GOODYEAR AEROSPACE
CORPORATION
AKRON, OHIO 44315

PROPOSAL FOR STUDY OF
APPLICATION OF ASSOCIATIVE
PROCESSORS TO AIRCRAFT COLLISION AVOIDANCE

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# TABLE OF CONTENTS

**LIST OF ILLUSTRATIONS.** vii

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>INTRODUCTION</td>
</tr>
<tr>
<td>1.</td>
<td>Purpose</td>
</tr>
<tr>
<td>2.</td>
<td>Projected Summary of Program Results.</td>
</tr>
<tr>
<td>3.</td>
<td>Approach to the Problem</td>
</tr>
<tr>
<td>a.</td>
<td>General</td>
</tr>
<tr>
<td>b.</td>
<td>Problem Analysis.</td>
</tr>
<tr>
<td>c.</td>
<td>Associative Processor Programming and Comparative Analysis</td>
</tr>
<tr>
<td>d.</td>
<td>Interface Studies</td>
</tr>
<tr>
<td>e.</td>
<td>Associative Processor Design</td>
</tr>
<tr>
<td>f.</td>
<td>Future Study Recommendations</td>
</tr>
<tr>
<td>4.</td>
<td>Background</td>
</tr>
<tr>
<td>II</td>
<td>TECHNICAL DISCUSSION</td>
</tr>
<tr>
<td>1.</td>
<td>Collision Avoidance Problem.</td>
</tr>
<tr>
<td>2.</td>
<td>Current Approaches</td>
</tr>
<tr>
<td>3.</td>
<td>Proposed Approach</td>
</tr>
<tr>
<td>a.</td>
<td>General</td>
</tr>
<tr>
<td>b.</td>
<td>Collision Avoidance Algorithms</td>
</tr>
<tr>
<td>c.</td>
<td>Effects of Radar Measurement Errors</td>
</tr>
<tr>
<td>III</td>
<td>STATEMENT OF WORK.</td>
</tr>
<tr>
<td>1.</td>
<td>General</td>
</tr>
<tr>
<td>2.</td>
<td>Study and Investigation Phases</td>
</tr>
<tr>
<td>a.</td>
<td>Problem Analysis.</td>
</tr>
<tr>
<td>b.</td>
<td>Associative Processor Programming</td>
</tr>
<tr>
<td>c.</td>
<td>Comparative Analysis</td>
</tr>
<tr>
<td>d.</td>
<td>Interface Studies</td>
</tr>
<tr>
<td>e.</td>
<td>Associative Processor Design</td>
</tr>
<tr>
<td>f.</td>
<td>Future Study Recommendations</td>
</tr>
<tr>
<td>3.</td>
<td>Reporting</td>
</tr>
<tr>
<td>a.</td>
<td>Contract Status Reports</td>
</tr>
<tr>
<td>b.</td>
<td>Technical Report</td>
</tr>
<tr>
<td>c.</td>
<td>Oral Reports.</td>
</tr>
</tbody>
</table>

Page 1
<table>
<thead>
<tr>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV</td>
</tr>
<tr>
<td>PROGRAM PLAN</td>
</tr>
<tr>
<td>1. Corporate Organization</td>
</tr>
<tr>
<td>2. Project Organization</td>
</tr>
<tr>
<td>3. Project Personnel</td>
</tr>
<tr>
<td>4. Program Schedule</td>
</tr>
<tr>
<td>V</td>
</tr>
<tr>
<td>RELATED EXPERIENCE</td>
</tr>
<tr>
<td>1. General</td>
</tr>
<tr>
<td>2. Contractual Experience</td>
</tr>
<tr>
<td>a. Hybrid Associative Computer Study</td>
</tr>
<tr>
<td>b. Advanced Computer Organization Study</td>
</tr>
<tr>
<td>c. ELINT Data Processing (Phase I)</td>
</tr>
<tr>
<td>d. ELINT Data Processing (Phase II)</td>
</tr>
<tr>
<td>e. Geographic Sorting Study</td>
</tr>
<tr>
<td>f. Associative Processor for SAEWS</td>
</tr>
<tr>
<td>g. RADC 2048-Word AM</td>
</tr>
<tr>
<td>h. RADC Associative List Selector</td>
</tr>
<tr>
<td>i. NDRO Memory</td>
</tr>
<tr>
<td>3. Research and Development</td>
</tr>
<tr>
<td>a. General</td>
</tr>
<tr>
<td>b. Associative Memory Materials and Devices (R and D)</td>
</tr>
<tr>
<td>c. Associative Processor Circuits and System Development</td>
</tr>
<tr>
<td>d. Associative Systems Research</td>
</tr>
<tr>
<td>e. Associative Processor Application Studies</td>
</tr>
<tr>
<td>f. Electronic Warfare R and D</td>
</tr>
</tbody>
</table>

Appendix

A  BASIC ASSOCIATIVE PROCESSOR ORGANIZATION  51
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tactical Air Control System Deployment</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>Maximum Equipment Configuration for CRC No. 1, Including Collision Avoidance</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>Collision Avoidance Algorithm for Method 1</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>Buildup of Word Structure for Given Target to Implement Collision Avoidance Algorithm for Method 1</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>Collision Avoidance Algorithm for Method 2</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>Goodyear Aerospace Corporate Organization</td>
<td>34</td>
</tr>
<tr>
<td>7</td>
<td>Goodyear Aerospace Engineering Organization</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>Program Schedule</td>
<td>38</td>
</tr>
<tr>
<td>9</td>
<td>Research and Development in Associative Systems and Techniques</td>
<td>47</td>
</tr>
<tr>
<td>A-1</td>
<td>Block Diagram of Associative Processor</td>
<td>52</td>
</tr>
<tr>
<td>A-2</td>
<td>Tentative Response Store</td>
<td>57</td>
</tr>
</tbody>
</table>
SECTION I - INTRODUCTION

1. PURPOSE

Goodyear Aerospace Corporation (GAC) proposes a 12-month study to investigate the feasibility and practicality of applying an associative processor to the problem of aircraft collision avoidance in a tactical control system. A scenario covering the air environment for an Advanced Tactical Command and Control Capability (ATCCC) will be provided by ESD. Since the associative processor is not expected to be heavily loaded in serving this collision avoidance function, GAC proposes in addition to suggest the application of the associative processor to other ATCCC functions to achieve the goal of maximum system cost-effectiveness.

2. PROJECTED SUMMARY OF PROGRAM RESULTS

Specific considerations and results of this program will involve determination of the feasibility and practicality of the associative processor in handling the flight plans and real-time collision avoidance functions. Specifically, the results will be the following:

1. Tactical control system characteristics and air environment affecting any collision avoidance solution technique will be determined and compiled.

2. A final conflict prediction technique will be developed and selected giving consideration to:
   a. Effect of prediction accuracy of typical radar data on avoidance times and false alarms
   b. Techniques for handling aircraft crossing sector boundaries
   c. Various possible modes of operation

3. Techniques for providing safe collision avoidance maneuvering commands for conflicting aircraft will be developed.

\textsuperscript{a}For Electronic Systems Division (ESD), L. G. Hanscom Field, Bedford, Mass.
4. Final algorithms and their subsequent translation into associative processor instructions will be developed.

5. Solution time of the associative processor program will be determined (ESD will provide a comparable general-purpose computer solution).

6. Comparative analysis of computation times (associative processor versus general-purpose computer) will be conducted.

7. Requirements imposed by interface of the associative processor with typical tactical control system equipment will be determined.

8. Final design of the associative processor on the block-diagram level will be developed.

9. Cost, weight, and size of the associative processor will be estimated, based on minimum through maximum system deployment.

Another major result will be a preliminary determination of applications of an associative processor to other tactical control system functions. Those functions that appear amenable to solution on the associative processor will be recommended for further study.

3. APPROACH TO THE PROBLEM

a. General

The annual number of midair collisions has almost doubled in the last two years. This is true of military as well as commercial aircraft. To meet this alarming increase, plans are in progress to provide automatic conflict prediction. However, the current approach involves placing rather extensive equipment aboard each aircraft. This costly approach may become a necessity if the single alternative is a conventional sequentially oriented ground-based radar data-processing system. For every radar scan in a ground-based system, effective control would require that position, speed, and direction of every aircraft detected be processed with respect to all other aircraft within the surveillance range of a given radar. Even with the
fastest computers available, the sequential approach would immediately place a relatively low limit on the number of aircraft that could be handled.

An associative processor is a digital processor capable of performing common arithmetic or logical operations on all words in its memory simultaneously, compared with the conventional digital processor performance of one operation on only two words at one time. With the parallel arithmetic capability provided by the associative processor, ground-based automatic conflict prediction becomes feasible and compatible with existing tactical air control (TAC) facilities. The technique will also provide protection for the entire spectrum of aircraft in the TAC environment.

Goodyear Aerospace for seven years has studied the application of associative techniques to Air Force data-processing problems, including those problems dealing specifically with conflict prediction. Further, Goodyear Aerospace has developed an associative processor design (see Appendix A) to fit selected command and control problems, including conflict prediction. Still, the design is flexible enough to apply to other data-processing problems within the TAC structure.

The GAC program approach will consist of the following work areas:

1. Problem analysis
2. Associative processor programming and comparative analysis
3. Interface studies
4. Associative processor design
5. Future study recommendations

b. Problem Analysis

In the problem analysis phase, the purpose will be to develop the requirements for high-confidence conflict prediction and collision avoidance. As a minimum, the following aspects of the problem will be analyzed:

1. Modes of operation and associated confidence levels
2. Collision avoidance maneuvering techniques
3. Retention of manual prediction art for degraded operation
4. Conflict prediction across sector boundaries
5. Effect of aircraft characteristics and aircraft population on techniques and confidence levels
6. Effect of prediction accuracy of typical radar data on avoidance time and false alarms

c. Associative Processor Programming and Comparative Analysis

Goodyear Aerospace already has developed two algorithms for conflict prediction that can be implemented on an associative processor (see Section II). These algorithms will be analyzed with respect to performance and compatibility with the air environment, sensors, and communications in the ATCCC. Based on this analysis, one of the algorithms will be selected and refined. The final algorithm will be programmed in sufficient detail to enable a determination of the problem solution time on an associative processor model. Solution time for the program will be determined as a function of the number of aircraft handled by the system. This result will be compared with the solution time required by a conventional computer. It is proposed that the conventional computer solution be supplied by ESD.

d. Interface Studies

In the study of interface problems, GAC will consider requirements for linking the associative processor with typical tactical control system equipment. The following aspects of the interface will be investigated:

1. Format and format conversion requirements
2. Electrical compatibility
3. Software requirements
4. Modes of operation and requirements of the operator

e. Associative Processor Design

Goodyear Aerospace already has developed an associative processor organization flexible enough for application to many problems, including conflict prediction. Because the associative processor is potentially applicable to
other tactical control system functions, no attempt will be made in the proposed program to modify the existing design to provide a special purpose collision avoidance data processor. However, the existing associative processor design will be reviewed and modified as necessary to meet the requirements imposed by the collision avoidance problem. In addition, estimates of size, weight, and cost of the GAC design will be provided, based on minimum tactical control system deployment through maximum deployment.

f. Future Study Recommendations

Much less than the full capability of the associative processor is likely to be required to process collision avoidance data. If the results of the study indicate this, time will be available for other tactical control system functions. Therefore, the study will include a review of processing requirements of other ATCCC functions. Those requirements that appear amenable to solution with an associative processor will be recommended for further study.

4. BACKGROUND

Goodyear Aerospace experience in associative techniques, applications, and hardware development under Air Force, Navy, and company sponsorship totals more than 200 professional man-years. The company believes it should be selected to carry out the proposed program because it has:

1. Extensive background in all aspects of associative system technology (see Section V)
2. In-depth understanding of collision avoidance processing and other command and control applications of the associative processor
3. A flexible associative processor organization to handle collision avoidance processing and other command and control system requirements (see Appendix A)
4. A continuing associative processor hardware development program that reduces lead time needed to produce final system hardware
5. Interface with users and system developers to achieve early system utilization
6. A vital corporate interest in furthering associative system technology
SECTION II - TECHNICAL DISCUSSION

1. COLLISION AVOIDANCE PROBLEM

In the period 1959 to 1962 the annual number of U.S. Air Force midair collisions was more or less static, varying between 21 and 22. Beginning in 1964, the following midair collisions occurred:

<table>
<thead>
<tr>
<th>Year</th>
<th>Collisions(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1964</td>
<td>22</td>
</tr>
<tr>
<td>1965</td>
<td>28</td>
</tr>
<tr>
<td>1966</td>
<td>41</td>
</tr>
</tbody>
</table>

The above list shows that the number of collisions doubled in two years time although the gross number of flying hours was being reduced during the same period. This alarming increase can be attributed for the most part to: (1) increasing numbers of aircraft, (2) increasing speeds, and (3) greater cockpit workloads. The collision hazard will continue to increase unless a satisfactory collision avoidance system is developed. Currently, only manual methods of aircraft conflict prediction are available. In light of increasing numbers and velocities of aircraft, automatic yet economical means must be found for conflict prediction.

Any collision avoidance system must incorporate the following functions:

1. Detection of potentially dangerous intruders
2. Evaluation of the actual existence or nonexistence of a collision threat
3. Determination of the precise maneuver, if any is needed
4. Indication of when the maneuver should be initiated to guarantee safe clearance

These functions must be performed within certain minimum system performance requirements. The system:

1. Must handle a large number of aircraft within communication range
2. Must be capable of resolving many potential aircraft conflict situations simultaneously
3. Must process hazards in essentially real time
4. Must operate in environments with varying aircraft performance characteristics

In addition, certain additional desirable features should be associated with an acceptable collision avoidance system. These are:

1. Complete protection against all hazardous situations
2. A system that works uniformly well in all traffic densities
3. A system that works uniformly well regardless of aircraft performance differences
4. A low-cost, highly reliable system
5. Use of avoidance maneuvers consistent with airframe capability, pilot confidence, and crew safety

2. CURRENT APPROACHES

Any conceivable approach to a collision avoidance system must involve either airborne or ground responsibility for resolution of conflicts.

Currently, the primary method for avoiding conflicts is still the see-and-avoid method. This is a do-it-yourself method with the pilot fully responsible for collision avoidance. Various methods of conspicuity enhancement such as special painting and lighting techniques currently are being investigated. This is an interim approach, oriented to increasing the probability of the pilot seeing an intruder.

Only one current situation relieves the pilot of the responsibility for avoiding collisions. When visibility is too poor for the see-and-avoid method, instruments are necessary. Under these conditions, instrument flight rules
(IFR) are imposed upon the pilot. These flight rules reduce the probability of conflict. In the vicinity of an air terminal or base, where traffic converges and becomes more dense, the ground-based air traffic control (ATC) system has the responsibility for collision avoidance. The traffic in the vicinity of the airbase is under the surveillance and control of the ATC radar. When the aircraft are under visual flight rules (VFR), this system is used as an assisted see-and-avoid method of collision avoidance. Here the air traffic controller tells the pilot where to look for an intruder. It is then the pilot's responsibility to see and avoid.

Several approaches to a device termed Pilot Warning Indicator (PWI) are being evaluated. This device can be placed into the assisted see-and-avoid classification. It is an electronic device that detects intruders and "tells" the pilot approximately where to look. It remains pilot responsibility to see-and-avoid.

These interim approaches satisfy only a few of the necessary requirements for a final solution of the collision-avoidance problem.

A more exotic collision avoidance system called EROS [Eliminate Range Zero (0) System] has been proposed and is currently under evaluation. This primarily is an airborne electronic system that takes on the responsibility for collision avoidance. The system detects and evaluates potential conflicts and indicates to the pilot the type of avoidance maneuver to execute and when to execute it. From a purely technical standpoint, this system may provide a functional solution, but other practical considerations indicate that it has serious shortcomings.

To provide a completely satisfactory solution, all aircraft must be equipped with a unit. The cost of each unit would be on the order of tens of thousands of dollars. This cost, when multiplied by the total number of aircraft, would be astronomical even if the cost of each unit could be somewhat reduced. To be practical, the cost of each airborne unit should cost no more than several hundreds of dollars.

Another shortcoming of this system would be the possibility of a large number of false alarms likely to be generated in crowded environments such as found at relatively low altitudes in the vicinity of air bases or terminals.
3. PROPOSED APPROACH

a. General

A practical solution to the collision avoidance problem has not yet been found. It appears that a primarily ground-based system would be a more practical and less costly approach to the problem.

An airborne system approach would require an equipment complex with every aircraft. In a ground-based system, each equipment complex could service many aircraft. The aircraft would require only equipment for warning and for reception and display of maneuver signals.

The greatest handicap in a ground-based approach has been the requirement that, for every radar scan, every aircraft detected must be processed with respect to all other aircraft, or a large subset of all, within the surveillance range of the ground-based radar. To use conventional sequential computers for this large processing task would be to place a restrictively low limit on the number of aircraft that could be handled by a ground-based system. Primarily for this reason, the ground-based approach to a collision avoidance system has not yet been seriously considered for implementation.

Associative processors can perform logical or arithmetic operations on all items or selected items in memory simultaneously. With this capability, the ground-based approach now may provide a practical solution to the problem of collision avoidance. It is estimated that a ground-based system using conventional sequential computers may only be able to handle an estimated 100 aircraft. Associative processors on the other hand would enable a ground-based system to handle several thousand aircraft. If the environment is restricted to a maximum of 500 aircraft, the conventional sequential computer might not be adequate. The associative processor, however, could perform the tasks, requiring only 25 percent of its processing capability. Approximately 75 percent of its processing capacity would be available for other system functions.

Figure 1 shows a potential system application of the associative processor for collision avoidance. A military tactical air control system is established in the country of Utopia, under attack by the country of Redland. Blackland and Blueland, Redland allies, have also entered the conflict.
Figure 1 - Tactical Air Control System Deployment
Figure 2 shows the equipment complex for a maximally equipped Control and Reporting Center (CRC). While some CRC and Control and Reporting Post (CRP) complexes may have more or less equipment than others, all are expected to have a radar and a data-processing module. The associative processor could be conveniently integrated into these data-processing modules to add collision avoidance capability with minimum modification to the existing system (see Figure 2). The deployment of CRC/CRP operations can provide sufficient coverage for a ground-based collision avoidance system. This coverage could include hostile territory if satisfactory communications could be maintained with the Forward Air Control Post (FACP) complexes (see Figure 1).

b. **Collision Avoidance Algorithms**

(1) **General**

One method for handling collision avoidance in an associative processor involves a two-dimensional equation of the point/slope form that describes the flight path for each target. The second method is a more sophisticated three-dimensional approach that considers the controlled airspace around each aircraft. This airspace is expanded for future predictions to take care of the expanding uncertainty in aircraft position. Neither method is final. Each should be considered a starting point for further detailed study.

(2) **Method 1**

Assume that the flight paths of two aircraft are described by the following:
Figure 2 - Maximum Equipment Configuration for CRC No. 1, Including Collision Avoidance
In the above, the equations are of the point/slope form where the slope 
"a" = Δy/Δx, but Δx = ˙x and Δy = ˙y. Hence, for the same time interval, 
a = ˙y/˙x.

If \( x_m \) and \( y_m \) are the measured coordinates of a target at any given time, 
"a" can be found as above and "b" can be found by \( b = y_m - ax_m \).

The potential point of collision \((x_c, y_c)\) can be found by solving this set of equations:

\[
\begin{align*}
  y - a_1 x &= b_1 \\
  y - a_2 x &= b_2
\end{align*}
\]

\[
x = \frac{b_2 - b_1}{a_2 - a_1}
\]

Only the solution for \( x_c \) is needed since only one of the two coordinates is 
used in the calculation of time for each target to intersect the paths of each 
of the other targets.

With the above background, consider Figure 3, the collision avoidance 
algorithm flow chart for Method 1, which has the following steps:

1. After a report has been correlated with an existing 
   track, say track T, determine if T is friendly; only 
   if T is friendly will collision avoidance determina-
   tions be made.
2. If T is friendly, calculate slope "a" and y-intercept 
   "b" for all targets.
3. Examine "a" for track T; if it is less than 1, proceed 
   to Step 5; if it is not, then go to Step 4; keeping "a" 
   less than 1 avoids the case where "a" approaches 
   infinity.
4. If "a" is not less than 1, switch T and all other tracks to equations having the form \( x = ay + b \) and solve for a and b.

5. Determine if \( |a_1 - a_T| < K \), where \( a_1 \) is the slope for each of the other targets, \( a_T \) is the slope for T, and K is some predetermined constant; this is done to determine if target i is flying a path that is parallel to that of T. If parallel, i.e., \( |a_1 - a_T| < K \), then proceed to Step 19; if not, proceed to Step 6.

6. Calculate \( x_{c_i} \), a potential coordinate of collision, for each of the other paths with the path of T:

\[
x_{c_i} = \frac{b_T - b_i}{a_1 - a_T}.
\]  

(3)

7. Calculate the time for track T to collide with all other tracks:

\[
t_{T_i} = \left| \frac{x_{c_i} - x_T}{x_T} \right|.
\]  

(4)

8. Determine if \( t_{T_i} < t_1 \) minutes in those tracks that are hostile or unknown; if yes, set flag bit \( \alpha \) for each response.

9. Determine if \( t_{T_1} < 2 \) minutes in those tracks that are friendly; if yes, set flag bit \( \alpha \) for each response.

10. Determine if a new \( t_1 \) has been calculated. If Step 10 rather than Step 15 is being performed, it has not been; if no, proceed to Step 11; if yes, proceed to Step 16.

11. Proceed to Step 12 only for those targets having flag bit \( \alpha \) set; if \( \alpha \) is not set for any target, exit.

12. If \( \alpha_i \) is set, calculate the time for all other tracks to collide with track T:
\[ t_i = \frac{x_c_i - x_i}{x_i}. \]  

13. Determine if \( t_i < 1 \) minute in those tracks that are hostile or unknown; if yes, set flag bit \( \beta \) for each response.

14. Determine if \( t_i < 2 \) minutes in those tracks that are friendly; if yes, set flag bit \( \beta \) for each response.

15. Determine if a new \( t_i \) has been calculated. If Step 15 rather than Step 10 is being performed, it has been; if yes, proceed to Step 16; if no, proceed to Step 11.

16. Proceed to Step 17 only for those targets having flag bit \( \beta \) set; if \( \beta \) is not set for any target, exit.

17. If \( \beta_i \) is set, determine if \( |t_{T_i} - t_i| < 1 \) minute in those tracks that are friendly; if yes, alert an operator of a potential collision.

18. If \( \beta_i \) is set, determine if \( |t_{T_i} - t_i| < t_2 \) minutes in those tracks that are hostile or unknown; if yes, alert the battle control officer of a hostile threat.

19. If flight paths are essentially parallel, determine if \( y \)-intercepts are less than 3 mi apart in those tracks that are friendly; if no, conflict is not imminent.

20. If less than 3 mi, determine if the time for the aircraft to meet is less than 1 minute; if yes, alert an operator of a potential collision.

In the above explanation, the following should be noted:

1. The values of the constants are merely best-guess at this time and can easily be changed.

2. Sign bits of velocity vectors can be accounted for, but for ease in understanding the method rather than minute details, they have not been considered in the explanation.
3. Altitude has not been included, but if it is desired to include this parameter in collision avoidance, the following calculation can be made for each target if a collision is imminent in x and y:

\[ z_{c_i} = z_{m_i} \pm z_i t_{c_i} \]  

(6)

where

- \( z_{c_i} \) = altitude at potential point of collision for target i,
- \( z_{m_i} \) = present measured value of altitude for target i,
- \( z_i \) = rate of altitude change of target i, and
- \( t_i \) = time to reach potential point of collision for target i.

If \[ |z_{c_i} - z_{c_T}| < k \], where \( z_{c_T} \) = altitude at potential point of collision for target T, and k is a predetermined threshold, then a collision is imminent in altitude.

Figure 4 shows the buildup of the associative processor word structure for a given target necessary to implement the collision avoidance algorithm. The step numbers and the resulting word correspond to the steps as numbered on the flow chart. Since position coordinates and velocity information is already in the associative processor word for other functions, they are not shown here.

(3) Method 2

Let one aircraft be at \( x_0, y_0, h_0 \) flying at velocities \( \dot{x}_0, \dot{y}_0, \dot{h}_0 \), and another aircraft be at \( x_1, y_1, h_1 \) flying at velocities \( \dot{x}_1, \dot{y}_1, \dot{h}_1 \). It is desired to determine if one aircraft will violate the airspace of the other within some time interval \( (0, T) \). The controlled airspace around one aircraft is \( \pm \Delta_x, \pm \Delta_y, \pm \Delta_h \) at current time and expands with velocities \( \dot{\Delta}_x, \dot{\Delta}_y, \dot{\Delta}_h \), to take care of the expanding uncertainty as aircraft positions are predicted farther into the future.
Figure 4 - Buildup of Word Structure for Given Target to Implement Collision Avoidance Algorithm for Method 1
There will be conflict if and only if the following three inequalities are simultaneously satisfied for some time \( t \geq 0 \):

\[
\begin{align*}
|x_i + \dot{x}_i t - x_o - \dot{x}_o t| & \leq \Delta_x + \dot{\Delta}_x t, \\
|y_i + \dot{y}_i t - y_o - \dot{y}_o t| & \leq \Delta_y + \dot{\Delta}_y t, \\
|h_i + \dot{h}_i t - h_o - \dot{h}_o t| & \leq \Delta_h + \dot{\Delta}_h t,
\end{align*}
\tag{7}
\tag{8}
\tag{9}
\]

The easiest way to find whether such a \( t \) exists is to compute a minimum and maximum \( t \) for each inequality and compare. Thus, if

\[
\begin{align*}
t_{\min 1} & = \text{minimum time satisfying (7)}, \\
t_{\max 1} & = \text{maximum time satisfying (7)}, \\
t_{\min 2} & = \text{minimum time satisfying (8)}, \\
t_{\max 2} & = \text{maximum time satisfying (8)}, \\
t_{\min 3} & = \text{minimum time satisfying (9), and} \\
t_{\max 3} & = \text{maximum time satisfying (9)},
\end{align*}
\]

then there is a conflict within the time interval \( 0 \) to \( T \) if and only if

\[
\text{Maximum } (0, t_{\min 1}, t_{\min 2}, t_{\min 3}) \leq \text{Min } (t_{\max 1}, t_{\max 2}, t_{\max 3}, T).
\tag{10}
\]

Inequality (7) is equivalent to the following pair of inequalities:

\[
\begin{align*}
-\Delta_x - \dot{\Delta}_x t & \leq x_i + \dot{x}_i t - x_o - \dot{x}_o t, \\
x_i + \dot{x}_i t - x_o - \dot{x}_o t & \leq \Delta_x + \dot{\Delta}_x t,
\end{align*}
\tag{11}
\tag{12}
\]

which can be modified to produce

\[
\begin{align*}
x_o - x_i - \Delta_x & \leq (\dot{x}_i - \dot{x}_o - \dot{\Delta}_x) t, \\
x_o - x_i + \Delta_x & \leq (\dot{x}_i - \dot{x}_o + \dot{\Delta}_x) t.
\end{align*}
\tag{13}
\]
Let

\[ A = \frac{x_0 - x_i - \Delta_x}{\dot{x}_i - \dot{x}_o - \Delta_x} \]  

(14)

and

\[ B = \frac{x_0 - x_i + \Delta_x}{\dot{x}_i - \dot{x}_o - \Delta_x} \]  

(15)

Note that \( A \) and \( B \) will always be defined if \( \Delta_x \) is fixed so that it is not a multiple of the resolution of \( \dot{x}_o \) and \( \dot{x}_i \).

Now consider three cases: \( \dot{x}_i - \dot{x}_o + \Delta_x < 0 \), \( -\Delta_x - \dot{x}_i - \dot{x}_o < \Delta_x \), and \( \dot{x}_i - \dot{x}_o - \Delta_x > 0 \).

In the first case \( (\dot{x}_i - \dot{x}_o + \Delta_x < 0) \), \( \dot{x}_i - \dot{x}_o - \Delta_x \) also < 0; (12) and (13) become \( B \leq t \leq A \) and

\[ t_{\text{min}} = B, \]  

(16)

\[ t_{\text{max}} = A. \]  

(17)

In the second case \( (-\Delta_x < \dot{x}_i - \dot{x}_o < \Delta_x) \), (12) and (13) become \( A \leq t \leq B \) and

\[ t_{\text{min}} = \text{max} (A, B), \]  

(18)

\[ t_{\text{max}} = \infty. \]  

(19)

If this case is divided into three subcases: \( x_0 - x_i < -\Delta_x \), \( -\Delta_x < x_0 - x_i < \Delta_x \), \( \Delta_x < x_0 - x_i \), then from the following,

<table>
<thead>
<tr>
<th>Case</th>
<th>( A )</th>
<th>( B )</th>
<th>( t_{\text{min}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_0 - x_i &lt; -\Delta_x )</td>
<td>-</td>
<td>+</td>
<td>( B )</td>
</tr>
<tr>
<td>( -\Delta_x &lt; x_0 - x_i &lt; \Delta_x )</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \Delta_x &lt; x_0 - x_i )</td>
<td>+</td>
<td>-</td>
<td>( A )</td>
</tr>
</tbody>
</table>
It is seen that the choice of \( A \) or \( B \) depends on \( x_o - x_i \). In the middle sub-case, it does not matter whether \( A \) or \( B \) is picked since both are negative and the effect of the choice will be masked out when inequality (10) is checked.

In the third case, \( \dot{x}_i - \dot{x}_o - \dot{\Delta}_x > 0 \), \( \dot{x}_i - \dot{x}_o + \dot{\Delta}_x > 0 \), and (12) and (13) become \( A = t = B \), so

\[
    t_{\min 1} = A, \quad (20)
\]

\[
    t_{\max 1} = B. \quad (21)
\]

Using (8) and (9) the same way, similar equations can be developed for \( t_{\min 2} \), \( t_{\max 2} \), \( t_{\min 3} \), and \( t_{\max 3} \). Figure 5 shows the resulting algorithm for collision avoidance. This can be performed in an associative processor in parallel (one plane \( x_o', y_o', h_o' \)) against all others (\( x_i', y_i', h_i' \)). Since the most significant digits of the quotient are formed first in division, the quotient formation can be combined with the maximum and minimum operations in Boxes 1, 2, 3, and 4 in Figure 5 and need not be explicitly stored. In general, some associative processor words will be in Box 1, others in Box 2, others in Box 3, and others in Box 4 simultaneously. The operations can be carried out simultaneously.

\section*{c. Effects of Radar Measurement Errors}

Both methods proposed for handling collision avoidance involve estimating the future positions of the aircraft based on present measurements. The precision with which a potential collision can be predicted therefore depends upon the accuracy of the aircraft track information. Position information presumably will come mainly from radar measurements. Prediction accuracy depends on accuracy of the radar measurements and the nature of the tracking algorithms.

In Method 1, described above, a determination is made of the difference in time required for different aircraft to reach the intersection of the straight line extrapolations of their flight paths. This time difference for Aircraft 1 and Aircraft 2 is given by
Figure 5 - Collision Avoidance Algorithm for Method 2
\[
\Delta t = \frac{(Y_2 - Y_1)(\dot{X}_1 - \dot{X}_2) + (X_2 - X_1)(\dot{Y}_2 - \dot{Y}_1)}{(a_1 - a_2) \dot{X}_1 \dot{X}_2}
\] (22)

where \(X_1, Y_1\) and \(X_2, Y_2\) are the present positions of the two aircraft.

Equation 22 shows the relationship of radar measurements and track slope to the time difference. Fortunately, the situation is somewhat mitigated by the fact that the effects of errors tend to be proportional to the time differences. Effects of error are greatest when the time difference is large and therefore collision less imminent.

In any event, errors increase the time difference needed for safe operation. The increases in time differences required can be determined by analysis of the measurement errors anticipated from typical radar systems. It may be desirable to make the increase a function of the track slope and mean target velocities because the effect of measurement errors depend upon these quantities. This refinement can be readily incorporated into the associative processor operation. These measurement errors also affect Method 2 since the expansion of the controlled airspace is a function of the velocities \(\Delta\).

The effect of track slope on the prediction of aircraft position can perhaps be optimized with an adaptive \(\alpha - \beta\) tracking algorithm under investigation by Goodyear Aerospace. In this tracking algorithm, the \(\alpha - \beta\) parameters are made functions of the track quality and the difference between measured and predicted positions. The results appear to be more rapid response to an aircraft maneuver and better filtering of radar noise in an established track.