1) Rank the following functions by order of growth. That is, find an arrangement \( f_1, f_2, \ldots \) of the functions satisfying \( f_1 \in O(f_2) \), \( f_2 \in O(f_3) \), \ldots. Partition your list into equivalence classes such that functions \( f_i \) and \( f_j \) are in the same class if and only if \( f_i \in \Theta(f_j) \).

\[
\begin{align*}
\sqrt{2}^{\log n} & \quad n^2 & \quad n! & \quad \left(\frac{3}{2}\right)^n & \quad \log^2 n \\
4^{\log n} & \quad n & \quad 2^n & \quad n \log n & \quad 2^{2^{n+1}}
\end{align*}
\]

2) Find two functions \( f(n) \) and \( g(n) \) that satisfy the following relationship. If no such \( f \) and \( g \) exist, shortly explain why.

a) \( f(n) \in o(g(n)) \) and \( f(n) \notin \Theta(g(n)) \)

b) \( f(n) \in \Theta(g(n)) \) and \( f(n) \in o(g(n)) \)

c) \( f(n) \in \Theta(g(n)) \) and \( f(n) \notin O(g(n)) \)

d) \( f(n) \in \Omega(g(n)) \) and \( f(n) \notin O(g(n)) \)

3) Use a recursion tree to determine a good asymptotic upper bound on the recurrence \( T \). Use the master theorem for a) and the substitution method for b) to verify your answer.

a) \( T(n) = 3T(n/2) + n \)

b) \( T(n) = 2T(n - 1) + 1 \)