Advanced Algorithms: Homework 1

Problem 1. 15.2-1 (p.338 in CLRS). This is the problem 16.1.1 in old edition.

Problem 2. 15.2-5 (p.339 in CLRS). This is the problem 16.1.4 in old edition.

Problem 3. 15.3-3 (p.349 in CLRS).
Consider a variant of the matrix-chain multiplication problem in which the goal is to parenthesize the sequence of matrices so as to maximize, rather than minimize, the number of scalar multiplications. Does this problem exhibit optimal substructure? Explain.

Problem 4. 15.3-5 (p.350 in CLRS).
As stated, in dynamic programming we first solve the subproblems and then choose which of them to use in an optimal solution to the problem. Professor Capulet claims that it is not always necessary to solve all the subproblems in order to find an optimal solution. She suggests that an optimal solution to the matrix-chain multiplication problem can be found by always choosing the matrix $A_k$ at which to split the subproduct $A_kA_{k+1}...A_j$ (by selecting $k$ to minimize the quantity $p_{i-1}p_kp_j$) before solving the subproblems. Find an instance of the matrix-chain multiplication problem for which this greedy approach yields a suboptimal solution.

Problem 5. 15.4-1 (p.355 in CLRS). This is the problem 16.3.1 in old edition.

Problem 6. 15.4-5 (p.356 in CLRS). This is the problem 16.3.5 in old edition.

Extra Credit Problem. 15-7 (p.369 in CLRS).

Scheduling to maximize profit: Suppose you have one machine and a set of $n$ jobs $a_1, a_2,...,a_n$ to process on that machine. Each job $a_j$ has a processing time $t_j$, a profit $p_j$, and a deadline $d_j$. The machine can process only one job at a time, and job $a_j$ must run uninterruptedly for $t_j$ consecutive time units. If job $a_j$ is completed by its deadline $d_j$, you receive a profit $p_j$, but if it is completed after its deadline, you receive a profit of 0. Give an algorithm to find the schedule that obtains the maximum amount of profit, assuming that all processing times are integers between 1 and $n$. What is the running time of your algorithm?