Some Worked-Out Exercises from Chapter 2

Page 90, exercise 13: Find

$$\lim_{x\to 3}(4x^2 - 10x + 2)$$

without using a graphing calculator or making tables.

Let $f(x) = 4x^2 - 10x + 2$. f is a continuous function, and thus,

$$\lim_{x\to 3}(4x^2 - 10x + 2) = \lim_{x\to 3} f(x) = f(3),$$

and since $f(3) = 4 \cdot 3^2 - 10 \cdot 3 + 2 = 36 - 30 + 2 = 8$, then

$$\lim_{x\to 3}(4x^2 - 10x + 2) = 8.$$

Page 91, exercise 23: Find

$$\lim_{x\to 2} \frac{x^2 - 4}{x - 2}$$

without using a graphing calculator or making tables.

We would like to employ the same method we used in exercise 13. We let $g(x) = \frac{x^2-4}{x-2}$, and we note that g is a continuous function. Thus, we'd like to say that

$$\lim_{x\to 2} \frac{x^2 - 4}{x - 2} = \lim_{x\to 2} g(x) = g(2).$$

Everything is okay with our plan except g is not defined at 2 because when $x = 2$, the denominator of g becomes 0. Although we were correct in saying that g is a continuous function, we failed to note that g is not defined at 2. Thus, of course, g is not continuous at 2. However, we can use g as follows: $\frac{x^2-4}{x-2} = \frac{(x+2)(x-2)}{x-2} = x + 2$. Let $h(x) = x + 2$. Thus, $g(x) = h(x)$ for all real values of x except for 2. Since $g(x) = h(x)$ except when $x = 2$, then

$$\lim_{x\to 2} g(x) = \lim_{x\to 2} h(x).$$

Remember when we are finding

$$\lim_{x\to 2} g(x),$$

we don't use the value $x = 2$. Thus, we can find

$$\lim_{x\to 2} g(x)$$

by finding

$$\lim_{x\to 2} h(x).$$

However, finding

$$\lim_{x\to 2} h(x),$$

is easy. Since h is a continuous function, then

$$\lim_{x\to 2} h(x) = h(2) = 2 + 2 = 4.$$

Hence,

$$\lim_{x\to 2} \frac{x^2 - 4}{x - 2} = \lim_{x\to 2} x + 2 = 4.$$

Page 106, the exercises 17 - 24 are all the same. For each one we need to calculate

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

for a given value of x. Exercises 25 - 44 are similar to exercises 17 - 24 except we do the calculations for an arbitrary x-value, and thus, our results are of the form

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

However, the work required is basically the same for exercises 17 - 44.

Page 106, exercise 21: Find the slope of the tangent line to the curve $f(x) = 2x^2 + x - 2$ at $x = 2$.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0} \frac{2\cdot(2+h)^2 + (2+h) - 2 - (2\cdot 2^2 + 2 - 2)}{h}$$

$$= \lim_{h\to 0} \frac{2\cdot(4 + 4h + h^2) + 2 + h - 2 - (2\cdot 4 + 2 - 2)}{h}$$

$$= \lim_{h\to 0} \frac{2\cdot(4 + 4h + h^2) + h - (2\cdot 4)}{h}$$

$$= \lim_{h\to 0} \frac{8 + 8h + 2h^2 + h - 8}{h}$$

$$= \lim_{h\to 0} \frac{9h + 2h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(9 + 2h)}{h}$$

$$= \lim_{h\to 0} 9 + 2h$$

$$= 9 + 2\cdot 0 = 9$$

Page 106, exercise 25: Find $f'(x)$ for $f(x) = x^2 - 3x + 5$.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) + 5 - (x^2 - 3x + 5)}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h + 5 - x^2 + 3x - 5}{h} \\
&= \lim_{h\to 0} \frac{2xh + h^2 - 3h}{h} \\
&= \lim_{h\to 0} \frac{h(2x + h - 3)}{h} \\
&= \lim_{h\to 0} 2x + h - 3 \\
&= 2x + 0 - 3 = 2x - 3
\end{aligned}$$

Page 120, exercise 29: Find the derivative function when $f(x) = \frac{x^2+x^3}{x}$.

$$f(x) = \frac{x^2 + x^3}{x} = \frac{x(x + x^2)}{x} = x + x^2$$

Thus,

$$f'(x) = 1 + 2x$$

Page 121, exercise 41a: Find the equation of the tangent line to $f(x) = x^3 - 3x^2 + 2x - 2$ at $x = 2$.

$$f'(x) = 3x^2 - 6x + 2$$

and

$$f'(2) = 3 \cdot 2^2 - 6 \cdot 2 + 2 = 2.$$

Since the tangent line touches the graph of f when $x = 2$, we find $f(2)$ to get the y-value of this common point.

$$f(2) = 2^3 - 3 \cdot 2^2 + 2 \cdot 2 - 2 = 8 - 12 + 4 - 2 = -2$$

Thus, the tangent line has slope 2 and contains the point $(2, -2)$. Therefore, an equation for the tangent line is

$$y - (-2) = 2(x - 2)$$

$$y + 2 = 2x - 4$$

$$y = 2x - 6$$

Page 136, exercise 9: Find the derivative of $f(x) = \sqrt{x}(6x+2)$ using the product rule.

First, we rewrite $f(x) = \sqrt{x}(6x+2)$ as $f(x) = x^{\frac{1}{2}} \cdot (6x+2)$. Then it's easier for us to calculate the derivative.

$$\begin{aligned}
f'(x) &= [\frac{d}{dx}x^{\frac{1}{2}}] \cdot (6x+2) + x^{\frac{1}{2}} \cdot [\frac{d}{dx}(6x+2)] \\
&= \frac{1}{2}x^{\frac{1}{2}-\frac{2}{2}} \cdot (6x+2) + x^{\frac{1}{2}} \cdot 6 \\
&= \frac{1}{2}x^{-\frac{1}{2}} \cdot (6x+2) + 6x^{\frac{1}{2}} \\
&= \frac{1}{2} \cdot 6 \cdot x^{-\frac{1}{2}} \cdot x + \frac{1}{2} \cdot 2 \cdot x^{-\frac{1}{2}} + 6x^{\frac{1}{2}} \\
&= \frac{1}{2} \cdot 6 \cdot x^{-\frac{1}{2}+1} + \frac{1}{2} \cdot 2 \cdot x^{-\frac{1}{2}} + 6x^{\frac{1}{2}} \\
&= 3x^{\frac{1}{2}} + x^{-\frac{1}{2}} + 6x^{\frac{1}{2}} \\
&= 9x^{\frac{1}{2}} + x^{-\frac{1}{2}} \\
&= 9x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}} \\
&= 9\sqrt{x} + \frac{1}{\sqrt{x}}
\end{aligned}$$

Page 136, exercise 21: Find the derivative of $f(t) = 6t^{\frac{4}{3}}(3t^{\frac{2}{3}}+1)$ using the product rule.

$$\begin{aligned}
f'(t) &= [\frac{d}{dt}6t^{\frac{4}{3}}] \cdot (3t^{\frac{2}{3}}+1) + 6t^{\frac{4}{3}} \cdot [\frac{d}{dt}(3t^{\frac{2}{3}}+1) \\
&= 6 \cdot \frac{4}{3}t^{\frac{4}{3}-\frac{3}{3}} \cdot (3t^{\frac{2}{3}}+1) + 6t^{\frac{4}{3}} \cdot (3 \cdot \frac{2}{3} \cdot t^{\frac{2}{3}-\frac{3}{3}} + 0) \\
&= 8t^{\frac{1}{3}} \cdot (3t^{\frac{2}{3}}+1) + 6t^{\frac{4}{3}} \cdot 2t^{-\frac{1}{3}} \\
&= 24t^{\frac{1}{3}+\frac{2}{3}} + 8t^{\frac{1}{3}} + 12t^{\frac{4}{3}-\frac{1}{3}} \\
&= 24t + 8t^{\frac{1}{3}} + 12t \\
&= 36t + 8t^{\frac{1}{3}}
\end{aligned}$$

Page 137, exercise 62: A company can produce computer flash memory devices at a cost of \$6 each, while fixed costs are \$50 per day. Therefore, the company's daily cost function is $C(x) = 6x + 50$.

a. Find the average cost function $AC(x) = \frac{C(x)}{x}$.

b. Find the marginal average cost function $MAC(x)$.

c. Evaluate $MAC(x)$ at $x = 25$ and interpret your answer.

a. Since $C(x) = 6x + 50$, then $AC(x) = \frac{6x+50}{x}$. For our purposes, it will be helpful to rewrite $AC(x)$ as

$$\begin{aligned} AC(x) &= \frac{6x}{x} + \frac{50}{x} = 6 + \frac{50}{x} \\ &= 6 + 50x^{-1} \end{aligned}$$

b. It then follows that

$$\begin{aligned} MAC(x) &= \frac{d}{dx}C(x) \\ &= \frac{d}{dx}(6 + \frac{50}{x}) \\ &= 50 \cdot (-1)x^{-2} \\ &= -\frac{50}{x^2} \end{aligned}$$

c.

$$MAC(25) = -\frac{50}{25^2} = -\frac{50}{625} = -0.08$$

This means that the average cost for producing the computer flash memory devices is dropping or decreasing by \$0.08 when 25 devices have been produced in one day. To verify this, we can calculate the average cost to produce 25 items in one day and the average cost to produce 26 items in one day, and the average cost for producing 26 items should be about \$0.08 less that the average cost for producing 25 items in one day. To calculate the average costs for producing 25 and 26 items, we use the average cost function AC. We get

$$AC(25) = 6 + \frac{50}{25} = 6 + 2 = 8$$

and

$$AC(26) = 6 + \frac{50}{26} = 6 + 1.923 = 7.923$$

Thus, the average cost for producing 25 items in one day is \$8.00 and average cost for producing 26 items in one day is \$7.923 which rounds to \$7.92. Thus, the change in the average cost to produce the computer flash memory devices does drop approximately \$0.08 when going from producing 25 items in one day to 26 items in one day.

Page 161, exercise 35: Use the Generalized Power Rule to find the derivative of $f(x) = 3x^2(2x + 1)^5$.

To answer this question, we need to use the product rule before using the generalized power rule. $f(x)$ is the product of two parts. They are $3x^2$ and $(2x + 1)^5$. Thus, our answer may be gotten as follows:

$$f'(x) = [\frac{d}{dx}\,3x^2] \cdot (2x+1)^5 + 3x^2 \cdot [\frac{d}{dx}\,(2x+1)^5]$$

$$= 6x(2x+1)^5 + 3x^2 \cdot 5(2x+1)^4 \cdot [\frac{d}{dx}\,(2x+1)]$$

$$= 6x(2x+1)^5 + 3x^2 \cdot 5(2x+1)^4 \cdot 2$$

$$= 6x(2x+1)^5 + 30x^2(2x+1)^4$$

This next step is just emphasizing that there are two big terms from which we will factor out $6x(2x+1)^4$.

$$= [6x(2x+1)^5] + [30x^2(2x+1)^4]$$

$$= 6x(2x+1)^4 \cdot [(2x+1) + 5x]$$

$$= 6x(2x+1)^4(7x+1)$$