eat."
f) "I am either dreaming or hallucinating." "I am not dreaming." "If I am hallucinating, I see elephantts running down the road."

Q4) Prove that the square of an even integer is an even integer using: [15 points]
a) A direct proof
b) a proof by contradiction

Q5) prove that if integer $k>0$ then $k^{2}+2 k+1$ is composite. [10 points]
Q6) Prove that if $n$ is positive integer, then n is even if and only if $7 n+4$ is even. [15 points]
Q7) Prove or disprove that the product of two irrational numbers is irrational. [10points]
Q8) Prove or disprove that the sum of any three consecutive integers is divisible by 3. (Two integers are consecutive if, and only if, one is one more than the other.) [10 points]

