Data Representation

- Number systems
  - Grouping systems
    - Roman numerals (e.g, C, L, X, V, I)
    - Coins (e.g., quarter, dime, nickel, penny)
    - Order is generally not important
    - No zero
  - Positional number systems
    - Base 10 (decimal)
    - Base 2 (binary)
    - Order is important
    - Zero is used as a placeholder
- Encoding = symbolic representation of a value, in some specified number of digits, in some specified alphabet
  - Number (integers)
  - Characters

Positional Number Systems

- Base 10 (decimal)

  $1396 = 1 \bullet 10^3 + 3 \bullet 10^2 + 9 \bullet 10^1 + 6 \bullet 10^0$

- Base 2 (binary)

  $1101_2 = 1 \bullet 2^3 + 1 \bullet 2^2 + 0 \bullet 2^1 + 1 \bullet 2^0$

  $= 1 \bullet 8 + 1 \bullet 4 + 0 \bullet 2 + 1 \bullet 1$

  $= 13_{10}$

- Base 8 (octal)

  $173_8 = 1 \bullet 8^2 + 7 \bullet 8^1 + 3 \bullet 8^0$

  $= 1 \bullet 64 + 7 \bullet 8 + 3 \bullet 1$

  $= 123_{10}$

- Base 16 (hexadecimal) (A=10, B=11…)

  $7B_{16} = 7 \bullet 16^1 + 11 \bullet 16^0$

  $= 123_{10}$

Converting Base 2 to Bases 8 or 16

- Keep in mind the conversions

  000 = 0, 001 = 1, 010 = 2, 011 = 3

  100 = 4, 101 = 5, 110 = 6, 111 = 7

  1000 = 8, 1001 = 9, 1010 = A, 1011 = B

  1100 = C, 1101 = D, 1110 = E, 1111 = F

- And remember that
  - 3 bits correspond to one digit in base 8
  - 4 bits correspond to one digit in base 16
- Then simply “repartition” to convert
  - Convert 1111011_2 to base 8

    001 / 111 / 011 = 173_8
  - Convert 173_8 to base 16

    0001 / 1110 / 1011 = $7B_{16}$

Converting Base 10 to Other Bases

- Converting to base 8 the hard way:
  - Figure out how many (say) 512s, how many 64s, how many 8s, how many 1s
- Better algorithm for converting to base 8:
  - Repeatedly divide (integer division) by 8
    - Use the quotient in the next division
    - Use the remainder to form the converted value, starting with the least significant digit
    - Stop after quotient of 0 is reached
  - Example: convert 123 to base 8
    - 123 / 8 = 15, remainder of 3
    - 15 / 8 = 1, remainder of 7
    - 1 / 8 = 0, remainder of 1
    - Answer: $123_{10} = 173_8$
- Converting to base 2 is similar (although more time-consuming)

Converting Base 10 Fractions to Other Bases

- In text, explanation on page 12 is overly complicated
- Converting a base 10 fraction to base 8:
  - Repeatedly *multiply* by 8
    - Use the *fractional part of the result* in the next multiplication
    - Use the *integer part of the result* to form the converted fraction, starting with the *most* significant digit
    - Stop after fractional part of result reaches a value of 0
  - Example: convert 0.6875 to base 8
    - 0.6875 x 8 = 5.5 so far: 0.5_{8}
    - 0.5 x 8 = 4.0 so far: 0.54_{8}
    - Answer: $0.6875_{10} = 0.54_{8}$
- Converting to base 2 is similar (although more time-consuming)

Worksheet

- Convert to decimal:

  1011011_{2} 24_{8}

  14_{16} $1F7_{16}$

  10.101_{2} 3.74_{8}

- Perform conversion shown:

  234_{10} to base 8

  $F3_{16}$ to base 2

  $27E_{16}$ to base 8

  0.9375_{10} to base 8

Homework #1 — Due 9/14/98 (Part 1)

- Convert the following to decimal:

  1101_{2} 244_{8} $2B9_{16}$

- Convert the following to binary:

  89_{10} $1C4_{16}$ 56_{8}

- Convert the following to octal:

  12.484375_{10} 1010_{2} $9E7_{16}$