Discrete Structures — Homework 2

Due: September 11.

Section 1.4

10 Let $C(x)$ be the statement “$x$ has a cat,” let $D(x)$ be the statement “$x$ has a dog,” and let $F(x)$ be the statement “$x$ has a ferret.” Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectivities. Let the domain consist of all students in your class. (4 pt)

a) A student in your class has a cat, a dog, and a ferret.

b) All students in your class have a cat, a dog, or a ferret.

c) Some student in your class has a cat and a ferret, but not a dog.

d) No student in your class has a cat, a dog, and a ferret.

e) For each of the three animals, cats, dogs, and ferrets there is a student in your class who has this animal as pet.

16 Determine the truth value of each of these statements if the domain of each variable consists of all real numbers. (4 pt)

a) $\exists x : x^2 = 2$

b) $\exists x : x^2 = -1$

c) $\forall x : x^2 + 2 \geq 1$

d) $\forall x : x^2 \neq x$

36 Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers. (3 pt)

a) $\forall x : x^2 \neq x$

b) $\forall x : x^2 \neq 2$

c) $\forall x : |x| > 0$

Section 1.5

32 Express the negations of each of these statements so that all negation symbols immediately precede predicates. (4 pt)

a) $\exists z \forall y \forall x T(x, y, z)$

b) $\exists x \exists y P(x, y) \land \forall x \forall y Q(x, y)$

c) $\exists x \exists y (Q(x, y) \iff Q(y, x))$

d) $\forall y \exists x \exists z (T(x, y, z) \lor Q(x, y))$

40 Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers. (3 pt)

a) $\forall x \exists y : x = 1/y$

b) $\forall x \exists y : y^2 - x < 100$

c) $\forall x \forall y : x^2 \neq y^3$

EC Which of the following statements is right, which is wrong? Explain your answer. (4 pt)

a) $\forall x F(x) \land \forall x G(x) \equiv \forall x (F(x) \land G(x))$

b) $\forall x F(x) \lor \forall x G(x) \equiv \forall x (F(x) \lor G(x))$

c) $\exists x F(x) \land \exists x G(x) \equiv \exists x (F(x) \land G(x))$

d) $\exists x F(x) \lor \exists x G(x) \equiv \exists x (F(x) \lor G(x))$