Some Worked-Out Exercises from Chapter 4

Section 4.1

Page 270, exercise 13: Find the value of \$1,000 deposited in a bank at 10% interest for 8 years compounded:

a. annually

$$FV = 1,000(1 + \frac{0.10}{1})^{1 \cdot 8} = 1,000(1.1)^8 = 2,143.59$$

b. quarterly

$$FV = 1,000(1 + \frac{0.1}{4})^{4 \cdot 8} = 1,000(1.025)^{32} = 2,203.76$$

c. continuously

$$FV = 1,000e^{0.1 \cdot 8} = 1,000e^{0.8} = 2,225.54$$

Page 270, exercise 15: A loan shark lends you \$100 at 2% compound interest per **week**.

a. How much will you owe after 3 years? Note if the interest rate is 2% per week, it is 104% per year.

$$FV = 100(1 + \frac{1.04}{52})^{52 \cdot 3} = 100(1.02)^{156} = 2,195.97$$

b. In "street" language, the profit on such a loan is known as "vigorish" or the "vig". Find the shark's vig.

$$2,195.97 - 100 = 2,095.97$$

Section 4.2

Page 286, exercise 11: Use the properties of natural logarithms to simplify the function.

$$f(x) = \ln(e^{5x}) - 2x - \ln 1 = 5x - 2x - 0 = 3x$$

Section 4.3

Page 301, exercise 1: Find the derivative of $f(x) = x^2 \ln x$.

$$f'(x) = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + \frac{x^2}{x} = 2x \ln x + x$$

Page 301, exercise 5: Find the derivative of $f(x) = \ln \sqrt{x}$.

We first rewrite f as follows: $f(x) = \ln x^{\frac{1}{2}} = \frac{1}{2} \ln x$. Then

$$f'(x) = \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2x}$$

Page 301, exercise 17: Find the derivative of $f(x) = x - e^{-x}$.

$$f'(x) = 1 - e^{-x} \cdot (-1) = 1 + e^{-x}$$

Page 301, exercise 21: Find the derivative of $f(x) = e^{1+e^x}$.

$$f'(x) = e^{1+e^x} \cdot (0 + e^x) = e^{1+e^x} \cdot e^x = e^{(1+e^x)+x} = e^{1+x+e^x}$$

Page 301, exercise 25: Find the derivative of $f(x) = e^3$.

Note that e^3 is just a number, which you can approximate using a caculator to get 20.08553. Thus, $f(x) = e^3$ is a constant function, and since it's a constant function (with no variables), its derviative is 0, i.e., $f'(x) = \frac{d}{dx} e^3 = 0$.