TIP-09353-2012.R1: Summary of Changes

The authors would like to thank the editors and the three anonymous reviewers for their constructive and pertinent comments. In the revised paper, we have very carefully followed all the comments and suggestions and cleared up every issue raised. We provide below a summary of changes that we have made in response to the comments that the editors and the reviewers have raised.

Response to Reviewer #1:

1. The given examples are not highly competitive to show the goodness of the paper. I would suggest the authors to provide special results and descriptions on how this particular method is better with its new design.

Authors’ reply:

Thanks for this constructive comment. We therefore introduced a supervised quantitative evaluation for the performance of the proposed algorithm using test images and the manual segmentation of the Berkeley dataset BSDS300. The evaluation criteria used are the Martin and the Hausdorff metrics [33]-[34]. The experimental results have been reported in the manuscript in Fig. 10 and Tables VI and VII.

2. LBM may be very fast on GPU, but the computing of image features, such as local average value and histogram which involve statistical information would be time consuming on GPU. This paper does not give a clear discussion on the performance. You should provide the speed data for each step in the whole segmentation process. Make it clear which parts of algorithm are computed on GPU and which parts are on CPU?

Authors’ reply:

Thanks for this pertinent suggestion. We addressed this problem by providing the executive time for each step in the segmentation process. Furthermore a flowchart diagram makes it clear which part of the algorithm is computed on CPU and which part is computed on GPU. The results have been reported in the manuscript as follows, and the changed parts are in blue.

Table I. Executive times and objective evaluation results.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Inter LN</th>
<th>Intra LN</th>
<th>Executive time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>GPU</td>
<td>Total</td>
</tr>
<tr>
<td>Our method</td>
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<td>3.1197e-06</td>
<td>0.4415</td>
</tr>
<tr>
<td>C-V method</td>
<td>0.0773</td>
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<td>0.0997</td>
</tr>
<tr>
<td>Li’s method</td>
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<td>3.6571e-06</td>
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</tr>
<tr>
<td>Chen’s method</td>
<td>0.0732</td>
<td>5.4410e-06</td>
<td>15.0292</td>
</tr>
</tbody>
</table>

Note: the size of test image is 700 X 438.
3. Parameter $g_{ij}$ in Eq. (15) isn’t properly defined.

**Authors’ reply:**

Thanks for this lighting comment. We revised the definition of the 2D histogram in the manuscript as follows:

The energy term $E_{2D-Hist}(\phi)$ is based on the 2D histogram information. Let consider the 2D gray-scale histogram of an $M \times N$ image where the X-axis represents the intensity and the Y-axis the local average. By analyzing it, we define...
three sectors as shown in Fig.1. The first sector is the zone surrounding the straight line $I = I_{\text{mean}}$, i.e., pixels of which the intensity is equal to the local average. Those pixels belong to the object, the background, and the boundaries which are not defined by edges. In this sector the pixels intensity is slowly varying, thus the gray level of a given pixel is not far from the local average. In the proposed LSE, one of the contributions of the term $-\frac{\partial E_{\text{img}}}{\partial \phi}$ is to allow the detection of objects which boundaries are not defined by edges and therefore has its pixels in sector I.

Response to Reviewer #2:

1. The introduction starts immediately with the description of the LSM, it is very difficult to understand why this technique is used here and no other well-known segmentation approaches. The goal of the segmentation is not defined at all. Do authors want to find a binary decomposition, subtract foreground from the background or find and identify uniquely all objects in the scene? It's not clear.

Authors’ reply:

Thanks for this constructive and judicious comment. The introduction has therefore been modified as follows, and the changed parts are in green.

I. INTRODUCTION

THE level set technique is a general framework for tracking dynamic interfaces and shapes. It was first developed as a way to model fluid boundaries, such as a flame front. In computer vision and pattern recognition the level set method (LSM) had been widely used for image segmentation [1]-[5]. The attractive advantage of the LSM is its ability to extract complex contours and to automatically handle topological changes, such as splitting and merging. The LSM belongs to the active contours models (ACMs) which is based on the Eulerian framework, i.e., the geometric representation of the active contour instead of the parametric representation which is based on the Lagrangian framework [6]-[7]. The basic idea of the LSM is to evolve the zero-level of the level set function (LSF) $\phi$ in the image domain until it reaches the boundaries of the regions of interest.

In this paper, we propose a two phase level set method which can allows the realization of a binary segmentation, and can be easily expanded to the multi-phase case. The method uses both edge and region information in order to effectively detect both objects defined by edges and without edges. The sensitivity to noise introduced by local information, i.e., edge information, is handled by the 2D gray-scale histogram based constraint.

2. Too many equations in the manuscript! Over 30 for 11 pages paper. Many equations are just taken from other sources and describe other (but, no question, relevant) techniques. For a reader without ACM background it is impossible to understand the whole math here. The mathematical part must be shortened and better explained. Many equations do not seem to be very important for the introduction of the current method.

Authors’ reply:
Thanks for this valuable comment. We therefore decreased the number of equations from 31 to 23. The Eqs. (15) and (16) have been removed, as well as Eqs. (26) to (31) which described the unsupervised evaluation criteria have been removed.

3. Where is the Chapman-Enskog method coming from? The reference is required.

Authors’ reply:

Thanks for this pertinent comment. The appropriate reference has been added as follows, and the changed parts are in green.

By performing the Chapman-Enskog expansion \[36\], the following diffusion equation can be recovered from LBM \[23\],

\[
\frac{\partial \rho}{\partial t} = \beta \text{div}(\nabla \rho) + F.
\]  \hspace{1cm} (5)

Substituting \( \rho \) by the signed distance function \( \phi \) in Eq. (5), the LSE can be recovered.


4. P.7, L. 34 - 54: I suggest replacing this part by a block diagram.

Authors’ reply:

Thanks for this lighting comment. We therefore added a flowchart diagram of the proposed algorithm in order to make clear all the process. The flowchart diagram is displayed by the following figure.

![Flowchart diagram of the proposed algorithm](image.png)

Fig. 2. Flowchart representing the process of the proposed algorithm.
5. Why should the data transferring time be subtracted? If authors decided to use GPUs for acceleration then the data transfer between CPU and GPU is a part of computations. It cannot be ignored like this.

Authors’ reply:

Thanks for this relevant comment. We therefore added all the data transferring in the CPU time. The experimental result can be seen in Tables I to V.

6. The shown results are not convincing. There is no quantitative evaluation for a large and famous image dataset (like Berkeley benchmark for example). The evaluation of image segmentation techniques is not straightforward (due to its subjectiveness), but numbers for 5 images do not show anything. It is not evaluation. Here I can recommend the following paper: Arbelaez et al., “Contour detection and hierarchical segmentation”, published in PAMI in 2011. The Berkeley dataset is very well-known in the community and it is a good practice to use it for evaluation and comparison. Images used in the paper also seem to come from some dataset, but 5 are definitely not enough for the evaluation.

Authors’ reply:

Thanks for this pertinent comment. We therefore introduced a supervised evaluation for the performance of the proposed method on some test images of the Berkeley dataset BSDS300. The evaluation criteria used are the Martin and the Hausdorff’ metrics [33]-[34]. The experimental section was therefore modified as follows, and the changed parts are given in **green**.

II. EXPERIMENTAL RESULTS AND ANALYSIS

This section demonstrates the performance of the proposed level set image segmentation method in terms of efficiency, speed and effectiveness.

In the first part, we compare, subjectively and objectively, the proposed method with the Li’s method [30], the C-V method [17] and the Chen’s method [18]. The objective evaluation is done by mean of two unsupervised criteria, the Levine and Nazif (LN) inter-region and intra-region contrast criterion [31]-[32]. The lower is the intra-region contrast, the better is segmentation result, and the higher is the inter-region contrast, the better is the segmentation result.

In the second part, we use two supervised well-known metrics in order to objectively evaluate our method, the Hausdorff’s distance and the Martin’s global consistency error (GCE) [33]-[34]. They measure the similarity between two images. The lower they are, the better is the segmentation result. In the supervised evaluation, all the images, and the manual segmentations used as ground truth, are from the Berkeley segmentation dataset BSDS300 [35]. The proposed method is compared to the method proposed in [17], [18], [30] and [37].

The method was implemented using the parallel computing toolbox of Matlab R2012a installed on a PC AMD Athlon (TM) 5200 processor with a clock speed of 2.31 GHz, 2 GB of RAM and possessing the NVIDIA GPU GT 430. We fixed
Fig. 10 shows the result of the supervised quantitative evaluation of the segmentation results. We used fifteen test images of the Berkeley segmentation dataset BSDS300. TABLE VI and TABLE VII display the evaluation results using respectively the Hausdorff and the Martin’s metric. It can be seen that in most of the cases, the proposed method has the lowest Hausdorff and Martin’s values. It therefore gives better segmentation result comparing to the Chan –Vese’s method, the Li’s method, the Chen’s method and the method used by the Aaron-Ye (A-Y) in [37]. We should nevertheless notice that the A-Y’s method is clearly faster than our method since the average inside and outside the evolving contour are not computed during the segmentation process.
TIP-09353-2012: GPU Accelerated Edge-Region Based Level Set Evolution Constrained by 2D Gray-scale Histogram

<table>
<thead>
<tr>
<th></th>
<th>Image 6</th>
<th>Image 7</th>
<th>Image 8</th>
<th>Image 9</th>
<th>Image 10</th>
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Fig. 10. Segmentation result using fifteen test images of BSDS300. Lines number 1: test images, Lines number 2: results of the AY’s method, Lines number 3: results of the Li’s method, Lines number 4: results of the Chen’s method, Lines number 5: results of the CV’s method, Lines number 6: results of the proposed method and, Lines number 7: results of the manual segmentation used as ground truth.

Table VI. RESULTS OF THE SUPERVISED QUALITATIVE EVALUATION USING THE HAUSSDORF’S CRITERION

<table>
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<tr>
<th>Images</th>
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<th>Chen</th>
<th>A-Y</th>
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Table VII. RESULTS OF THE SUPERVISED QUALITATIVE EVALUATION USING THE MARTIN’S CRITERION

<table>
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<tr>
<th>Images</th>
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<td>0.8849</td>
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</table>

7. Are authors sure that Li's method, the C-V method, and Chen's method are used in a proper way? Did authors try to adjust parameters of these methods as they did it for their own approach? I cannot believe that the state-of-the-art approaches have such a poor performance. By the way, are the C-V method from 2001 and Chen's method from 2007 still the most up-to-date approaches?

Authors’ reply:

Thanks for this constructive comment. We therefore tried our best to adjust the parameter of the methods used for comparison in order to get fair result. We furthermore added a more recent method, the A-Y method from 2009, in the supervised quantitative evaluation.

8. Many references are quite old, thus it is difficult to say which methods are the state-of-the-art in this domain.

Authors’ reply:

Thanks for this pertinent remark. The reference section was therefore modified as follows, and the changed parts are given in green.

REFERENCES

Response to Reviewer #3:

1. Section II.B design of energy for 2D histogram is very confusing. I can understand the energy function (17), but I don't find it has anything to do with the histogram or number of bins. All the explanation about the histogram makes me confused. In my opinion, it acts more like a filter, and it filters out the noisy pixels or edges inside the region. Please try to rewrite this section and make it crystal clear for readers, as I think it is a major novelty for your paper.

Authors’ reply:

Thanks for this pertinent and relevant comment. The section II has therefore been modified as follows, the changed parts are given in red. We also added a flowchart diagram (Fig. 2) which clearly explains step by step the process of the proposed algorithm.

III. THE PROPOSED LEVEL SET METHOD

This section details the conception of the proposed level set algorithm and its implementation. Let $\phi: \Omega \rightarrow \mathbb{R}$ be a LSF defined on a domain $\Omega$. The energy functional that we propose to minimize, using the level set framework, is defined by

$$E(\phi) = g(I)E_{\text{reg}}(\phi) + E_{2D-\text{hist}}(\phi) + vE_{\text{reg}}(\phi),$$

(6)
where \( g(I) \) is an edge detection function, \( E_{\text{img}}(\phi) \) is the external energy depending upon the image in the case of image segmentation process, \( E_{\text{2D-hist}}(\phi) \) is an 2D gray-scale histogram based energy and \( E_{\text{reg}}(\phi) \) is the contour regularization term with \( \nu > 0 \) a constant coefficient. By using the gradient descent method, the level set equation can be recovered from the above defined energy functional

$$\frac{\partial \phi}{\partial t} = -\frac{\partial E}{\partial \phi},$$  \hspace{1cm} (7)

where \( \frac{\partial E}{\partial \phi} \) is the Gâteaux derivative \([27]\) of \( E \). According to Eq. (6), Eq. (7) is equivalent to the following evolution equation

$$\frac{\partial \phi}{\partial t} = -g(I) \frac{\partial E_{\text{img}}}{\partial \phi} - \frac{\partial E_{\text{2D-hist}}}{\partial \phi} - \nu \frac{\partial E_{\text{reg}}}{\partial \phi}.$$  \hspace{1cm} (8)

The common used edge detector is

$$g(I) = \frac{1}{1 + |\nabla G_\sigma \ast I|^r},$$  \hspace{1cm} (9)

where \( n \) is a positive parameter, and the convolution with the Gaussian kernel \( G_\sigma \) is used to smooth \( I \) in order to reduce the influence of noise, but it has also the disadvantage to weaken object edges and thus making \( g(I) \) less efficient.

In this paper, the 2D image gray-scale histogram information is used to reduce the noise influence; we can then omit the convolution product making \( g(I) \) more efficient and simpler

$$g(I) = \frac{1}{(1 + |I|^r)}.$$  \hspace{1cm} (10)

B. Design of \( E_{\text{2D-hist}}(\phi) \)

The energy term \( E_{\text{2D-hist}}(\phi) \) is based on the 2D histogram information. Let consider the 2D gray-scale histogram of an \( M \times N \) image where the X-axis represents the intensity and the Y-axis the local average. By analyzing it, we define three sectors as shown by Fig.1. The first sector is the zone surrounding the straight line \( I = I_{\text{mean}} \), i.e., pixels of which the intensity is equal to the local average. Those pixels belong to the object, the background, and the boundaries which are not defined by edges. In this sector the pixels intensity is slowly varying, thus the gray level of a given pixel is not far from the local average. In the proposed LSE, one of the contributions of the term \( -\frac{\partial E_{\text{img}}}{\partial \phi} \) is to allow the detection of objects which boundaries are not defined by edges and therefore has its pixels in sector I.

Sector II is the region of medium edges, the difference between the pixel intensity and the local average intensity is higher than in sector I. The term \( -\frac{\partial E_{\text{img}}}{\partial \phi} \) enable also the segmentation in this sector, which is enhanced by the edge stop function \( g(I) \). But the function \( g(I) \) also decreases the robustness against noise.

Sector III is the zone of strong edges and noise, the difference between the pixel intensity and the local average intensity is the highest one. The energy term \( E_{\text{2D-hist}} \) is designed with the aim of preventing the contour to stop in this
region, wherein the pixel's information is more likely corrupted by noise. It therefore eliminates the weakness to noise introduced by the edge based stop function $g(I)$. Furthermore, a good segmentation result should minimize $E_{2D-hist}$ inside the right object contours and maximize it outside the contours, thus $E_{2D-hist}$ can be defined as

$$E_{2D-hist} = \int_{\Omega_{\text{inside}}} (am_1 + \beta m_2 - I) \cdot \exp\{\mu[I-I_{\text{mean}}]-\eta\} \, dx \, dy.$$  \hfill (17)

The form of the function $\exp\{\mu[I-I_{\text{mean}}]-\eta\}$ is displayed in Fig.1. The LSF can therefore be introduced as follows

$$E_{2D-hist}(\phi) = \int_{\Omega} (am_1 + \beta m_2 - I) \cdot \exp\{\mu[I-I_{\text{mean}}]-\eta\} \cdot H(\phi) \, dx \, dy,$$  \hfill (18)

where $\eta$ and $\mu$ are positive parameters. The parameter $\eta$ mostly decides the width of sector I and II, and is fixed according to the kind of image we want to segment. The multiplication with the expression $am_1 + \beta m_2 - I$ has the advantage to free us to add another controlling parameter, and to constrain $-\partial E_{\text{reg}}/\partial \phi$ and $-\partial E_{2D-hist}/\partial \phi$ to have always the same sign. If it arrives but they have different sign they can annihilate each other in some regions and cause the contour to stop on false boundaries. We can remark that if we are in sector I the effect of the above defined energy is insignificant. But if we are in Sector III which is the sector of noise, the above defined $2(D)\cdot (\phi)$ act to constrain the motion of the active contour so that it does not consider noise like boundaries. The derivative of $E_{2D-hist}$ with respect to $\phi$ is therefore

$$\frac{\partial E_{2D-hist}}{\partial \phi} = (am_1 + \beta m_2 - I) \exp\{\mu[I-I_{\text{mean}}]-\eta\} \delta(\phi).$$  \hfill (19)

The regularization term in Eq. (8), used as a constraint on the evolving contour, can be expressed as in [28]

$$E_{\text{reg}}(\phi) = \int_{\Omega} |\nabla H(\phi)| \, dx \, dy.$$  \hfill (20)

Its derivative with respect to $\phi$ is

$$\frac{\partial E_{\text{reg}}}{\partial \phi} = -\delta(\phi) \nabla\left( \frac{\nabla \phi}{|\nabla \phi|} \right).$$  \hfill (21)

Thus, our proposed level set equation is
As defined, the proposed method should be effective when segmenting objects with medium edges and without edges in a noisy environment. This theoretical prevision will be demonstrated subjectively and objectively by experiments on various kinds of images.

\[
\frac{\partial \phi}{\partial t} = \delta(\phi)(I - (\alpha I_m + \beta I_s))(\frac{1}{1 + |\nabla I|^2})
+ \exp\left\{\mu[I - I_{\text{mean}}]|-\eta|\right\} + v \text{div}(\nabla \phi)
\]

s.t. \( \alpha + \beta = 1 \) with \( \alpha > 0 \) and \( \beta > 0 \).

2. If my understanding is right, the algorithm should suppress the edge inside the region where there is high contract between pixel intensity and local average. However, the algorithm can clearly recover the butterfly's antennae in Fig.2 that against the effect of this energy term.

Authors' reply:

Thanks for this relevant remark. The detection butterfly's antennae was made possible thanks to the energy term 
\(-\partial E_{\text{seg}} / \partial \phi\), since their boundaries are defined by pixels which can be classified in the sector II.

3. The "GPU" in the title is very eye catching. Unfortunately, the author only mentioned few sentences about GPU in the Experiment section. More detailed explanation about parallel implementation is desirable.

Authors' reply:

Thanks for this pertinent and valuable remark. We therefore modified some part of sections I and II in order to give more details about the implementation of our method using the GPU. Again, in section II, the flowchart diagram we added shows the way we use the GPU to accelerate the proposed method. Section I was modified as follows. The changes are given in \textcolor{red}{red} color.

Lately, the GPU has been developed for general-purpose computing. Since increasing the clock speeds drive transistors to thermal limits, the multi-core technique became the evident solution to increase the processing speed of computing hardware. The GPU has, therefore, been recognized as one of the most promising techniques to accelerate scientific computations. The GPU architecture favors dense data and local computations because the communications between microprocessor is time consuming. Since the majority of LBM is local, it is thus suitable for GPU-based computation. In this paper, the proposed algorithm is accelerated using an NVIDIA graphics processing units. The proposed method is efficient and effective when detecting object with well-defined edges, with weak edges and without edges. Furthermore the method is robust against noise, although it uses edge information, and fast to enable real-time image segmentation.

4. An closely related literature about GPU implementation of LBM is missing.
Authors’ reply:

Thanks for this valuable and judicious remark. We therefore added reference [37]. Furthermore, we use the method introduced by this paper in the supervised quantitative evaluation of the proposed method.