CHAPTER 2

Geometric Searching

- Problems frequently arise in
  - geographic applications
  - database management

- Four separate cost measures

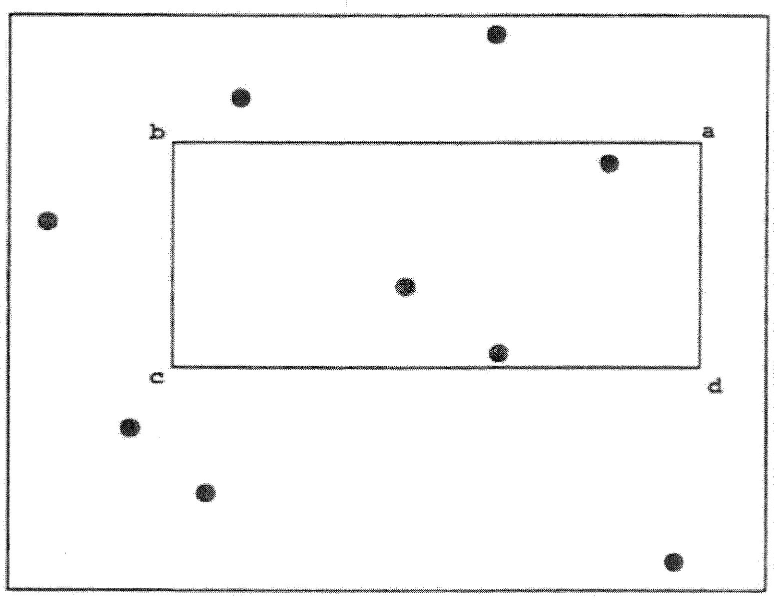
1. *Query time.* How much time is required, in both the average and worst cases, to respond to a single query?
2. *Storage.* How much memory is required for the data structure?
3. *Preprocessing time.* How much time is needed to arrange the data for searching?
4. *Update time.* Given a specific item, how long will it take to add it to or to delete it from the data structure?

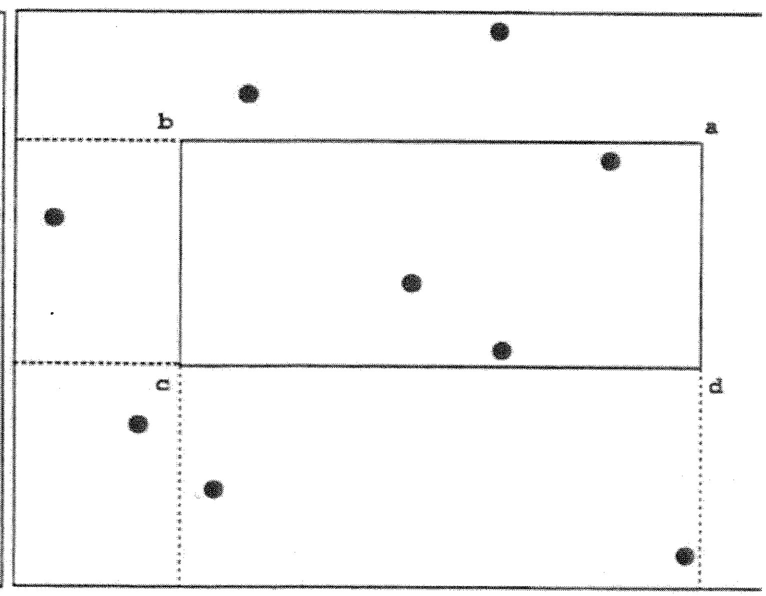
- First problem

PROBLEM S.1 (RANGE SEARCHING—COUNT). Given N points in the plane, how many lie in a given rectangle with sides parallel to the coordinate axes?

Search - Counting

- **Given:** n points in the plane.
- **Find:** $N(a,b,c,d)$ = number of points within the rectangle defined by corners a, b, c, d.

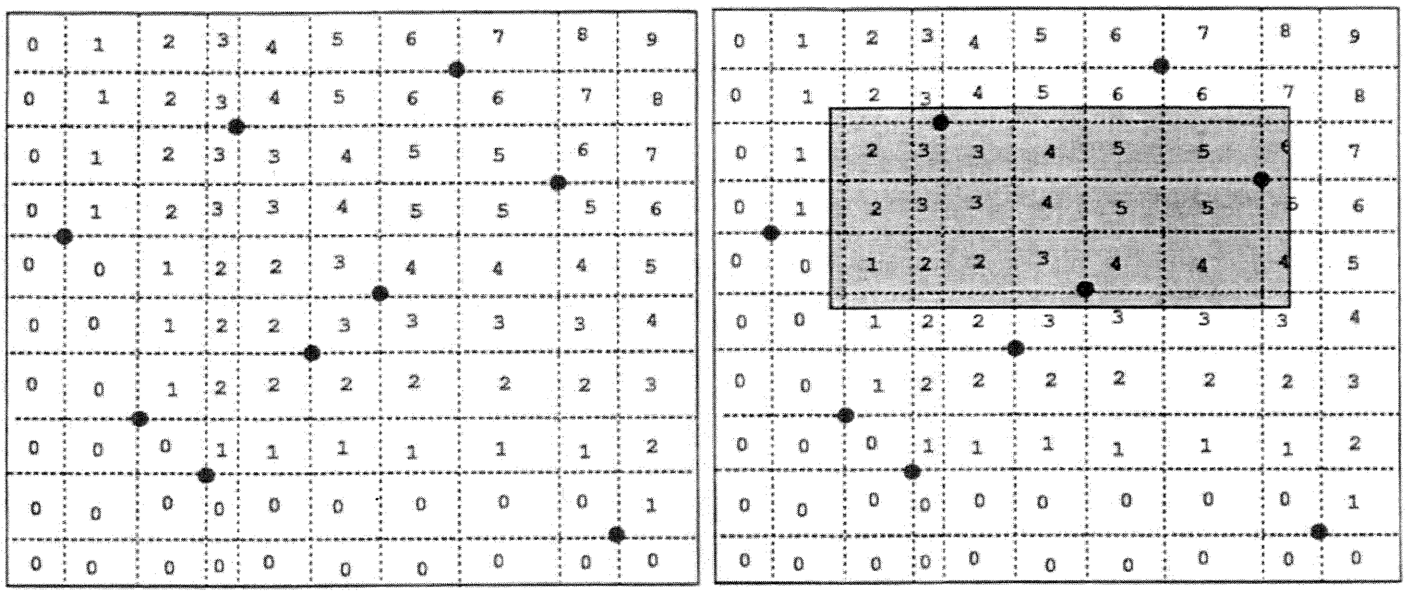




Related Problem

- **Given:** n points in the plane.
- **Find:** $Q(p)$ = number of points with smaller x- and y-coordinates than respectively $p.x$ and $p.y$.
- $N(a,b,c,d) = Q(a) - Q(b) - Q(d) + Q(c)$

Search - Counting; Locus Strategy



- Subdivision can be determined in $O(n^2)$ time.
- Space required: $O(n^2)$.
- Query time: $O(\log n)$.

Table I

Query	Storage	Preprocessing	Comments
$O(\log N)$	$O(N^2)$	$O(N^2)$	Above method
$O(\log^2 N)$	$O(N \log N)$	$O(N \log N)$	4
$O(N)$	$O(N)$	$O(N)$	No preprocessing

1. *Location problems*, where the file represents a partition of the geometric space into regions, and the query is a point. Location consists of identifying the region the query point lies in.
2. *Range-search problems*, where the file represents a collection of points in space, and the query is some standard geometric shape arbitrarily translatable in space (typically, the query in 3-space is a ball or a box). Range-search consists either of retrieving (*report* problems) or of counting (*census* or *count* problems) all points contained within the query domain.

2.2 Point-Location Problems

PROBLEM S.3 (CONVEX POLYGON INCLUSION). Given a convex polygon P and a point z, is z internal to P?

PROBLEM S.2 (POLYGON INCLUSION). Given a simple polygon P and a point z, determine whether or not z is internal to P.

Theorem 2.1. *Whether a point z is internal to a simple N-gon P can be determined in $O(N)$ time, without preprocessing.*

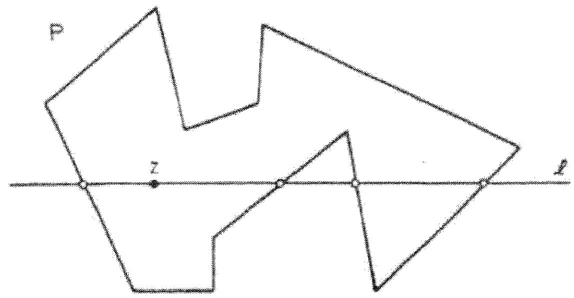
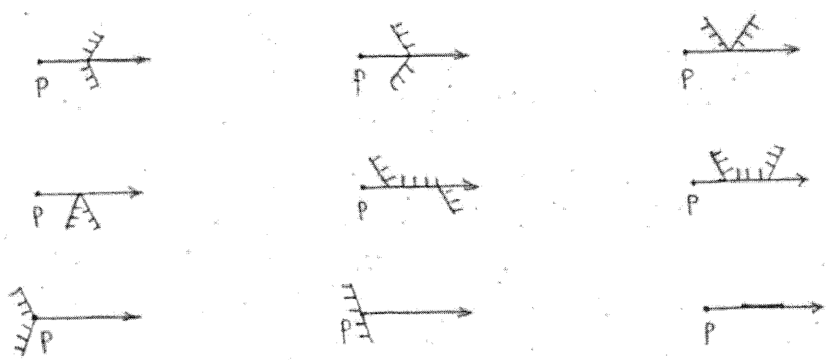
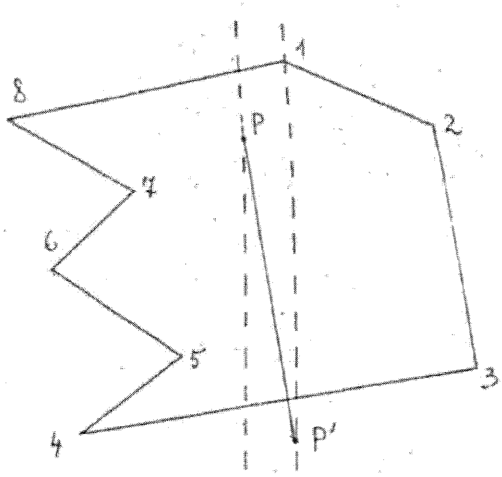


Figure 2.5 Single-shot inclusion in a simple polygon. There is one intersection of l with P to the left of z, so z is inside the polygon.

- degenerate situations



- to avoid them choose $p' = (x', y')$

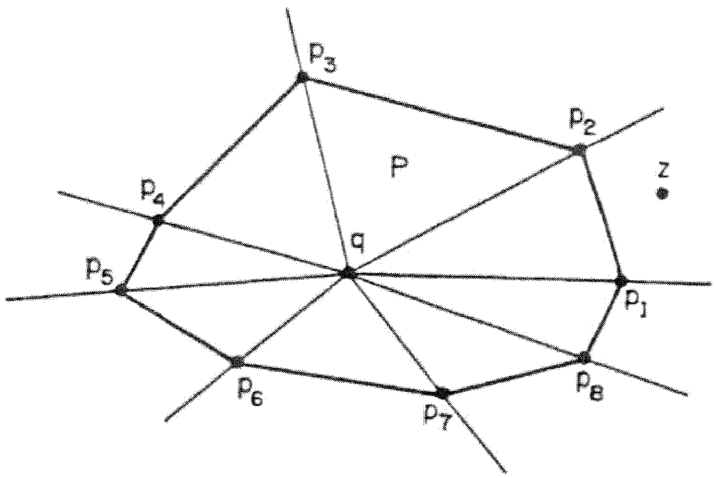


$y' = (\min_i y_i) - 1$

$x' = x_{i*}$, where

$x_{i*}: \ |x_{i*} - x| = \min_{\substack{x_i - x \neq 0 \\ i = \overline{1,n}}} |x_i - x|$

Here $p = (x, y)$ is a query poi

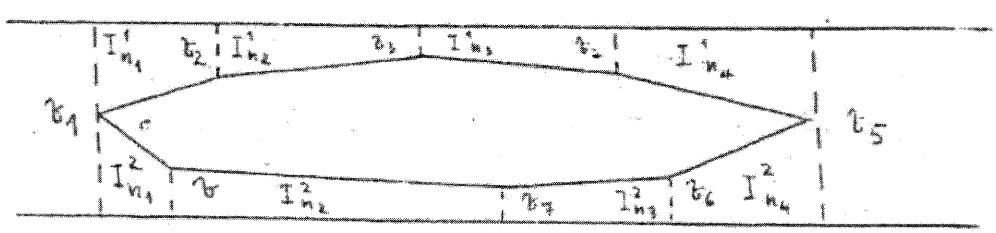


• wedge method

Figure 2.6 Division into wedges for the convex inclusion problem. 1. By binary search we learn that z lies in wedge p_1qp_2. 2. By comparing z against edge $\overline{p_1p_2}$ we find that it is external.

Theorem 2.2. *The inclusion question for a convex* N-gon *can be answered in* $O(\log N)$ *time, given* $O(N)$ *space and* $O(N)$ *preprocessing time.*

• interval method



- extension to star-shaped polygons (wedge method)

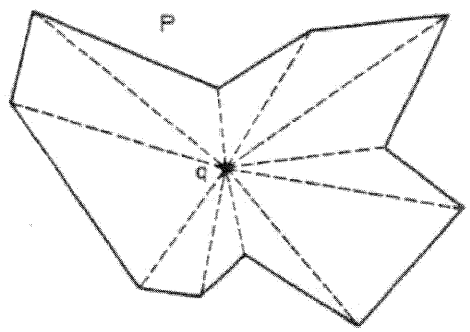
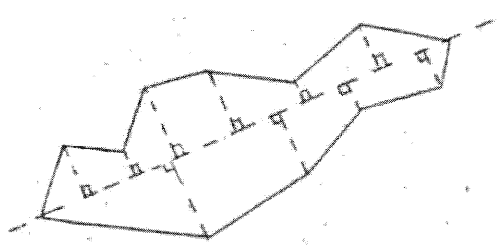


Figure 2.7 A star-shaped polygon.

- The kernel of a simple N-gon can be found in $O(N)$ time (later)

Theorem 2.3. *The inclusion question for an N-vertex star-shaped polygon can be answered in $O(\log N)$ time and $O(N)$ storage, after $O(N)$ preprocessing time.*

- extension to monotone polygons (interval method)



- - query in $O(\log n)$
  - storage in $O(n)$
  - prepr. in $O(n)$

CONVEX $\subset$ STAR-SHAPED $\subset$ GENERAL.

$\subset$ monotone $\subset$