Section 1.6

6 Use rules of inference to show that the hypotheses “If it does not rain or it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,” “If the sailing race is held, then the trophy will be awarded,” and “The trophy was not awarded” imply the conclusion “It rained.”

Please use the following variables:
- \( r \) - It rains.
- \( f \) - It is foggy.
- \( s \) - The sailing race will be held.
- \( l \) - The life saving demonstration will go on.
- \( t \) - The trophy will be awarded.

24 Identify the error or errors in this argument that supposedly shows that if \( \forall x (P(x) \lor Q(x)) \) is true then \( \forall x P(x) \lor \forall x Q(x) \) is true.

(1) \( \forall x (P(x) \lor Q(x)) \) Premise
(2) \( P(c) \lor Q(c) \) Universal instantiation from (1)
(3) \( P(c) \) Simplification from (2)
(4) \( \forall x P(x) \) Universal generalisation from (3)
(5) \( Q(c) \) Simplification from (2)
(6) \( \forall x Q(x) \) Universal generalisation from (5)
(7) \( \forall x P(x) \lor \forall x Q(x) \) Conjunction from (4) and (6)

28 Use rules of inference to show that if \( \forall x (P(x) \lor Q(x)) \) and \( \forall x ((\neg P(x) \land Q(x)) \rightarrow R(x)) \) are true, then \( \forall x (\neg R(x) \rightarrow P(x)) \) is also true, where the domains of all quantifiers are the same.

Section 1.7

6 Use a direct proof to show that the product of two odd numbers is odd.

26 Prove that if \( n \) is a positive integer, then \( n \) is even if and only if \( 7n + 4 \) is even.

32 Show that these statements about the real number \( x \) are equivalent:

(i) \( x \) is rational
(ii) \( x/2 \) is rational
(iii) \( 3x - 1 \) is rational.
Section 1.8

12 Show that the product of two of the numbers $65^{1000} - 8^{2001} + 3^{177}, 79^{1212} - 9^{2399} + 2^{2001}$, and $24^{4493} - 5^{8192} + 7^{177}$ is nonnegative. Is your proof constructive or nonconstructive? (2 pt)

*Hint.* Ignore the value of the given numbers and don’t try to find out if they are positive or negative.

20 Prove that given real number $x$ there exist unique numbers $n$ and $\epsilon$ such that $x = n + \epsilon$, $n$ is an integer, and $0 \leq \epsilon < 1$ (2 pt)

34 Prove that $\sqrt{2}$ is irrational. (2 pt)

*Hint.* Example 10 of section 1.7 in the textbook (page 86) proves that $\sqrt{2}$ is irrational.

EC Prove the following statement: If a line $L$ does not intersect a diagonal of a convex polygon $P$ then $L$ can intersect only one of the two subpolygons defined by that diagonal. (2 pt)