Synchronous Computations

In a (fully) synchronous application, all the processes are synchronized at regular points.

A *barrier*, a basic mechanism for synchronizing processes - inserted at the point in each process where it must wait.

All processes can continue from this point when all the processes have reached it (or, in some implementations, when a stated number of processes have reached this point).

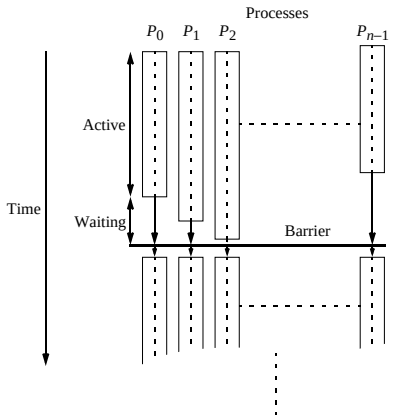


Figure 6.1 Processes reaching the barrier at different times.

In message-passing systems, barriers are often provided with library routines:

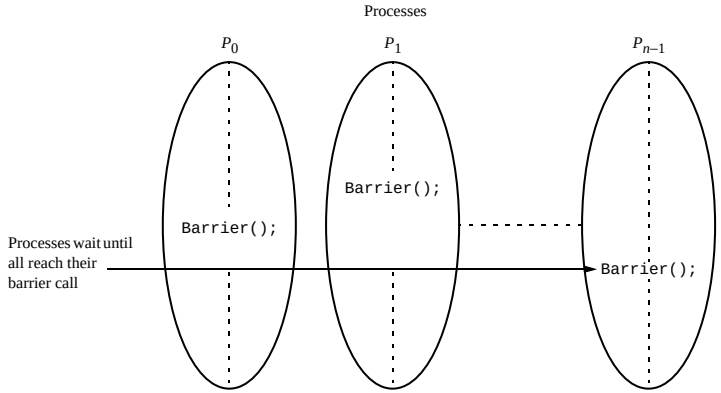


Figure 6.2 Library call barriers.

MPI has the barrier routine, `MPI_Barrier()`, with a named communicator being the only parameter.

`MPI_Barrier()` is called by each process in the group, blocking until all members of the group have reached the barrier call and only returning then.

PVM has a similar barrier routine, `pvm_barrier()`, which is used with a named group of processes.

PVM has the unusual feature of specifying the number of processes that must reach the barrier to release the processes.

Implementation

Centralized counter implementation (sometimes called a *linear barrier*):

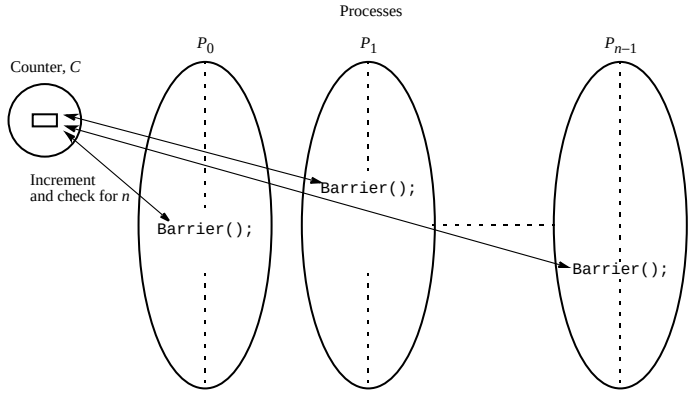


Figure 6.3 Barrier using a centralized counter.

Counter-based barriers often have two phases:

> A process enters *arrival phase* and does not leave this phase until all processes have arrived in this phase.
>
> Then processes move to *departure phase* and are released.

Good implementations of a barrier must take into account that a barrier might be used more than once in a process. It might be possible for a process to enter the barrier for a second time before previous processes have left the barrier for the first time. The two-phase design handles this scenario.

Example code:

Master:

```
for (i = 0; i < n; i++)  /* count slaves as they reach their barrier */
  recv(P_any);
for (i = 0; i < n; i++)  /* release slaves */
  send(P_i);
```

Slave processes:

```
send(P_master);
recv(P_master);
```

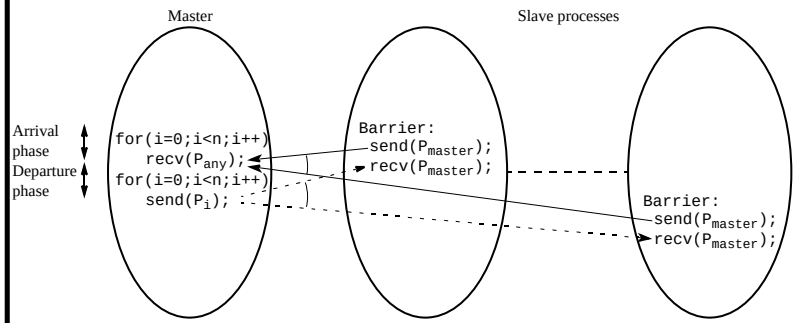


Figure 6.4 Barrier implementation in a message-passing system.

Tree Implementation

More efficient. Suppose there are eight processes, P_0, P_1, P_2, P_3, P_4, P_5, P_6, and P_7:

First stage: P_1 sends message to P_0; (when P_1 reaches its barrier)
P_3 sends message to P_2; (when P_3 reaches its barrier)
P_5 sends message to P_4; (when P_5 reaches its barrier)
P_7 sends message to P_6; (when P_7 reaches its barrier)

Second stage: P_2 sends message to P_0; (P_2 and P_3 have reached their barrier)
P_6 sends message to P_4; (P_6 and P_7 have reached their barrier)

Third stage: P_4 sends message to P_0; (P_4, P_5, P_6, and P_7 have reached their barrier)
P_0 terminates arrival phase; (when P_0 reaches barrier and has received message from P_4)

Release with a reverse tree construction.

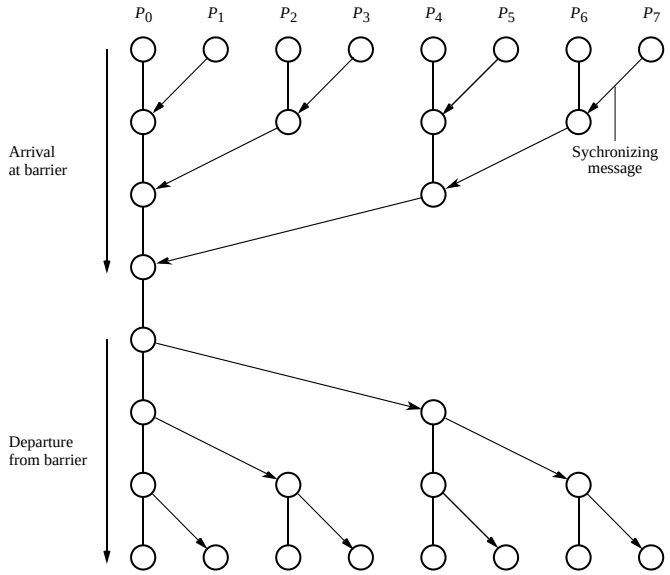


Figure 6.5 Tree barrier.

Butterfly Barrier

The tree construction can be developed into a so-called butterfly, in which pairs of processes synchronize at each stage:

First stage $P_0 \leftrightarrow P_1, P_2 \leftrightarrow P_3, P_4 \leftrightarrow P_5, P_6 \leftrightarrow P_7$
Second stage $P_0 \leftrightarrow P_2, P_1 \leftrightarrow P_3, P_4 \leftrightarrow P_6, P_5 \leftrightarrow P_7$
Third stage $P_0 \leftrightarrow P_4, P_1 \leftrightarrow P_5, P_2 \leftrightarrow P_6, P_3 \leftrightarrow P_7$

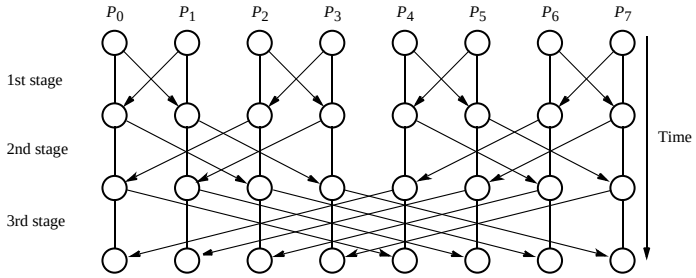
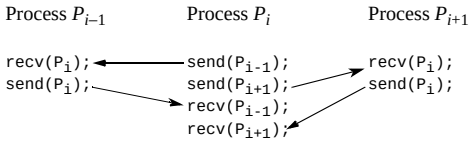


Figure 6.6 Butterfly construction.

Local Synchronization

Example

Suppose a process P_i needs to be synchronized and to exchange data with process P_{i-1} and process P_{i+1} before continuing:



Not a perfect three-process barrier because process P_{i-1} will only synchronize with P_i and continue as soon as P_i allows. Similarly, process P_{i+1} only synchronizes with P_i.

Deadlock

When a pair of processes each send and receive from each other, deadlock may occur.

Deadlock will occur if both processes perform the send, using synchronous routines first (or blocking routines without sufficient buffering). This is because neither will return; they will wait for matching receives that are never reached.

A Solution:

Arrange for one process to receive first and then send and the other process to send first and then receive.

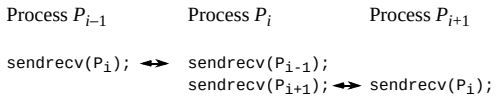
Example

Linear pipeline, deadlock can be avoided by arranging so the even-numbered processes perform their sends first and the odd-numbered processes perform their receives first.

Combined deadlock-free blocking `sendrecv()` routines

MPI provides routine `MPI_Sendrecv()` and `MPI_Sendrecv_replace()`.

Example



Synchronized Computations

Data Parallel Computations

In a data parallel computation, the same operation is performed on different data elements simultaneously; i.e., in parallel.

Particularly convenient because:

Ease of programming (essentially only one program).

Can scale easily to larger problem sizes.

Many numeric and some non-numeric problems can be cast in a data parallel form.

Example of a data parallel computation

To add the same constant to each element of an array:

```
for (i = 0; i < n; i++)
  a[i] = a[i] + k;
```

The statement `a[i] = a[i] + k` could be executed simultaneously by multiple processors, each using a different index `i` ($0 < i \leq n$).

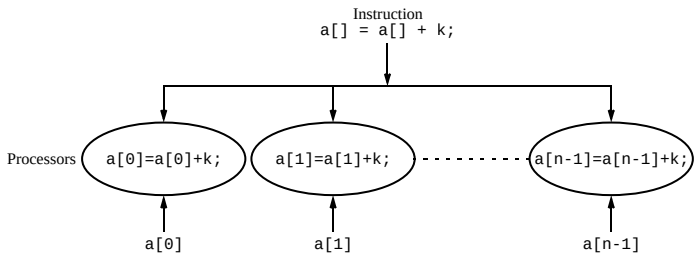


Figure 6.7 Data parallel computation.

Forall construct

Special "parallel" construct in parallel programming languages to specify data parallel operations

Example

```
forall (i = 0; i < n; i++) {
  body
}
```

states that *n* instances of the statements of the body can be executed simultaneously.

One value of the loop variable `i` is valid in each instance of the body, the first instance has `i` = 0, the next `i` = 1, and so on.

To add `k` to each element of an array, `a`, we can write

```
forall (i = 0; i < n; i++)
  a[i] = a[i] + k;
```

Data parallel technique applied to multiprocessors and multicomputers - Example:

To add `k` to the elements of an array:

```
i = myrank;
a[i] = a[i] + k;       /* body */
barrier(mygroup);
```

where `myrank` is a process rank between 0 and $n - 1$.

Prefix Sum Problem

Given a list of numbers, x_0, …, x_{n-1}, compute all the partial summations (i.e., $x_0 + x_1$; $x_0 + x_1 + x_2$; $x_0 + x_1 + x_2 + x_3$; …).

The prefix calculation can also be defined with associative operations other than addition; for example, subtraction, multiplication, maximum, minimum, and logical (Boolean) operations (AND, OR, exclusive OR, etc.).

Widely studied in connection with various computational models. Practical applications in areas such as processor allocation, data compaction, sorting, and polynomial evaluation.

The sequential code for the prefix problem could be

```
for(i = 0; i < n; i++) {
   sum[i] = 0;
   for (j = 0; j <= i; j++)
      sum[i] = sum[i] + x[j];
}
```

This is an O(n^2) algorithm.

Data parallel method of adding all partial sums of 16 numbers

Sequential code might be written as

```
for (j = 0; j < log(n); j++)          /* at each step */
   for (i = 2^j; i < n; i++)          /* add to accumulating sum */
      x[i] = x[i] + x[i - 2^j];
```

Parallel code:

```
for (j = 0; j < log(n); j++)          /* at each step */
   forall (i = 0; i < n; i++)         /* add to accumulating sum */
      if (i >= 2^j) x[i] = x[i] + x[i - 2^j];
```

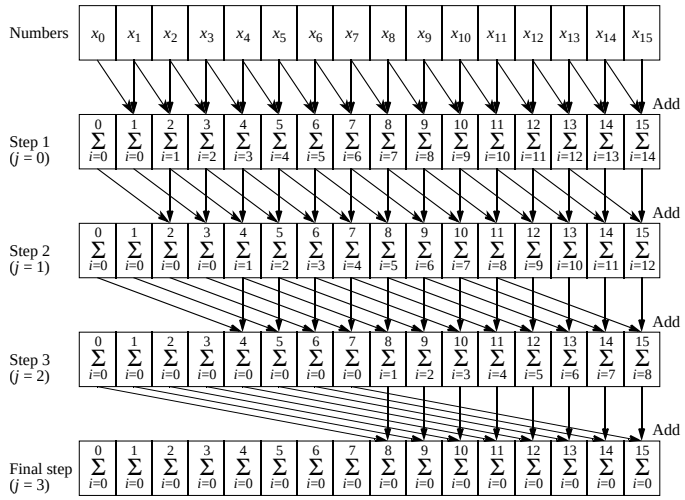


Figure 6.8 Data parallel prefix sum operation.

Synchronous Iteration

The term *synchronous iteration* or *synchronous parallelism* is used to describe solving a problem by iteration where each iteration is composed of several processes that start together at the beginning of each iteration and the next iteration cannot begin until all processes have finished the previous iteration.

The `forall` construct could be used to specify the parallel bodies of the synchronous iteration:

```
for (j = 0; j < n; j++)            /* for each synchronous iteration */
  forall (i = 0; i < N; i++) {     /* N processes each executing */
    body(i);                       /* body using specific value of i */
  }
```

In our case:

```
for (j = 0; j < n; j++) {          /* for each synchronous iteration */
  i = myrank;                      /* find value of i to be used */
  body(i);                         /* body using specific value of i */
  barrier(mygroup);
}
```


Solving a System of Linear Equations by Iteration

Suppose the equations are of a general form with n equations and n unknowns

$$\begin{aligned}
a_{n-1,0}x_0 + a_{n-1,1}x_1 + a_{n-1,2}x_2 \ldots &\quad + a_{n-1,n-1}x_{n-1} = b_{n-1}\\
&\quad \vdots\\
a_{2,0}x_0 + a_{2,1}x_1 + a_{2,2}x_2 \quad \ldots &\quad + a_{2,n-1}x_{n-1} = b_2\\
a_{1,0}x_0 + a_{1,1}x_1 + a_{1,2}x_2 \quad \ldots &\quad + a_{1,n-1}x_{n-1} = b_1\\
a_{0,0}x_0 + a_{0,1}x_1 + a_{0,2}x_2 \quad \ldots &\quad + a_{0,n-1}x_{n-1} = b_0
\end{aligned}$$

where the unknowns are x_0, x_1, x_2, … x_{n-1} ($0 \le i < n$).

One way to solve these equations for the unknowns is by iteration. By rearranging the ith equation:

$$a_{i,0}x_0 + a_{i,1}x_1 + a_{i,2}x_2 \quad \ldots \quad + a_{i,n-1}x_{n-1} = b_i$$

to

$$x_i = (1/a_{i,i})[b_i - (a_{i,0}x_0 + a_{i,1}x_1 + a_{i,2}x_2 \ldots a_{i,i-1}x_{i-1} + a_{i,i+1}x_{i+1} \ldots + a_{i,n-1}x_{n-1})]$$

or

$$x_i = \frac{1}{a_{i,i}}\left[b_i - \sum_{j \ne i} a_{i,j}x_j\right]$$

This equation gives x_i in terms of the other unknowns and can be be used as an iteration formula for each of the unknowns to obtain better approximations.

The iterative method described is called a *Jacobi iteration* – all values of x are updated together.

It can be proven that the Jacobi method will converge if the diagonal values of a have an absolute value greater than the sum of the absolute values of the other a's on the row (the array of a's is *diagonally dominant*) i.e. if

$$\sum_{j \ne i} |a_{i,j}| < |a_{i,i}|$$

This condition is a sufficient but not a necessary condition.

Termination

A simple, common approach is to compare values computed in each iteration to the values obtained from the previous iteration, and then to terminate the computation in the *t*th iteration when all values are within a given tolerance; i.e., when

$$\left|x_i^t - x_i^{t-1}\right| < \text{error tolerance}$$

for all i, where x_i^t is the value of x_i after the tth iteration and x_i^{t-1} is the value of x_i after the $(t-1)$th iteration.

However, this does not guarantee the solution to that accuracy.

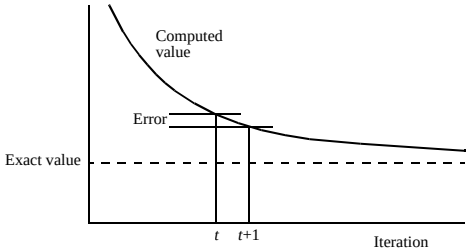


Figure 6.9 Convergence rate.

Sequential Code

Given the arrays `a[][]` and `b[]` holding the constants in the equations, `x[]` holding the unknowns, and a fixed number of iterations:

```
for (i = 0; i < n; i++)
  x[i] = b[i];                                  /*initialize unknowns*/
for (iteration = 0; iteration < limit; iteration++) {
  for (i = 0; i < n; i++) {                     /* for each unknown */
    sum = 0;
    for (j = 0; j < n; j++)       /* compute summation of a[][]x[] */
      if (i != j) sum = sum + a[i][j] * x[j];
    new_x[i] = (b[i] - sum) / a[i][i];          /* compute unknown */
  }
  for (i = 0; i < n; i++)
    x[i] = new_x[i];                            /* update values */
}
```

Slight more efficient sequential code:

```
for (i = 0; i < n; i++)
  x[i] = b[i];                                  /*initialize unknowns*/
for (iteration = 0; iteration < limit; iteration++) {
  for (i = 0; i < n; i++) {                     /* for each unknown */
    sum = -a[i][i] * x[i];
    for (j = 0; j < n; j++)                     /* compute summation */
      sum = sum + a[i][j] * x[j];
    new_x[i] = (b[i] - sum) / a[i][i];          /* compute unknown */
  }
  for (i = 0; i < n; i++) x[i] = new_x[i];      /* update values */
}
```

Parallel Code

Process P_i could be of the form

```
x[i] = b[i];                          /*initialize unknown*/
for (iteration = 0; iteration < limit; iteration++) {
   sum = -a[i][i] * x[i];
   for (j = 0; j < n; j++)            /* compute summation */
      sum = sum + a[i][j] * x[j];
   new_x[i] = (b[i] - sum) / a[i][i];  /* compute unknown */
   broadcast_receive(&new_x[i]);      /* broadcast value */
   global_barrier();                  /* wait for all processes */
}
```

The broadcast routine, `broadcast_receive()`, sends the newly computed value of `x[i]` from process *i* to every other process and collects data broadcast from the other processes.

An alternative simple solution is to return to basic `send()`s and `recv()`s, for `broadcast_receive()`; i.e., process *i* might have

```
for (j = 0; j < n; j++) if (i != j) send(&x[i], Pj);
for (j = 0; j < n; j++) if (i != j) recv(&x[j], Pj);
```


Broadcast and gather values in one composite construction - Allgather

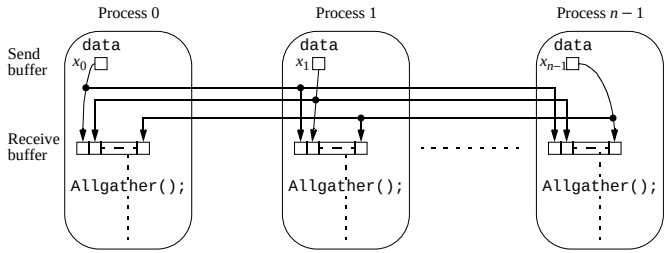


Figure 6.10 Allgather operation.

Typically, we want to Iterate until the approximations are sufficiently close, rather than for a fixed number of times (which may not provide a sufficiently accurate solution).

Each process could check its own computed value with, say,

```
x[i] = b[i];                            /*initialize unknown*/
iteration = 0;
do {
  iteration++;
  sum = -a[i][i] * x[i];
  for (j = 1; j < n; j++)               /* compute summation */
    sum = sum + a[i][j] * x[j];
  new_x[i] = (b[i] - sum) / a[i][i];    /* compute unknown */
  broadcast_receive(&new_x[i]);         /* broadcast value and wait */
} while (tolerance() && (iteration < limit));
```

where `tolerance()` returns `FALSE` if ready to terminate; otherwise it returns `TRUE`.

Partitioning

Usually the number of processors is much fewer than the number of data items to be processed (computing unknowns in this case).

Normally partition the problem so that processors take on more than one data item. In the problem at hand, each process can be responsible for computing a group of unknowns.

block allocation – allocate unknowns to processors in simple increasing order; i.e., with p processors and n unknowns.

cyclic allocation – processors are allocated one unknown in order; i.e., processor P_0 is allocated $x_0, x_p, x_{2p}, \ldots, x_{((n/p)-1)p}$, processor P_1 is allocated $x_1, x_{p+1}, x_{2p+1}, \ldots, x_{((n/p)-1)p+1}$, and so on.

Cyclic allocation no particular advantage here (Indeed, may be disadvantageous because the indices of unknowns have to be computed in a more complex way).

Analysis

Suppose there are n equations and p processors.

A processor operates upon n/p unknowns.

Suppose there are τ iterations.

One iteration has a computational phase and a broadcast communication phase.

Computation.

$$t_{comp} = n/p(2n + 4)\tau$$

Communication.

$$t_{comm} = p(t_{startup} + (n/p)t_{data})\tau = (pt_{startup} + nt_{data})\tau$$

Overall.

$$t_p = (n/p(2n + 4) + pt_{startup} + nt_{data})\tau$$

The resulting total execution time has a minimum value.

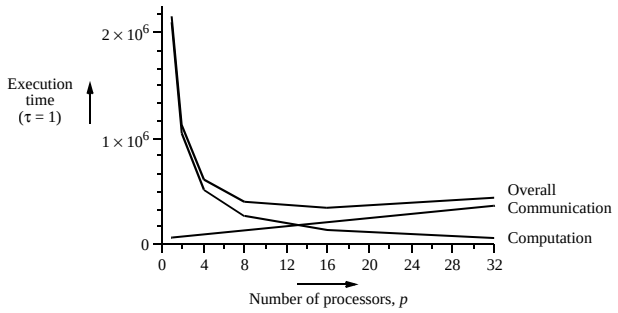


Figure 6.11 Effects of computation and communication in Jacobi iteration.

Heat Distribution Problem

Consider a square metal sheet that has known temperatures along each of its edges. The temperature of the interior surface of the sheet will depend upon the temperatures around it.

We can find the temperature distribution by dividing the area into a fine mesh of points, $h_{i,j}$.

The temperature at an inside point can be taken to be the average of the temperatures of the four neighboring points.

Convenient to describe the edges by points adjacent to the interior points. The interior points of $h_{i,j}$ are where $0 < i < n$, $0 < j < n$ [$(n - 1) \times (n - 1)$ interior points].

Compute the temperature of each point by iterating the equation

$$h_{i,j} = \frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}}{4}$$

($0 < i < n$, $0 < j < n$) for a fixed number of iterations or until the difference between iterations of a point is less than some very small prescribed amount.

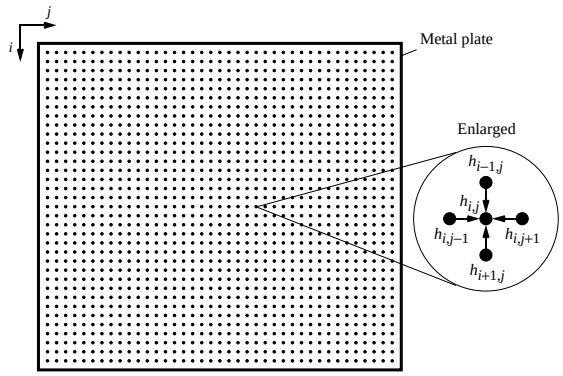


Figure 6.12 Heat distribution problem.

Actually, we are solving a system of linear equations.

Each point is an unknown dependent upon a few other unknowns, rather than all the other unknowns in the general case.

To clarify this relationship, consider the array of points as numbered in so-called *natural order*, starting at zero at the top left corner and in rows of k points:

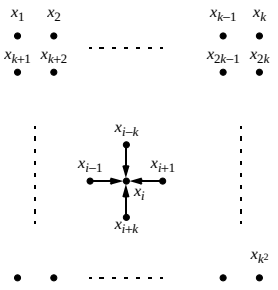


Figure 6.13 Natural ordering of heat distribution problem.

The points are numbered from 1 for convenience and include those representing the edges.

Each point will then use the equation

$$x_i = \frac{x_{i-1} + x_{i+1} + x_{i-k} + x_{i+k}}{4}$$

This could be written as a linear equation containing the unknowns x_{i-k}, x_{i-1}, x_{i+1}, and x_{i+k}:

$$x_{i-k} + x_{i-1} - 4x_i + x_{i+1} + x_{i-k} = 0$$

Known as the *finite difference* method.

We are also solving Laplace's equation.

Sequential Code

Using a fixed number of iterations

```
for (iteration = 0; iteration < limit; iteration++) {
  for (i = 1; i < n; i++)
    for (j = 1; j < n; j++)
      g[i][j] = 0.25 * (h[i-1][j] + h[i+1][j] + h[i][j-1] + h[i][j+1]);
  for (i = 1; i < n; i++)          /* update points */
    for (j = 1; j < n; j++)
      h[i][j] = g[i][j];
}
```

To stop at some precision:

```
do {
  for (i = 1; i < n; i++)
    for (j = 1; j < n; j++)
      g[i][j] = 0.25*(h[i-1][j] + h[i+1][j] + h[i][j-1] + h[i][j+1]);

  for (i = 1; i < n; i++)       /* update points */
    for (j = 1; j < n; j++)
      h[i][j] = g[i][j];

  continue = FALSE;             /* indicates whether to continue */
  for (i = 1; i < n; i++)       /* check each pt for convergence */
    for (j = 1; j < n; j++)
      if (!converged(i,j) {   /* point found not converged */
        continue = TRUE;
        break;
      }

} while (continue == TRUE);
```

Parallel Code

Version with a fixed number of iterations, process $P_{i,j}$ (except for the boundary points):

```
for (iteration = 0; iteration < limit; iteration++) {
  g = 0.25 * (w + x + y + z);
  send(&g, P_{i-1,j});           /* non-blocking sends */
  send(&g, P_{i+1,j});
  send(&g, P_{i,j-1});
  send(&g, P_{i,j+1});
  recv(&w, P_{i-1,j});           /* synchronous receives */
  recv(&x, P_{i+1,j});
  recv(&y, P_{i,j-1});
  recv(&z, P_{i,j+1});
}
```

Local barrier

after suitable initialization of `w`, `x`, `y`, and `z`.

Each process has its own iteration loop. The number of iterations must be sent to each process.

It is important to use `send()`s that do not block while waiting for the `recv()`s; otherwise the processes would deadlock, each waiting for a `recv()` before moving on.

The `recv()`s must be synchronous and wait for the `send()`s. Each process will be synchronized with its four neighbors by the `recv()`s.

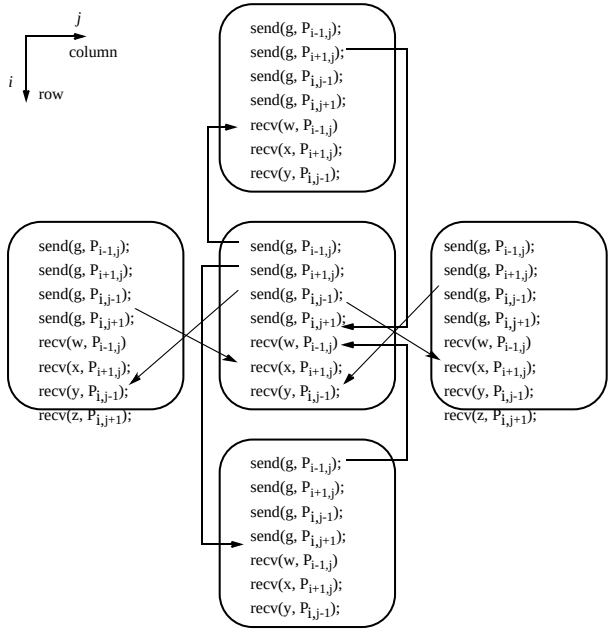


Figure 6.14 Message passing for heat distribution problem.

Version where processes stop when they reach their required precision:

```
iteration = 0;
do {
   iteration++;
   g = 0.25 * (w + x + y + z);
   send(&g, P_{i-1,j});                  /* locally blocking sends */
   send(&g, P_{i+1,j});
   send(&g, P_{i,j-1});
   send(&g, P_{i,j+1});
   recv(&w, P_{i-1,j});                  /* locally blocking receives */
   recv(&x, P_{i+1,j});
   recv(&y, P_{i,j-1});
   recv(&z, P_{i,j+1});
} while((!converged(i, j)) || (iteration < limit));
send(&g, &i, &j, &iteration, P_master);
```

To handle the processes operating at the edges, we could use the process ID to determine the location of the process in the array, leading to code such as

```
if (last_row) w = bottom_value;
if (first_row) x = top_value;
if (first_column) y = left_value;
if (last_column) z = right_value;
iteration = 0;
do {
   iteration++;
   g = 0.25 * (w + x + y + z);
   if !(first_row) send(&g, P_{i-1,j});
   if !(last_row) send(&g, P_{i+1,j});
   if !(first_column) send(&g, P_{i,j-1});
   if !(last_column) send(&g, P_{i,j+1});
   if !(last_row) recv(&w, P_{i-1,j});
   if !(first_row) recv(&x, P_{i+1,j});
   if !(first_column) recv(&y, P_{i,j-1});
   if !(last_column) recv(&z, P_{i,j+1});
} while((!converged) || (iteration < limit));
send(&g, &i, &j, iteration, P_master);
```

.

Partitioning

Normally allocate more than one point to each processor, because there would be many more points than processors. The mesh of points could be partitioned into square blocks or strips (columns):

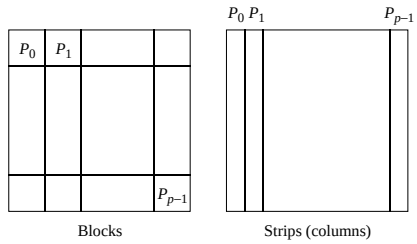


Figure 6.15 Partitioning heat distribution problem.

Block partition:

Four edges where data points are exchanged. Communication time is given by

$$t_{commsq} = 8\left(t_{startup} + \sqrt{\frac{n}{p}}t_{data}\right)$$

This equation is only valid for $p \geq 9$ when at least one block has four communicating edges.

Strip partition

Two edges where data points are exchanged. Communication time is given by

$$t_{commcol} = 4(t_{startup} + \sqrt{n}t_{data})$$

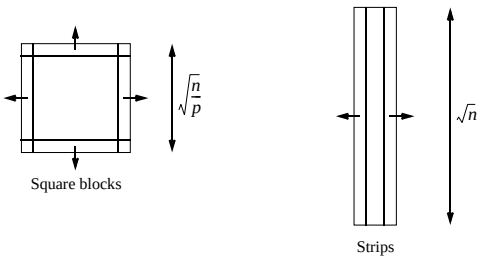


Figure 6.16 Communication consequences of partitioning.

Optimum

In general, the strip partition is best for a large startup time, and a block partition is best for a small startup time. With the previous equations, the block partition has a larger communication time than the strip partition if

$$8\left(t_{startup} + \sqrt{\frac{n}{p}}t_{data}\right) > 4(t_{startup} + \sqrt{n}t_{data})$$

or

$$t_{startup} > \sqrt{n}\left(1 - \frac{2}{\sqrt{p}}\right)t_{data}$$

($p \geq 9$).

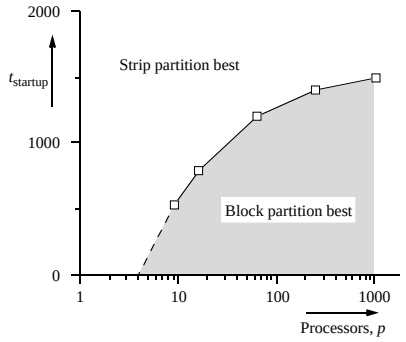


Figure 6.17 Startup times for block and strip partitions.

Ghost Points

Convenient to an additional row of points at each edge, called *ghost points*, that hold the values from the adjacent edge. Each array of points is increased to accommodate the ghost rows.

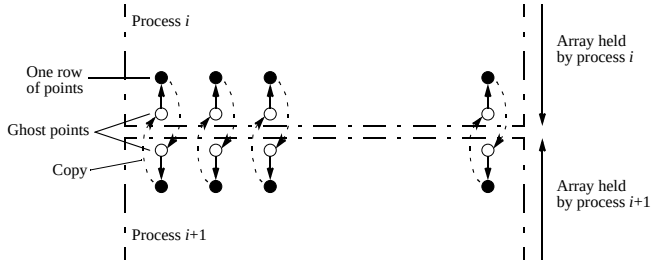


Figure 6.18 Configurating array into contiguous rows for each process, with ghost points.

Safety and Deadlock

When all processes send their messages first and then receive all of their messages, as in all the code so far, is described as "unsafe" in the MPI literature because it relies upon buffering in the `send()`s. The amount of buffering is not specified in MPI.

If a send routine has insufficient storage available when it is called, the implementation should be such to delay the routine from returning until storage becomes available or until the message can be sent without buffering.

Hence, the locally blocking `send()` could behave as a synchronous `send()`, only returning when the matching `recv()` is executed. Since a matching `recv()` would never be executed if all the `send()`s are synchronous, deadlock would occur.

A way of making the code safe is to alternate the order of the `send()`s and `recv()`s in adjacent processes. This is so that only one process performs the `send()`s first.

Then even synchronous `send()`s would not cause deadlock. In fact, a good way you can test for safety is to replace message-passing routines in a program with synchronous versions.

Safe code, by alternating the `send()`s and `recv()`s, could be of the form

```
if ((myid % 2) == 0) {                    /* even processes */
   send(&g[1][1], &m, P_{i-1});
   recv(&h[1][0], &m, P_{i-1});
   send(&g[1,m], &m, P_{i+1});
   recv(&h[1][m+1], &m, P_{i+1});
} else {                                  /* odd numbered processes */
   recv(&h[1][0], &m, P_{i-1});
   send(&g[1][1], &m, P_{i-1});
   recv(&h[1][m+1], &m, P_{i+1});
   send(&g[1,m], &m, P_{i+1});
}
```

MPI Safe message Passing Routines

MPI offers several alternative methods for safe communication:

- Combined send and receive routines: `MPI_Sendrecv()` (which is guaranteed not to deadlock)
- Buffered send()s: `MPI_Bsend()` — here the user provides explicit storage space
- Nonblocking routines: `MPI_Isend()` and `MPI_Irecv()`— here the routine returns immediately, and a separate routine is used to establish whether the message has been received (`MPI_Wait()`, `MPI_Waitall()`, `MPI_Waitany()`, `MPI_Test()`, `MPI_Testall()`, or `MPI_Testany()`)

A pseudocode segment using the third method is

```
isend(&g[1][1], &m, P_{i-1});
isend(&g[1,m], &m, P_{i+1});
irecv(&h[1][0], &m, P_{i-1});
irecv(&h[1][m+1], &m, P_{i+1});
waitall(4);
```

Essentially, the wait routine becomes a barrier, waiting for all the message-passing routines to complete.

Cellular Automata

In this approach, the problem space is first divided into cells.

Each cell can be in one of a finite number of states.

Cells are affected by their neighbors according to certain rules, and all cells are affected simultaneously in a "generation."

The rules are reapplied in subsequent generations so that cells evolve, or change state, from generation to generation.

The most famous cellular automata is the "Game of Life" devised by John Horton Conway, a Cambridge mathematician, and published by Gardner (Gardner, 1967).

The Game of Life

Board game; the board consists of a (theoretically infinite) two-dimensional array of cells.

Each cell can hold one "organism" and has eight neighboring cells, including those diagonally adjacent.

Initially, some of the cells are occupied in a pattern.

The following rules apply:

1. Every organism with two or three neighboring organisms survives for the next generation.
2. Every organism with four or more neighbors dies from overpopulation.
3. Every organism with one neighbor or none dies from isolation.
4. Each empty cell adjacent to exactly three occupied neighbors will give birth to an organism.

These rules were derived by Conway "after a long period of experimentation."

Simple Fun Examples of Cellular Automata

"Sharks and Fishes" in the sea, each with different behavior rules.

An ocean could be modeled as a three-dimensional array of cells.

Each cell can hold one fish or one shark (but not both).

Fish might move around according to these rules:

1. If there is one empty adjacent cell, the fish moves to this cell.
2. If there is more than one empty adjacent cell, the fish moves to one cell chosen at random.
3. If there are no empty adjacent cells, the fish stays where it is.
4. If the fish moves and has reached its breeding age, it gives birth to a baby fish, which is left in the vacating cell.
5. Fish die after *x* generations.

The sharks might be governed by the following rules:

1. If one adjacent cell is occupied by a fish, the shark moves to this cell and eats the fish.
2. If more than one adjacent cell is occupied by a fish, the shark chooses one fish at random, moves to the cell occupied by the fish, and eats the fish.
3. If no fish are in adjacent cells, the shark chooses an unoccupied adjacent cell to move to in a similar manner as fish move.
4. If the shark moves and has reached its breeding age, it gives birth to a baby shark, which is left in the vacating cell.
5. If a shark has not eaten for *y* generations, it dies.

Similar examples: "foxes and rabbits" -The behavior of the rabbits is to move around happily whereas the behavior of the foxes is to eat any rabbits they come across.

Serious Applications for Cellular Automata

Examples - fluid/gas dynamics, the movement of fluids and gases around objects or diffusion of gases, biological growth, airflow across an airplane wing, erosion/movement of sand at a beach or riverbank.

PROBLEMS

Scientific/Numerical

6-1. Implement the counter barrier described in Figure 6.4, and test it. Is it necessary to use blocking or synchronous routines for both send and receive? Explain.

6-2. Write a barrier, `barrier(procno)`, which will block until `procno` processes reach the barrier and then release the processes. Allow for the barrier to be called with different numbers of processes and with different values for `procno`.

6-3. Investigate the time that a barrier takes to operate by using code such as

```
t1 = time();
Barrier(group);
t2 = time();
printf("Elapsed time = %d", difftime(t2, t1));
```

(In MPI the barrier routine is `MPI_Barrier(Communicator)`. The time routine is `MPI_Wtime()`.) Investigate different numbers of processes.

6-4. Write code to implement an eight-process barrier using the tree construction described in Section and compare with any available barrier calls (e.g., in MPI `MPI_Barrier()`).

6-5. Implement the butterfly barrier described in Section 6.1.4, and compare with any available barrier calls.

6-6. Determine experimentally at what point in your system the limit to buffering is reached when using nonblocking sends. Establish the effects of requesting more buffering than is available. (It may be that the amount of buffering available is related to the amount of memory being used for other purposes.)

6-7. Can noncommutative operators such as division be used in the prefix calculation of Figure 6.8?

6-8. Determine the efficiency of the prefix calculation of Figure 6.8.

6-9. Given a fixed rectangular area with sides x and y and a communication that is proportional to the perimeter, $2(x + y)$, show that the minimum communication is given by $x = y$ (i.e., a square).

6-10. Write a parallel program to solve the one-dimensional problem based upon finite difference equation

$$x_i = \frac{x_{i-1} + x_{i+1}}{2}$$

for $0 \leq i \leq 1000$, given that $x_0 = 10$ and $x_{1000} = 250$.

6-11. In the text, we have assumed a square array for the heat distribution problem of Section 6.3.2. What are the mathematical conditions for choosing blocks or strips as the partition if the array has a length of n points and a width of m points?

6-12. Investigate the accuracy of convergence of the heat distribution problem using different termination methods described in Section . Determine whether it is sufficient to use the difference between the present and next values of the points or whether it is necessary to use a more complex termination calculation. The basic question being investigated here is, "If each point is computed until each is within 1% (say) of its previous computed value, what is the accuracy of the solution?"

6-13. Write a parallel program to simulate the Game of Life as described in Section and experiment with different initial populations.

Real Life

6-14. Figure 6.19 shows a room that has four walls and a fireplace. The temperature of the wall is 20°C, and the temperature of the fireplace is 100°C. Write a parallel program using Jacobi iteration to compute the temperature inside the room and plot (preferably in color) temperature contours at 10°C intervals using Xlib calls or similar graphics calls as available on your system. Instrument the code so that the elapsed time is displayed. (This programming assignment is convenient after a Mandelbrot assignment because it can use the same graphics calls.)

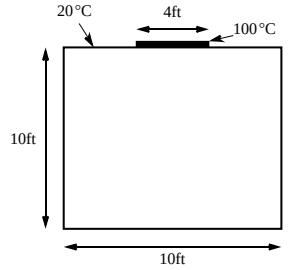


Figure 6.19 Room for Problem 6-14.

6-15. Repeat Problem 6-14 but with a round room of diameter 20 ft and a point heat source in the center at 100°C; the walls are at 20°C.

6-16. Simulate a road junction controlled by traffic lights as shown in Figure 6.20. Vehicles come from all four directions along the roads and either wish to pass straight through the junction to the other side, or turn left, or turn right. On average, 70% of vehicles wish to pass straight through, 10% wish to turn right, and 20% wish to turn left. Each vehicle moves at the same speed up to the junction. Develop a set of driving rules to solve this problem by a cellular automata approach, and implement them in a parallel program using your own test data (vehicle numbers and positions).

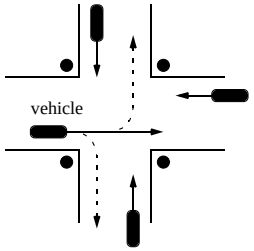


Figure 6.20 Road junction for Problem 6-16.

6-17. Write a parallel program to simulate the actions of the sharks and fish as described in Section . The parameters that are input are size of ocean, number of fish and sharks, their initial placement in the ocean, breeding ages, and shark starvation time. Adjacent cells do not include diagonally adjacent cells. Therefore, there are six adjacent cells, except for the edges. For every generation, the fishes' and sharks' ages are incremented by one. Modify the simulation to take into account currents in the water.

6-23. (A research assignment) Develop the rules necessary to model the airflow across a wing as shown in Figure 6.21 (two dimensions). Select your own dimensions for the solution space and object. Select the number of grid points and write code to solve the problem.

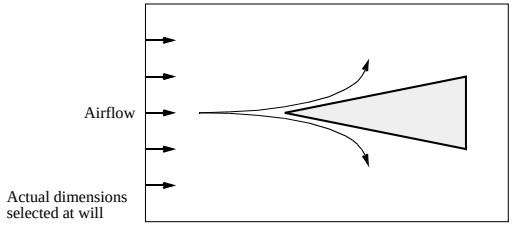


Figure 6.21 Figure for Problem 6-23.