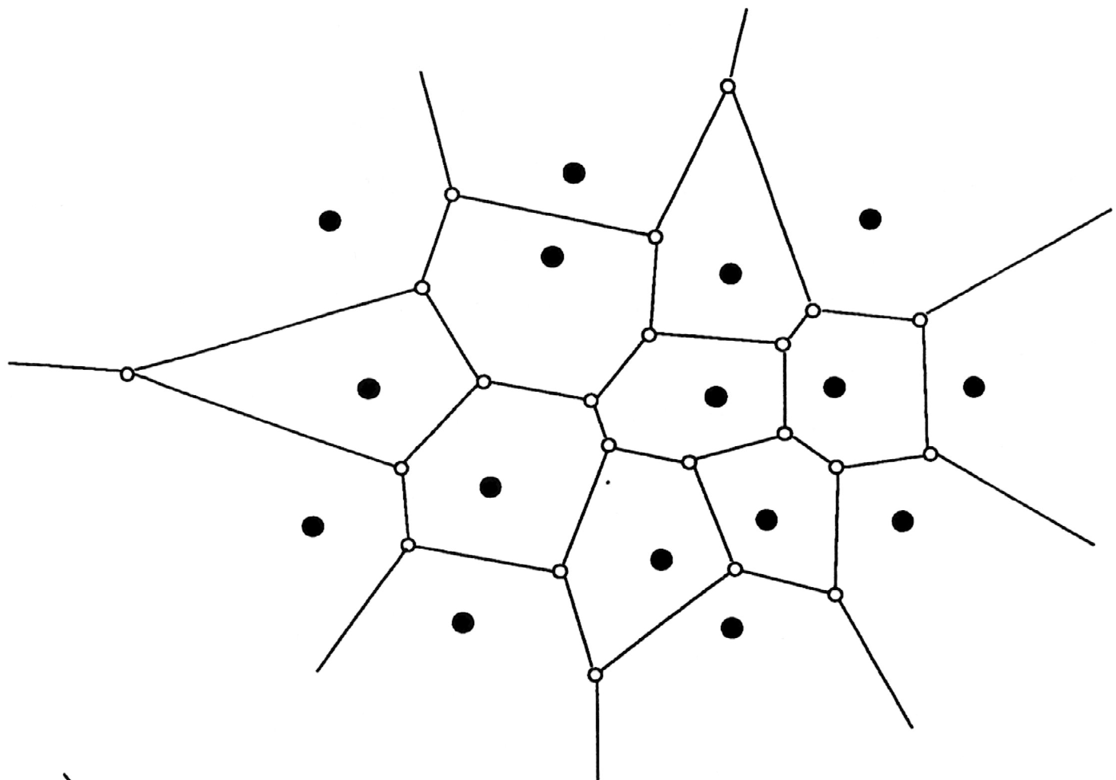
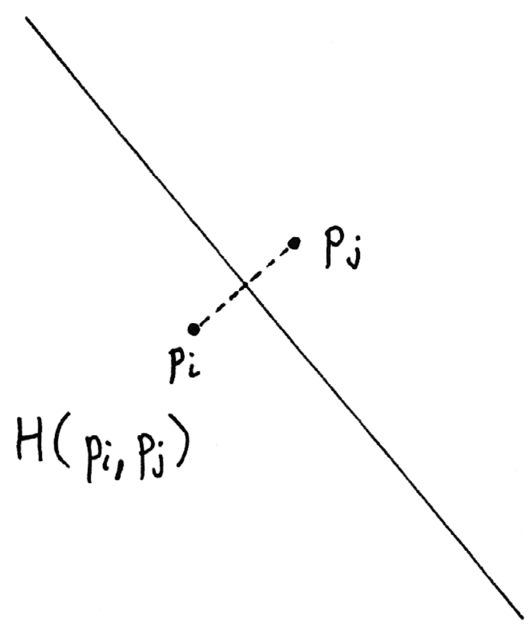
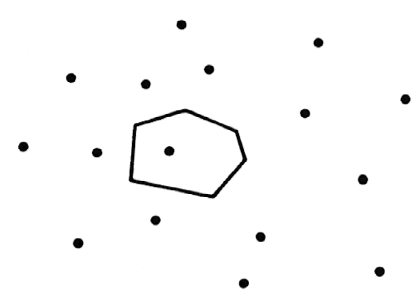
Voronoi Diagrams





$H(p_i, p_j)$ is a half-plane containing p_i.

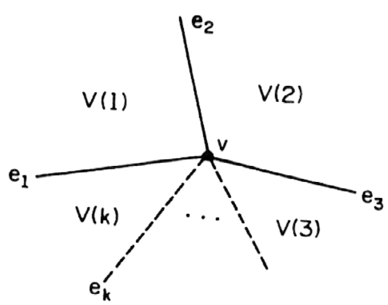


A Voronoi polygon

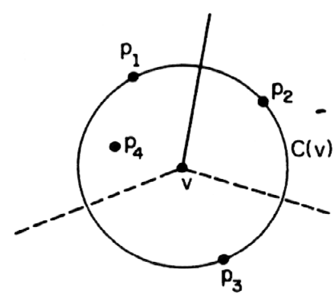
$$V(i) = \bigcap_{i \neq j} H(p_i, p_j)$$

Simplifying Assumptions and Basic Properties

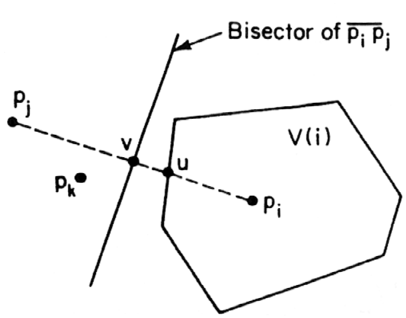
- No 4 points are on a common circle.
- Each Voronoi point is incident with 3 edges.
- Each Voronoi point v has exactly 3 points on a common circle with no points in its interior.
- Each nearest neighbor to a point $s \in S$ defines an edge in $VD(S)$.
- $VP(s)$ is unbounded iff $s \in CH(S)$



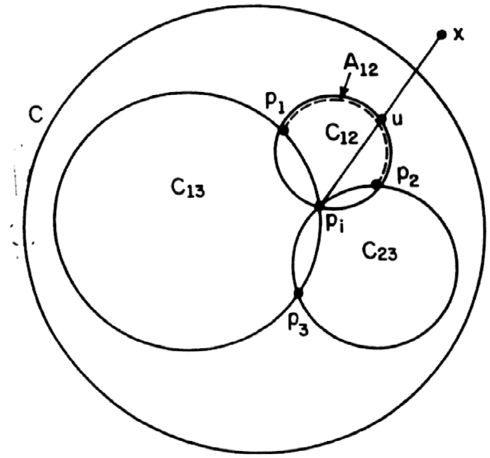
Voronoi edges incident on a Voronoi vertex.



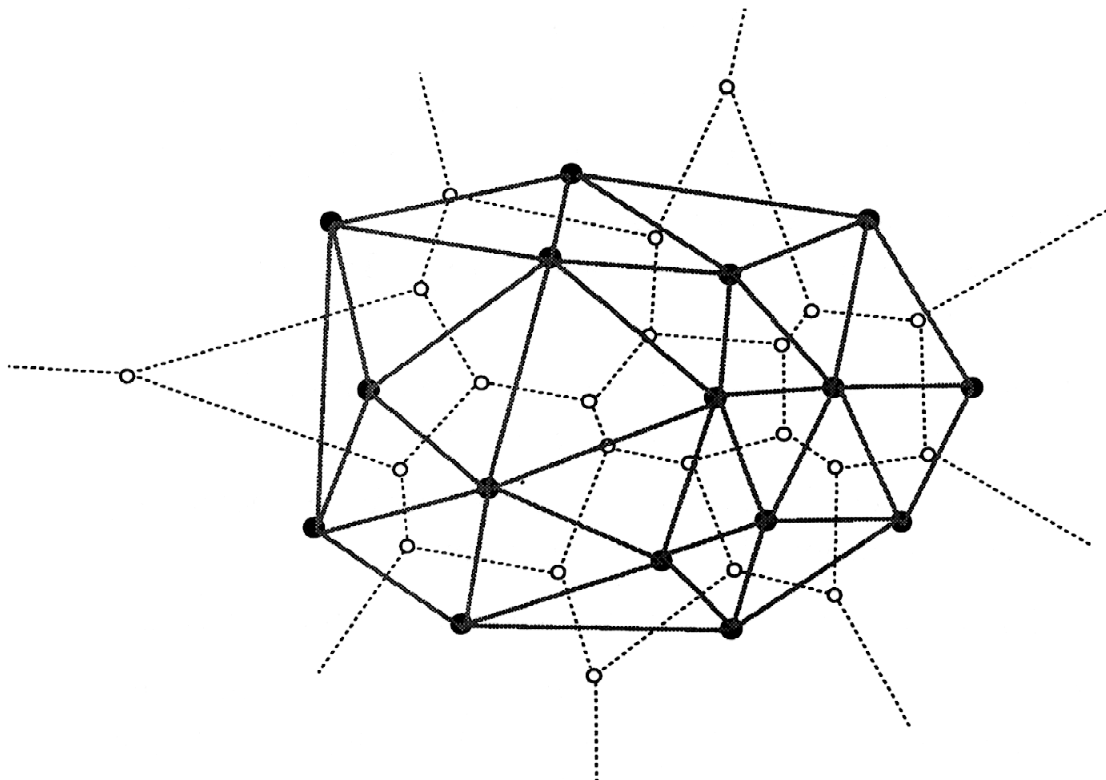
The circle $C(v)$ contains no other point of S.



Every nearest neighbor of p_i defines an edge of $V(i)$.

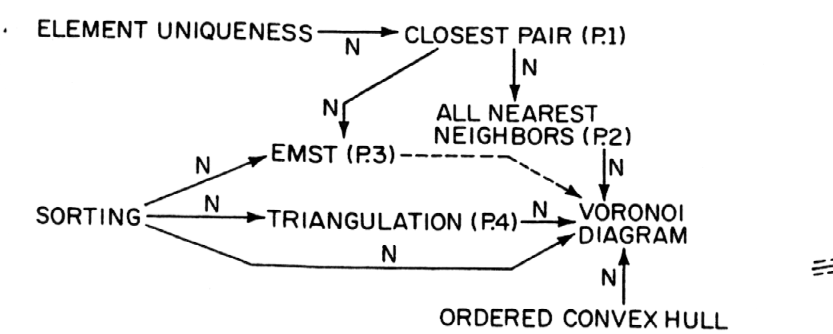


Delaunay Triangulations

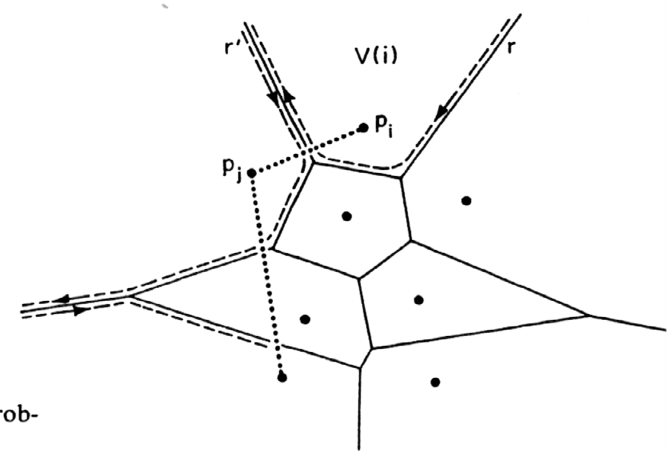


Use of Voronoi Diagrams and Dalaunay Triangulations

- Nearest neighbor: Closest pair of points defines an edge in the Voronoi diagram. Scan all edges (up to $3n-6$).
- All nearest neighbor: For each vertex, scan the edges of its Voronoi polygon. Each of up to $3n-6$ edges is scanned twice.
- Nearest neighbor search: Point location in the Voronoi diagram can be done in $O(\log n)$ time.
- k nearest neighbor search requires a structure which is a generalization of Voronoi diagram.
- Triangulation is given by the Delaunay triangulation.
- Delaunay triangulation contains at least one minimum spanning tree. Minimum spanning trees in planar graphs can be determined in $O(n)$ time.
- Convex hull CH(S)
- lower bound for Voronoi Diagram is $\Omega(n \log n)$

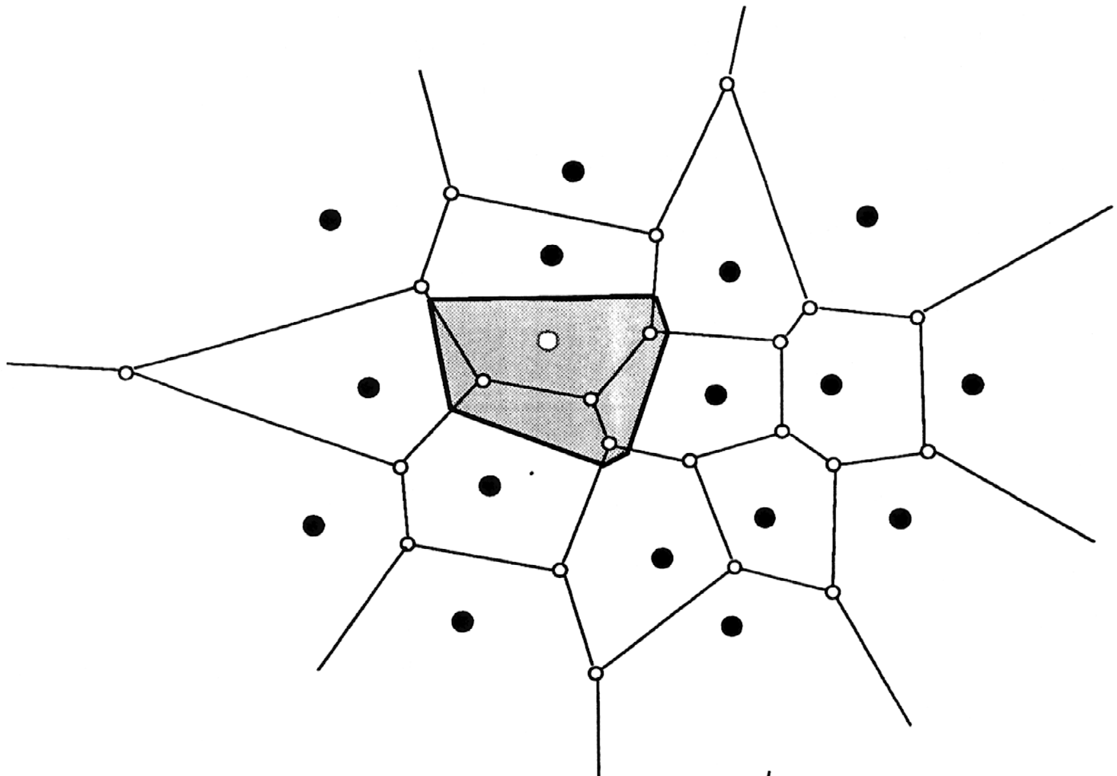


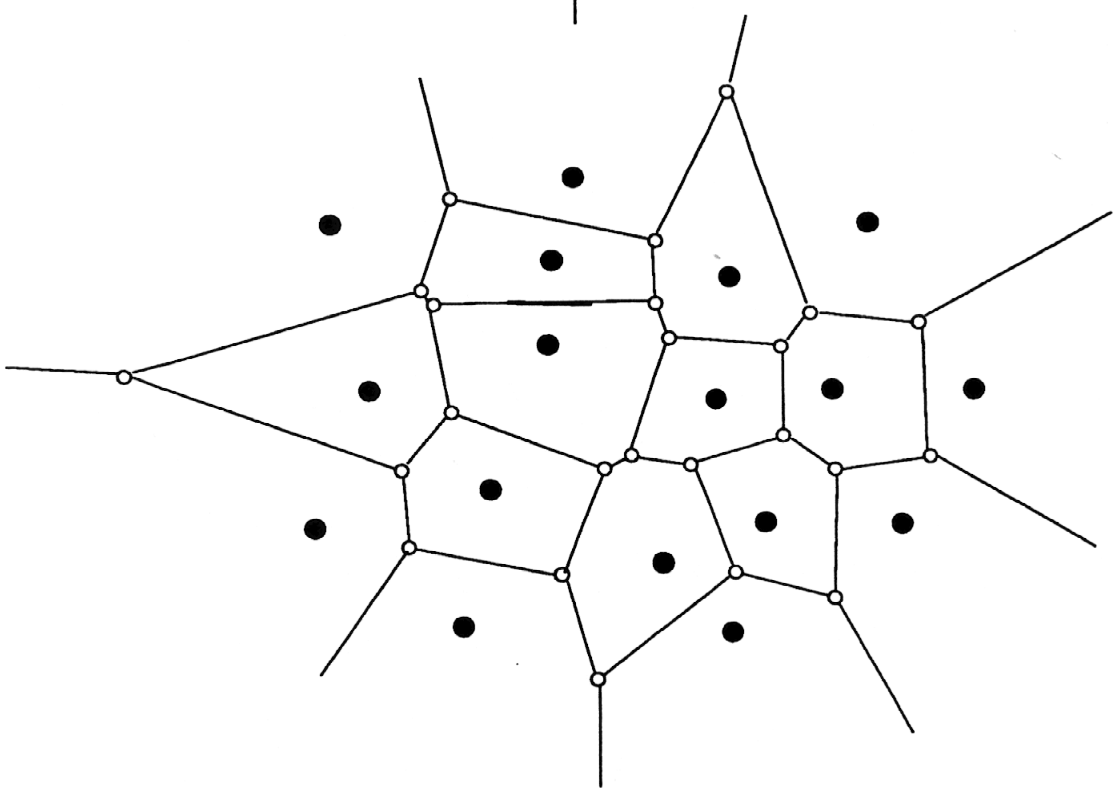
Relationship among computational prototypes and proximity problems.



Construction of the convex hull from the Voronoi di

Voronoi Diagrams - Incremental Algorithm

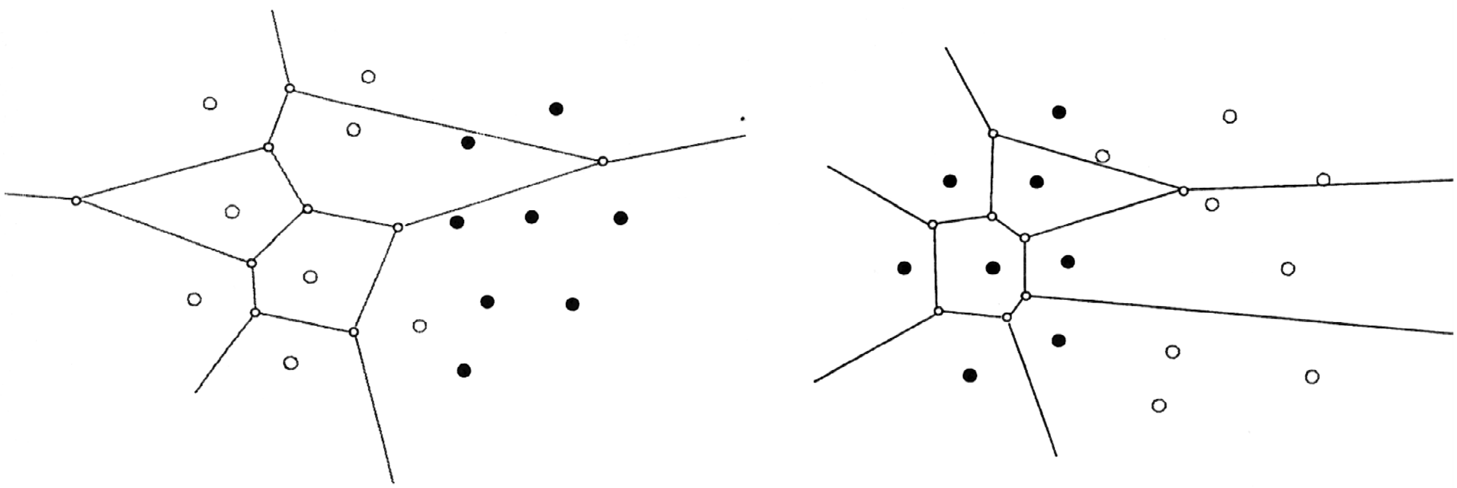


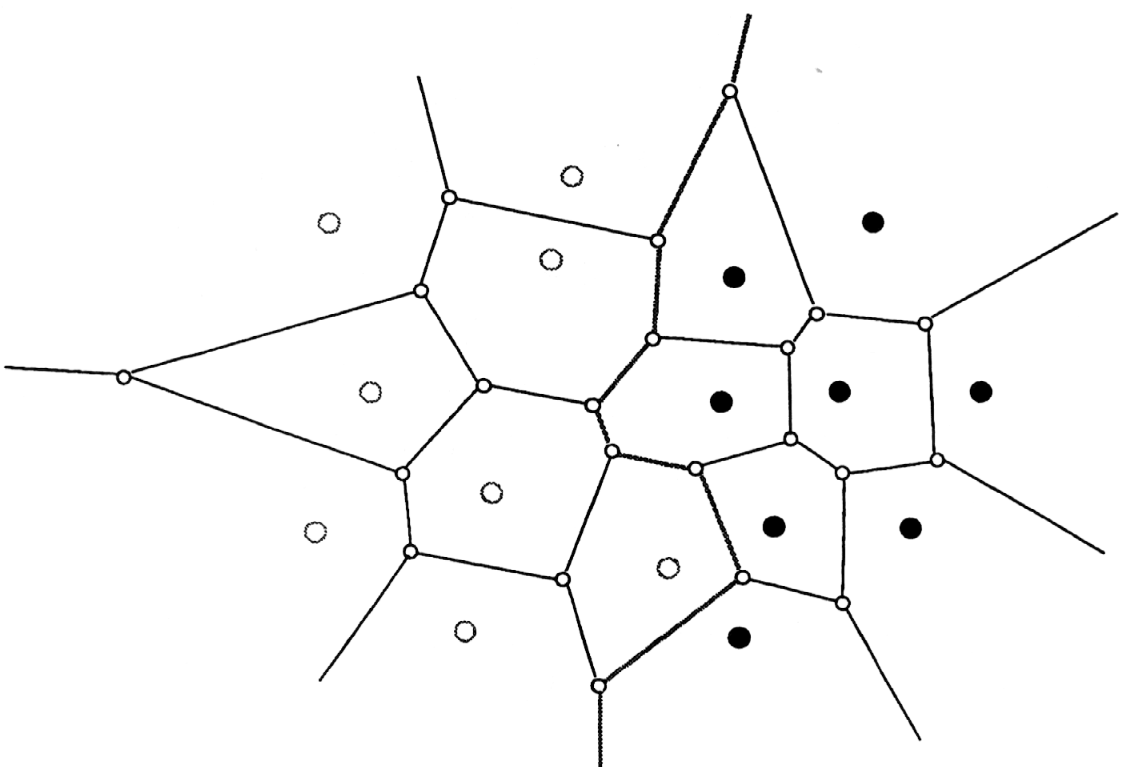


- Naive approach $O(n^2 \log n)$.

Voronoi Diagrams - Divide-and-Conquer

- Partition the set S of points in two equal size subsets S_1 and S_2 such that $s_1.x \leq s_2.x$. Solve directly if $|S| \leq 3$.
- Solve the problem for the two subsets.
- Merge the two solutions together.





Higher Order Voronoi Diagrams

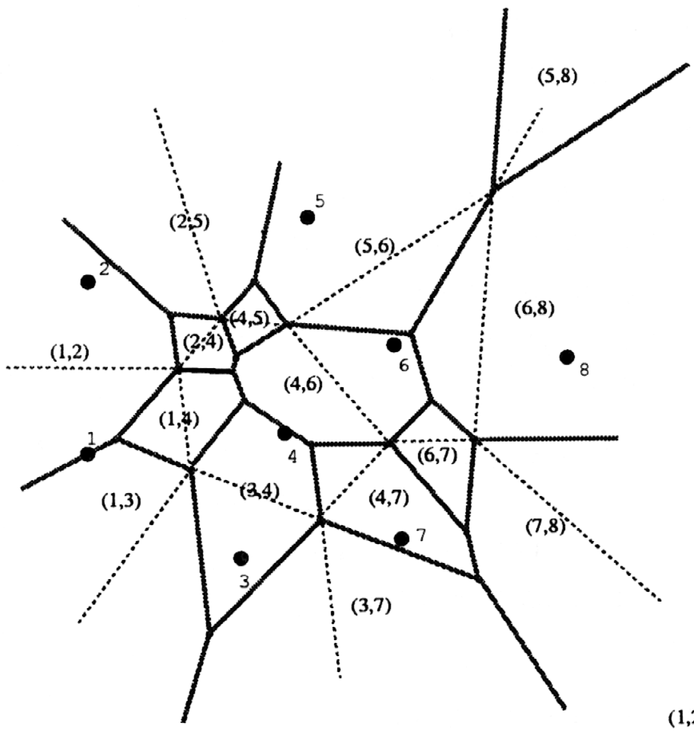
- Voronoi diagrams of 1. order: A polygon $VP(s_i)$ is associated with each point $s_i \in S$. It contains all points closer to s_i than to any other S-point.

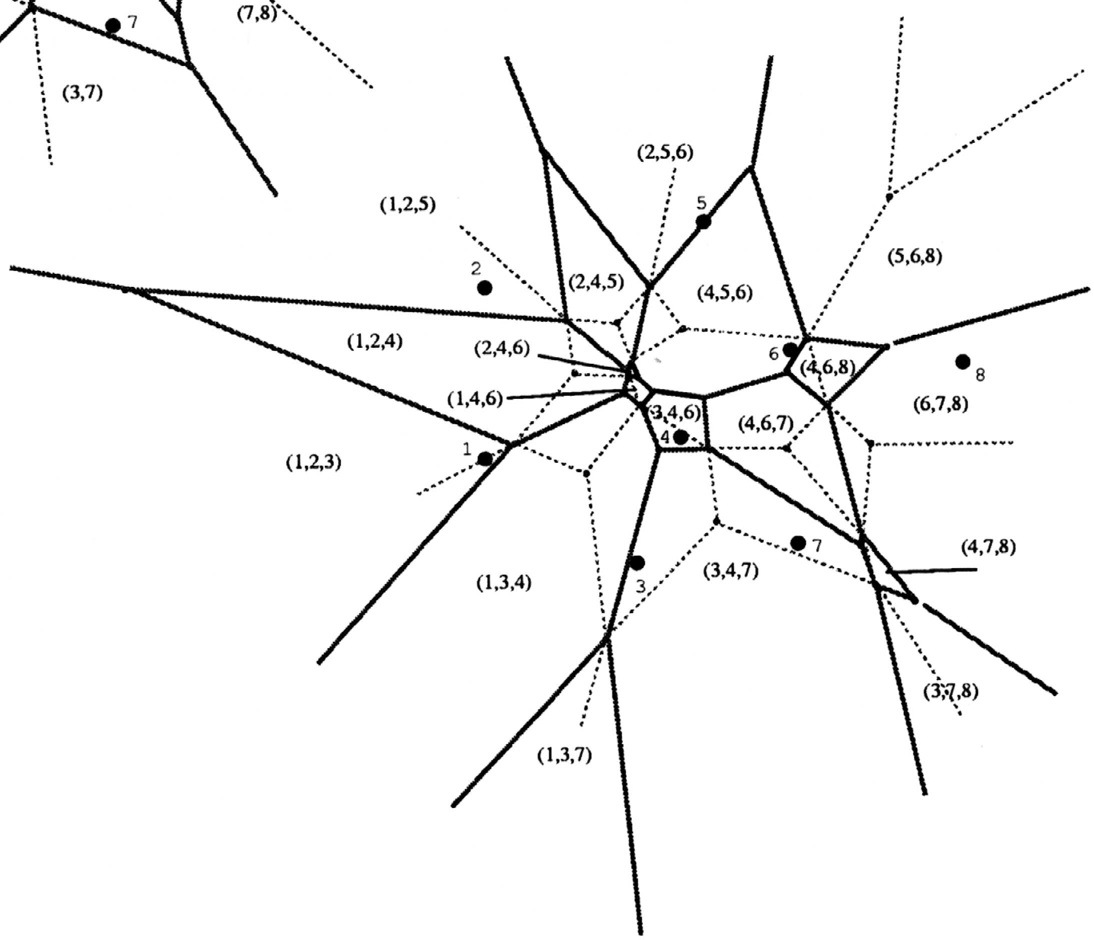
- Voronoi diagram of 2. order: A polygon $VP(s_i, s_j)$ is associated with each pair of points $s_i, s_j \in S$. It contains all points closest to s_i and second-closest to s_j (or vice versa). Note that $VP(s_i, s_j)$ can be empty and polygons partition the entire plane.

- Voronoi diagram of n-1 order: A polygon $VP(S - s_i)$ is associated with each subset $S - s_i$, $s_i \in S$. It contains points closer to all points in $S - s_i$ than to s_i. In other words, $VP(S - s_i)$ contains points farther away from s_i than from any other point in S.

- The number of non-empty Voronoi polygons of all orders is $O(n^3)$. Possibility to obtain polynomial algorithms.

Voronoi Diagrams of Second and Third Order





Voronoi Diagrams of (n-1)-st Order