Discrete Structures — Homework 9

Due: April 22.

Section 9.1

6 Determine whether the relation $R$ on the set of all real numbers is reflexive, symmetric, anti-
symmetric, and/or transitive, where $(x, y) \in R$ if and only if

a) $x + y = 0$

b) $x = \pm y$

c) $x - y \in \mathbb{Q}$

d) $x = 2y$

e) $xy \geq 0$

f) $xy = 0$

g) $x = 1$

h) $x = 1$ or $y = 1$

10 Give an example of a relation on a set that is

a) both symmetric and antisymmetric.

b) neither symmetric nor antisymmetric.

Section 9.3

22 Draw the directed graph that represents the relation

$$\{(a, a), (a, b), (b, c), (c, b), (c, d), (d, a), (d, b)\}.$$ (2 pt)

Section 9.5

2 Which of these relations on the set of all people are equivalence relations? Determine the prop-
erties of an equivalence relation that the others lack.

a) $\{(a, b) \mid a$ and $b$ are the same age$\}$

b) $\{(a, b) \mid a$ and $b$ have the same parents$\}$

c) $\{(a, b) \mid a$ and $b$ share a common parent$\}$

d) $\{(a, b) \mid a$ and $b$ have met$\}$

e) $\{(a, b) \mid a$ and $b$ speak a common language$\}$

14 Let $R$ be the relation consisting of all pairs $(x, y)$ such that $x$ and $y$ are strings of uppercase and
lowercase English letters with the property that for every positive integer $n$, the $n$th characters
in $x$ and $y$ are the same letter, either uppercase or lowercase. Show that $R$ is an equivalence relation.

42 Which of these collections of subsets are partitions of $\{-3, -2, -1, 0, 1, 2, 3\}$?

a) $\{-3, -1, 1, 3\}, \{-2, 0, 2\}$

b) $\{-3, -2, -1, 0\}, \{0, 1, 2, 3\}$

c) $\{-3, 3\}, \{-2, 2\}, \{-1, 1\}, \{0\}$

d) $\{-3, -2, 2, 3\}, \{-1, 1\}$
2 Which of these relations on \{0, 1, 2, 3\} are partial orderings? Determine the properties of a partial ordering that the others lack. (5 pt)

a) \{(0, 0), (2, 2), (3, 3)\}
b) \{(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 3)\}
c) \{(0, 0), (1, 1), (1, 2), (2, 2), (3, 1), (3, 3)\}
d) \{(0, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (2, 3), (3, 0), (3, 3)\}
e) \{(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (3, 3)\}

6 Which of these are posets? (4 pt)

\((X, \circ) = \{(a, b) | a, b \in X, a \circ b\}\)

a) \((\mathbb{R}, =)\)
b) \((\mathbb{R}, <)\)
c) \((\mathbb{R}, \leq)\)
d) \((\mathbb{R}, \neq)\)

24 Draw the Hasse diagram for inclusion on the set \(P(S)\), where \(S = \{a, b, c\}\). (2 pt)

\(P(S)\) is the power set of \(S\).