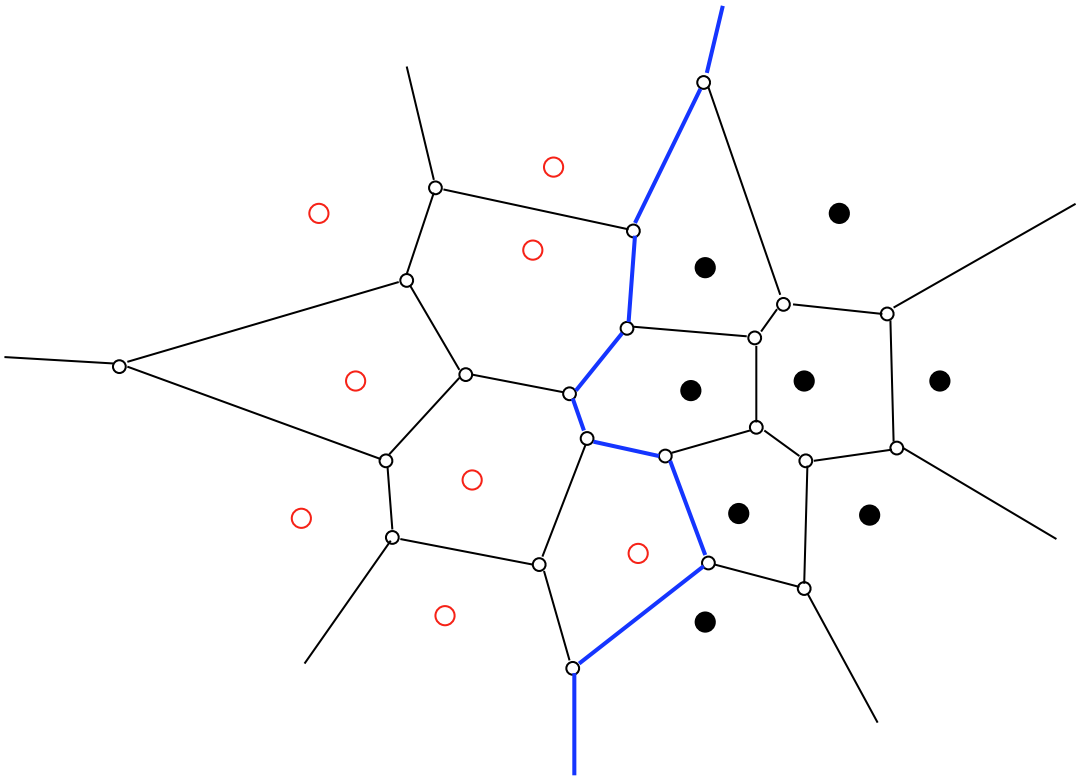
Voronoi Diagrams - Divide-and-Conquer

- Partition the set S of points in two equal size subsets S_1 and S_2 such that $s_1.x \leq s_2.x$. Solve directly if $|S| \leq 3$.
- Solve the problem for the two subsets.
- Merge the two solutions together.

Voronoi Diagrams - Separating Chain

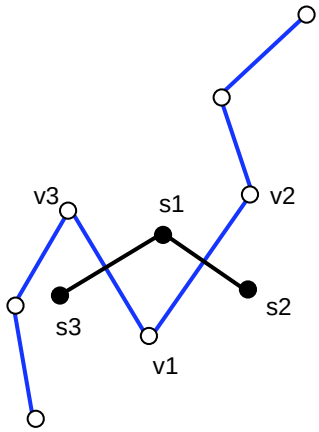
Assume that $VD(S)$ is given. Let σ be the set of segments in $VD(S)$ formed by bisectors of one point in S_1 and one point in S_2.



- Claim: σ is always a monotone (w.r.t. y-axis).
- How to determine σ?
- How to construct $VD(S)$ when σ, $VD(S_1)$ and $VD(S_2)$ are given?

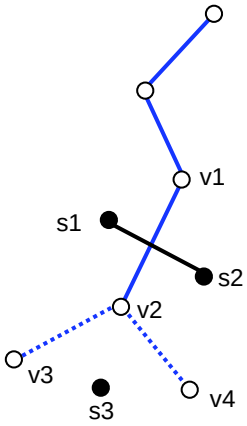
Separating Chain - Proving y-Monotonicity

- Separating chain turns upward.
- Separating chain stops.



v1v2 : bisector of s1 and s2
v1v3 : bisector of s1 and s3

s1 in S1, s2, s3 in S2
s3.x <= s1.x <= s2.x, a contradiction

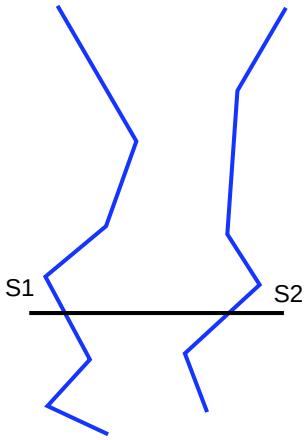


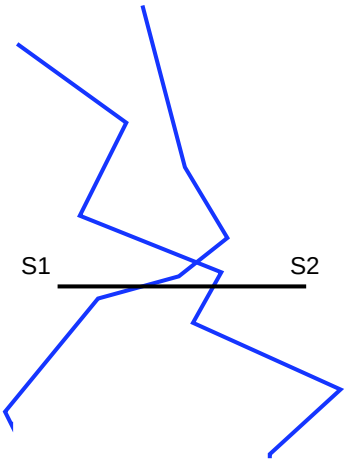
v1v2 : bisector of s1 and s2
v1v3 : bisector of s1 and s3
v2v4 : bisector of s2 and s3

s1 and s2 not both in S1 nor S2.
s3 either in S1 or S2, a contradiction

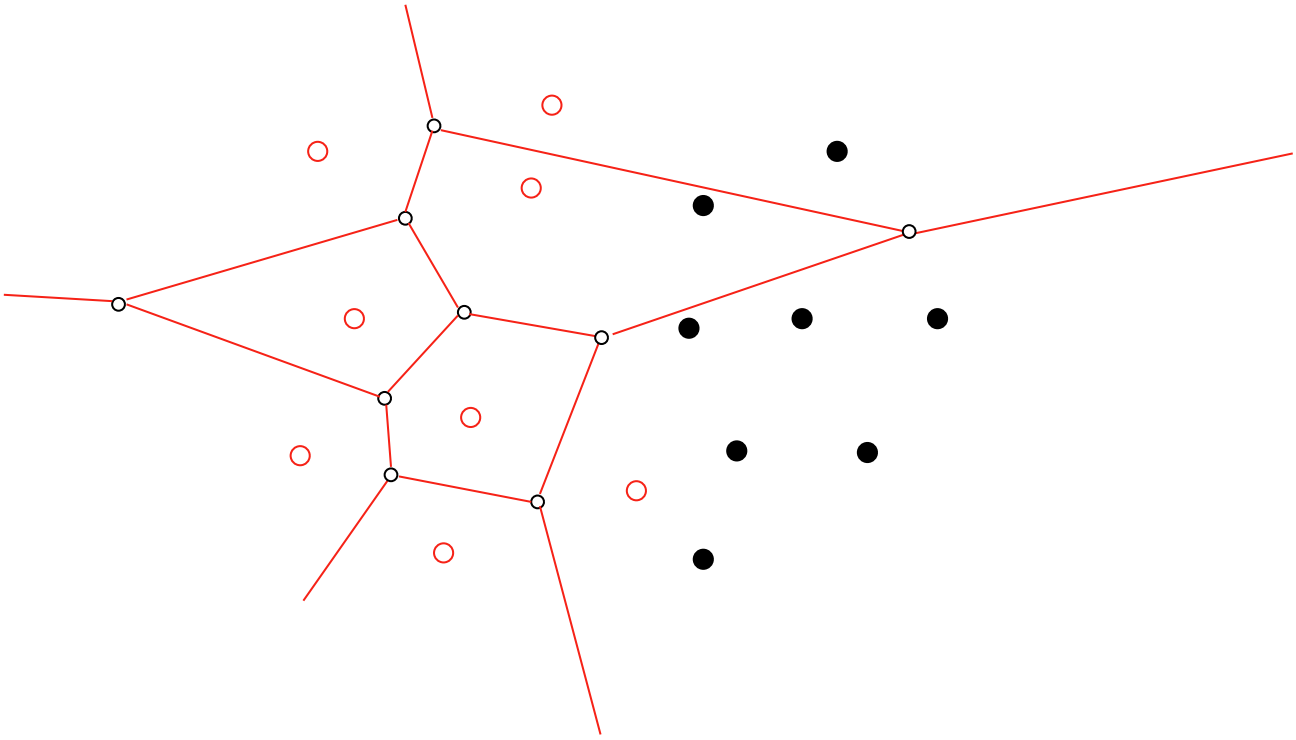
Separating Chain - Proving y-Monotonicity

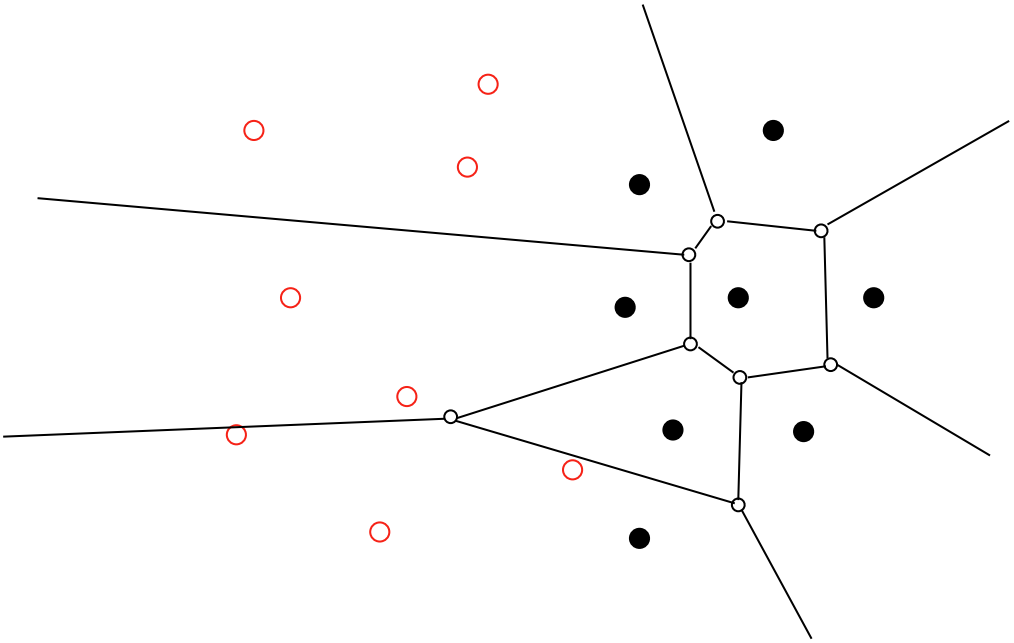
- Two non-crossing separating chains.
- Two crossing separating chains.



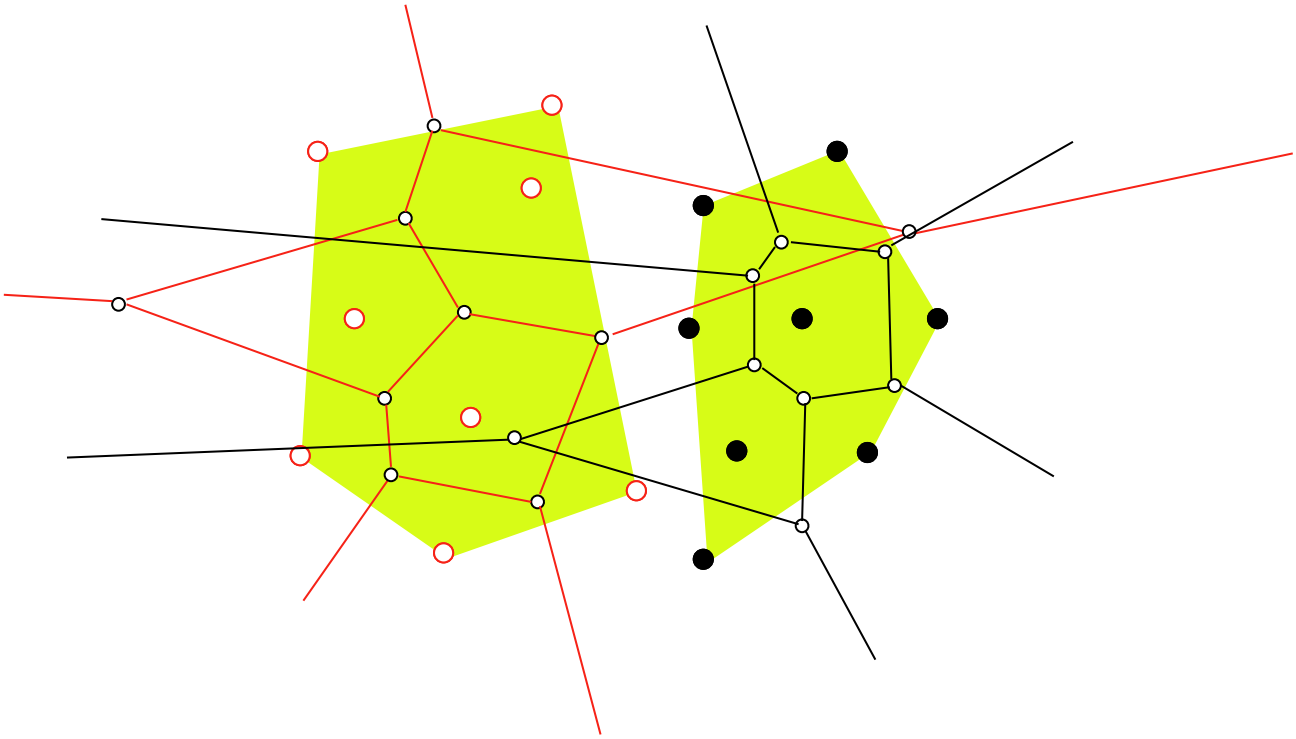


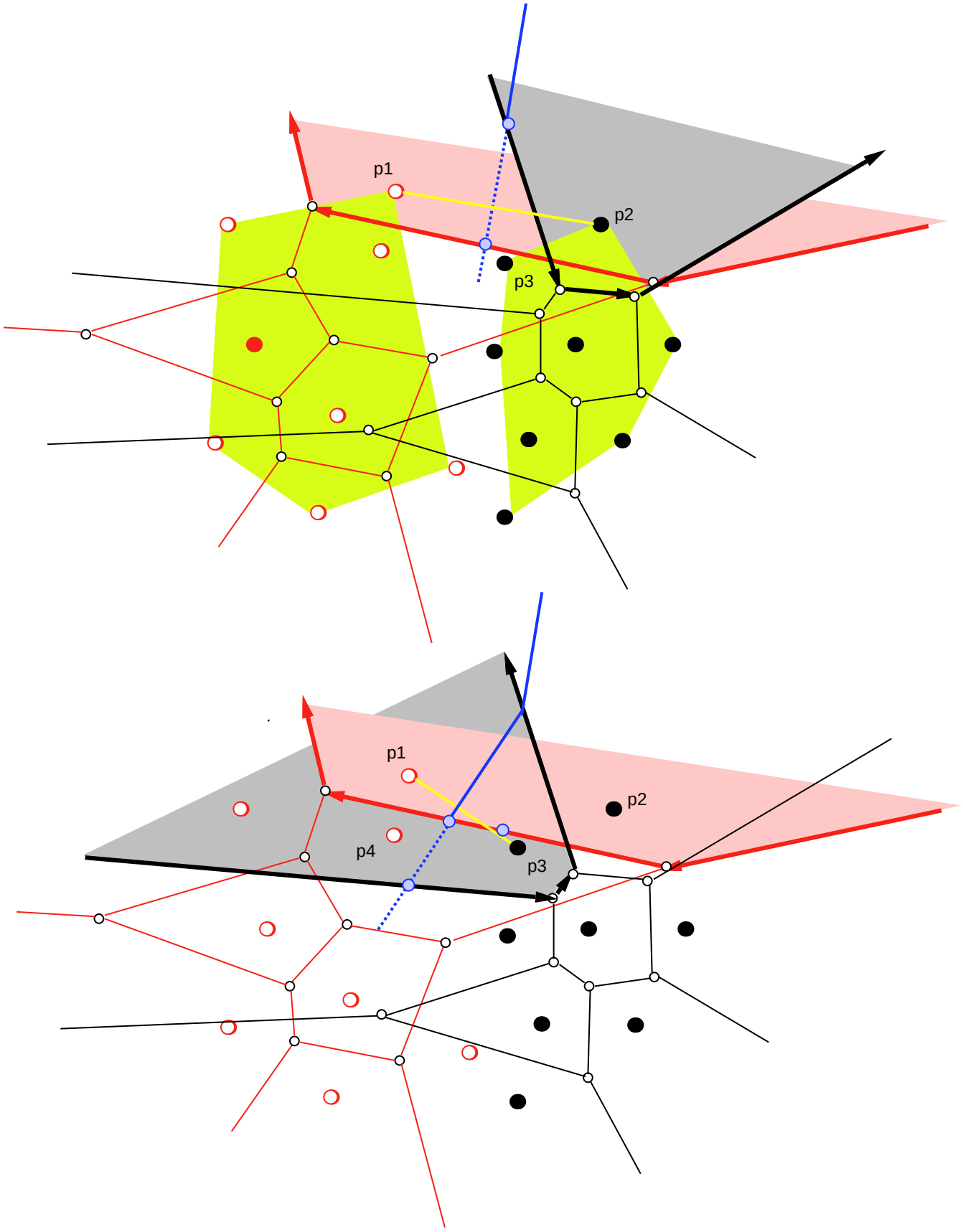
Voronoi Diagrams - Divide-and-Conquer





Voronoi Diagrams - Divide-and-Conquer

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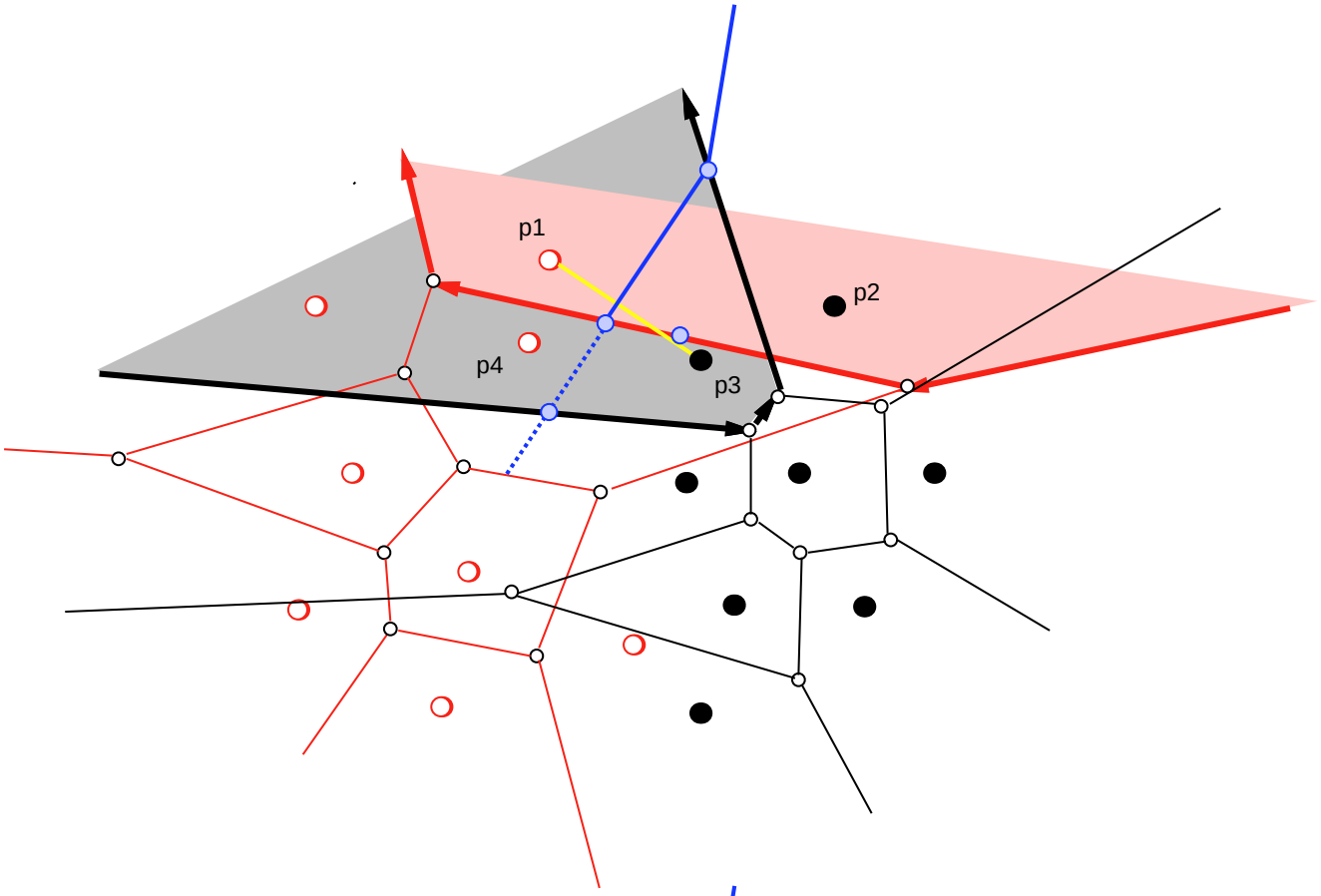
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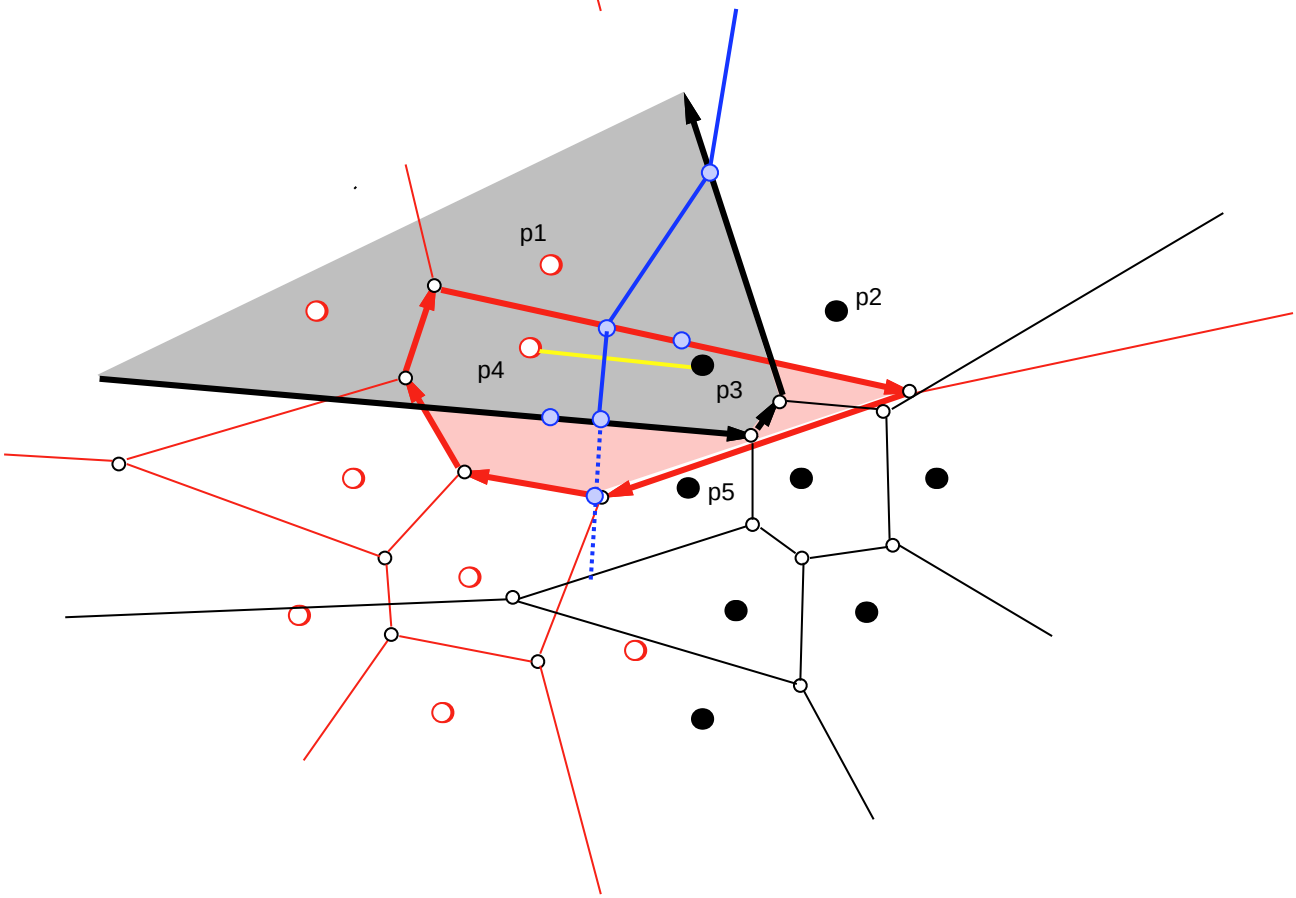
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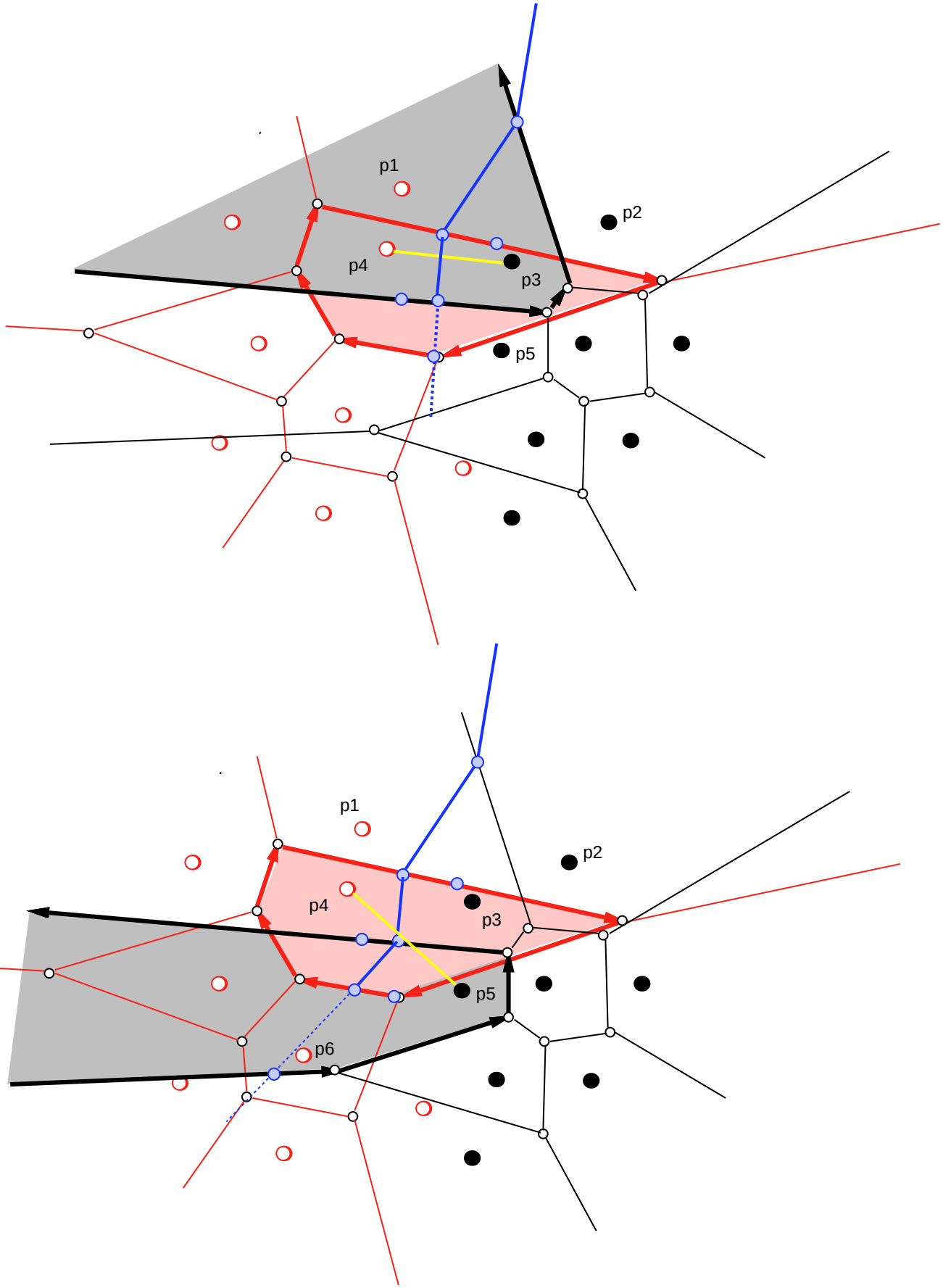
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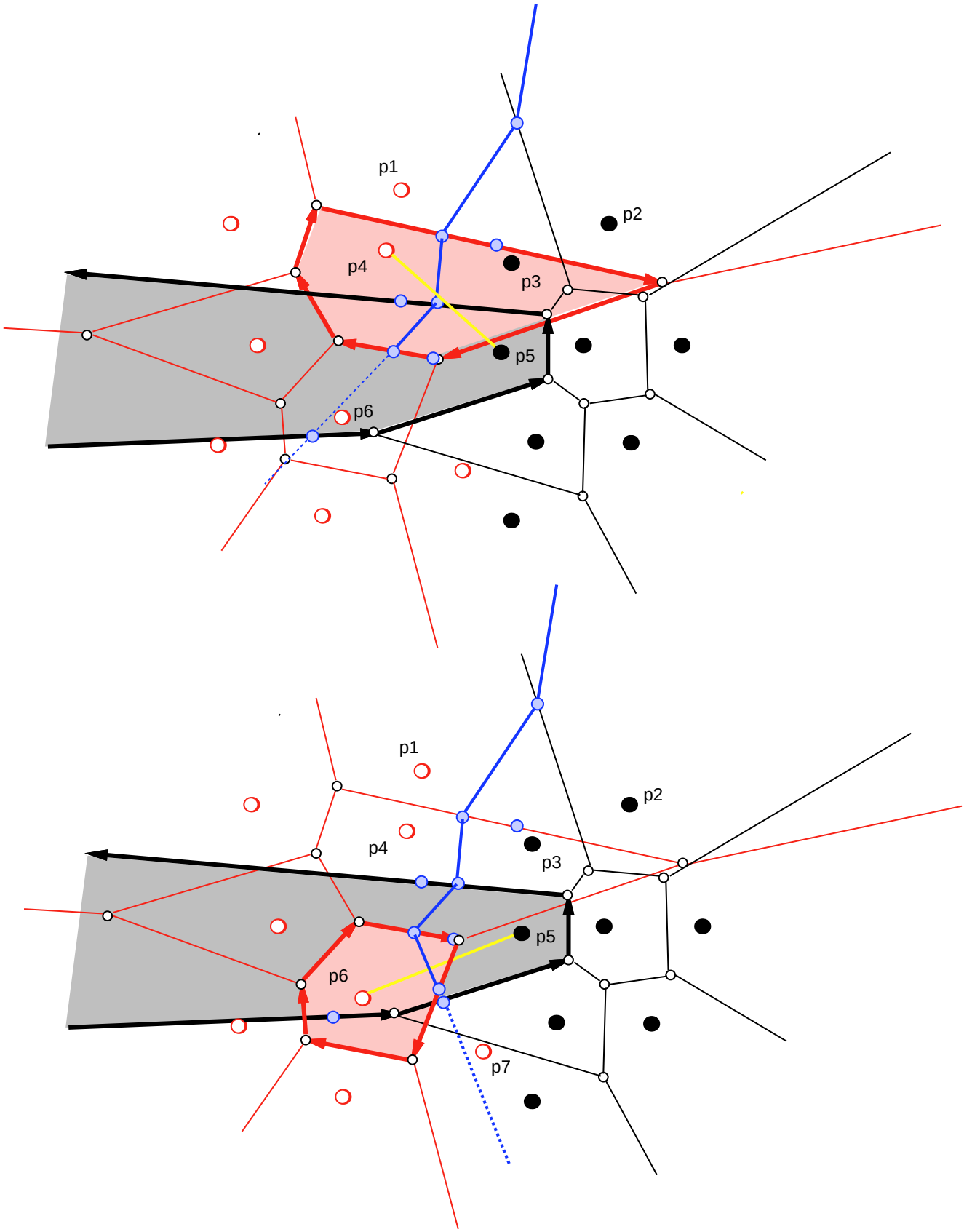
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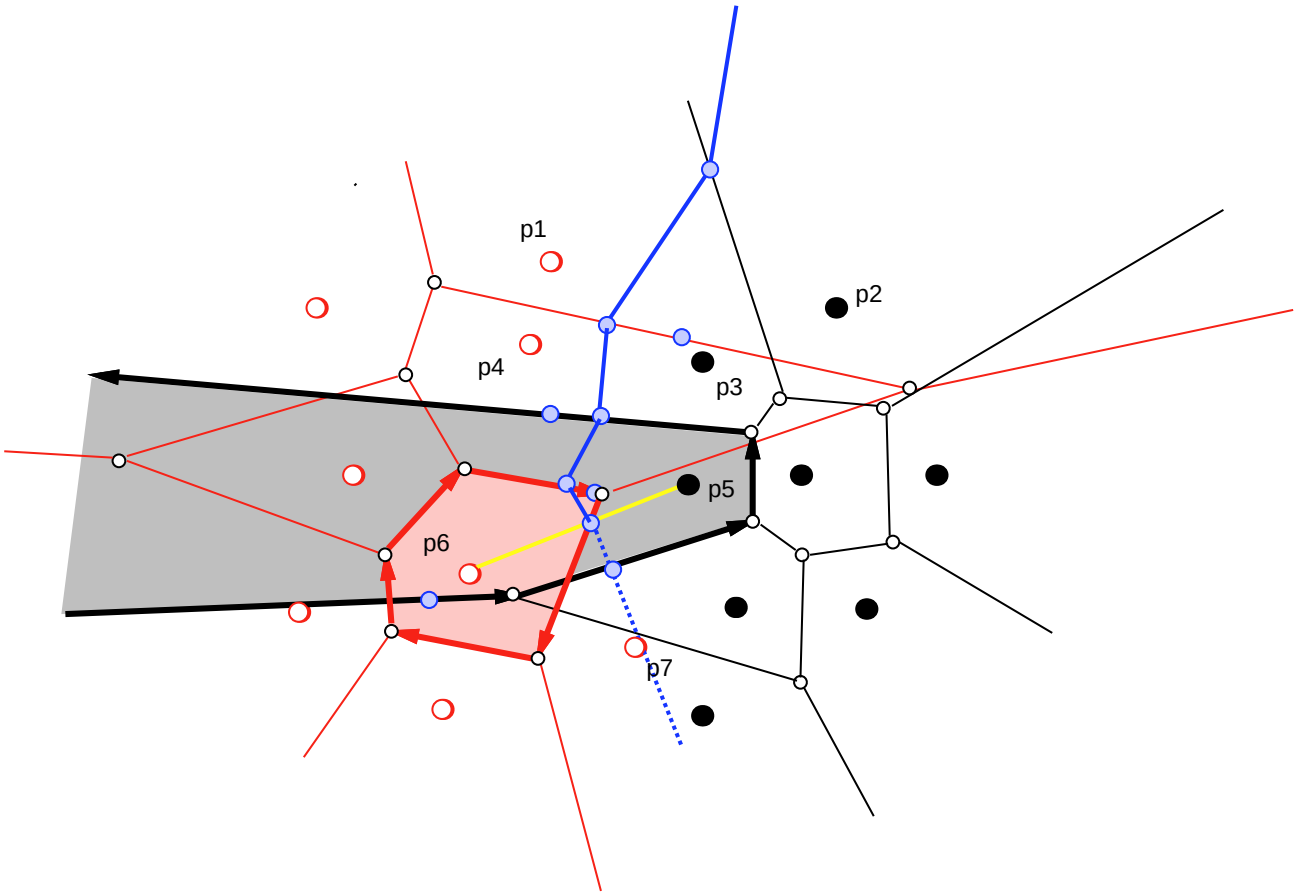
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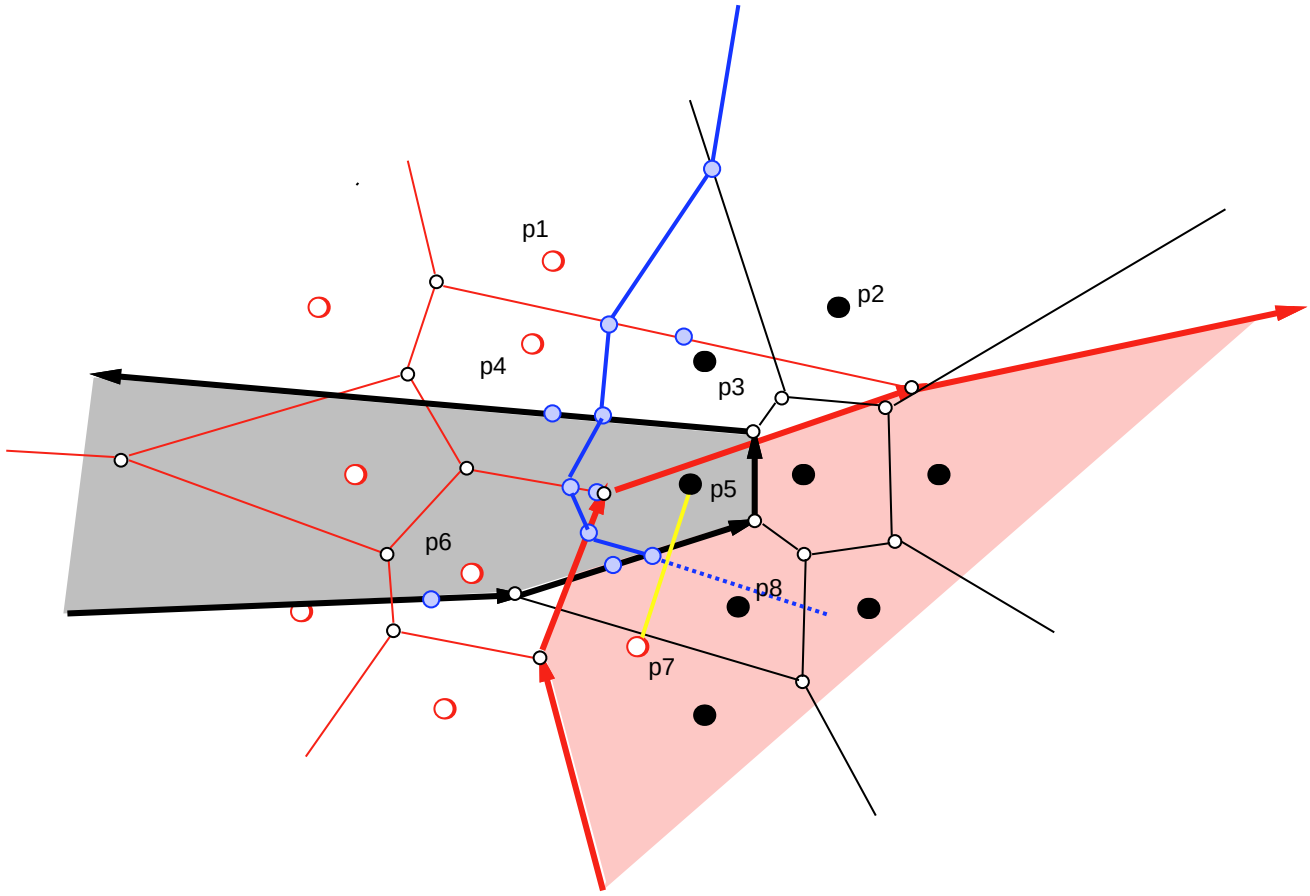
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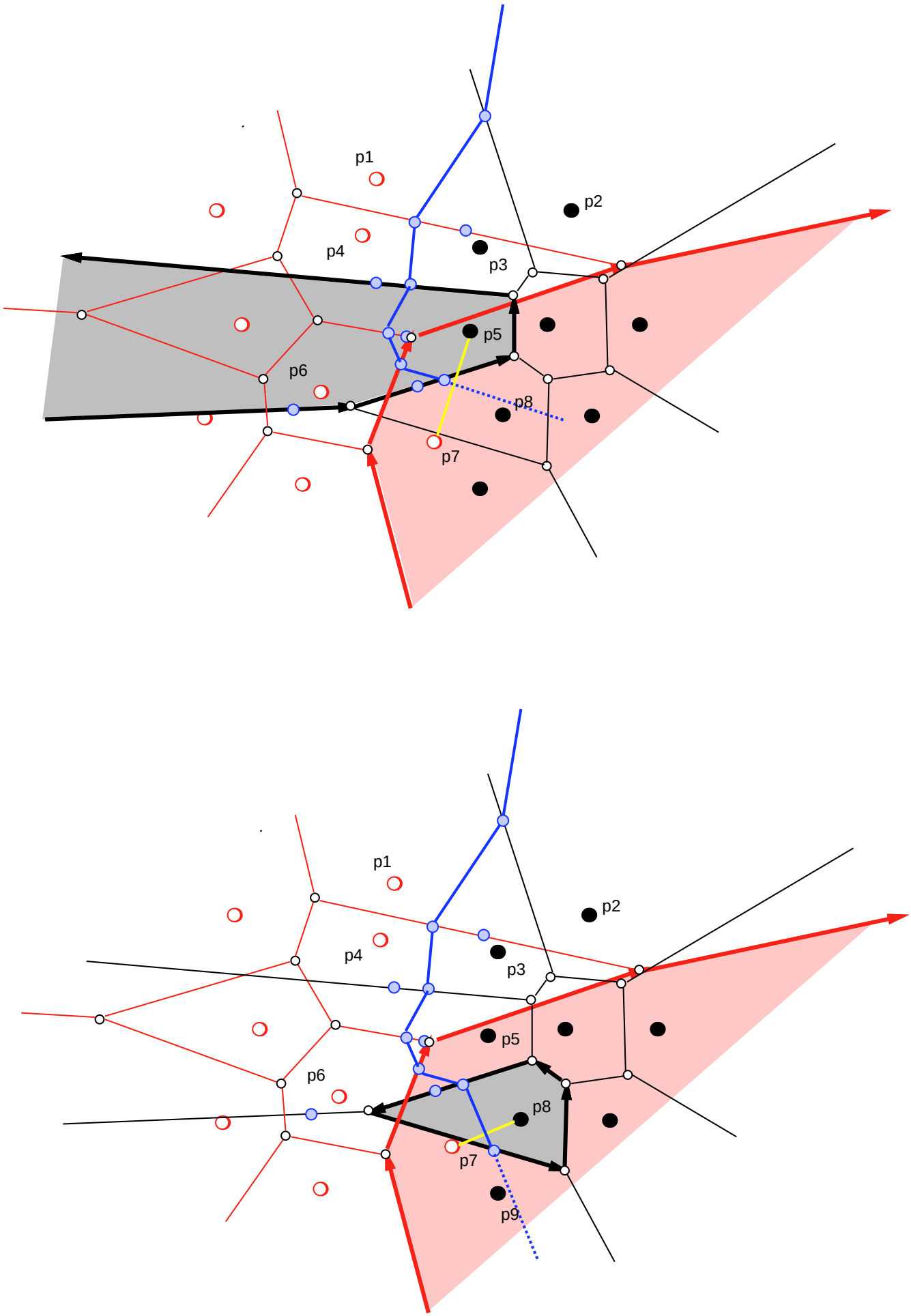
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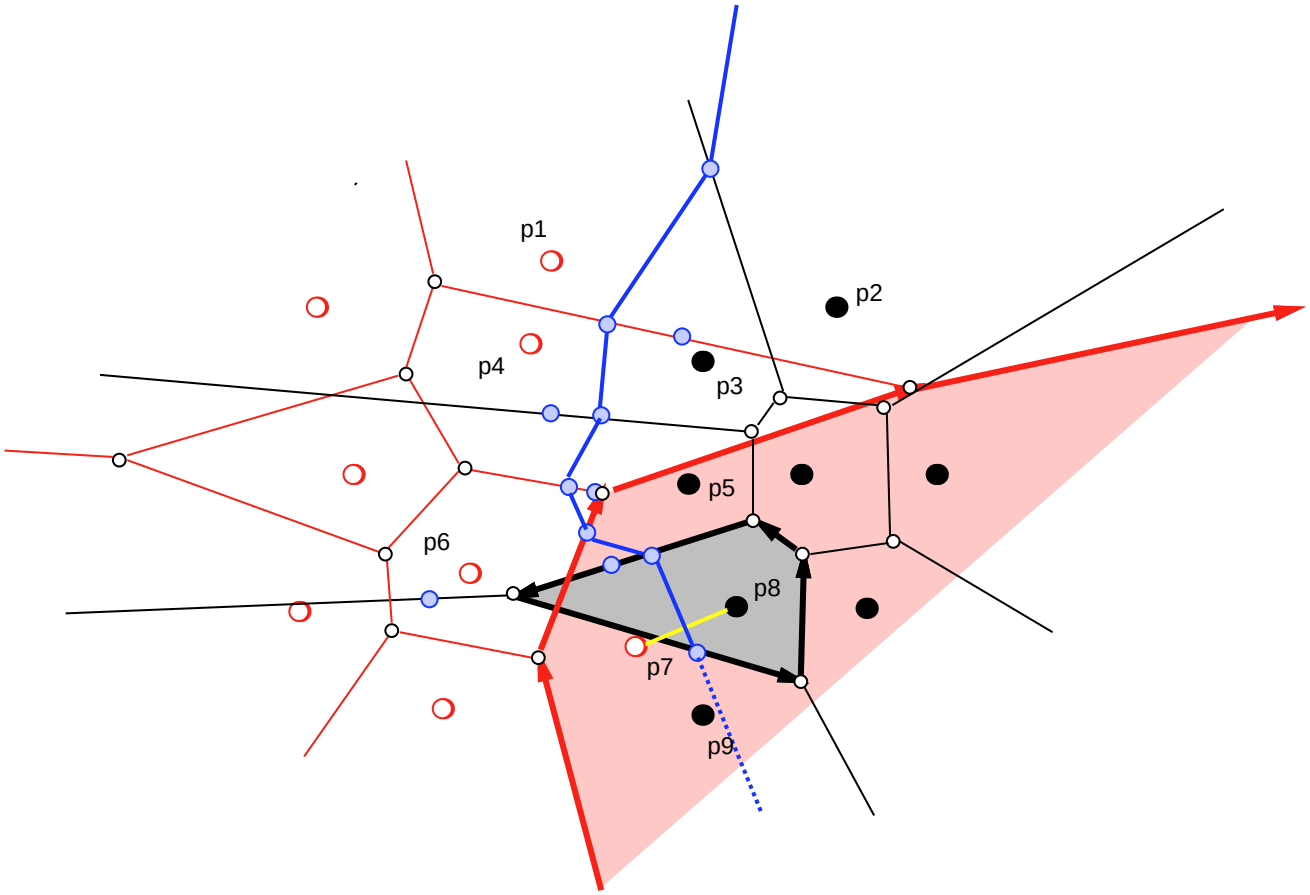
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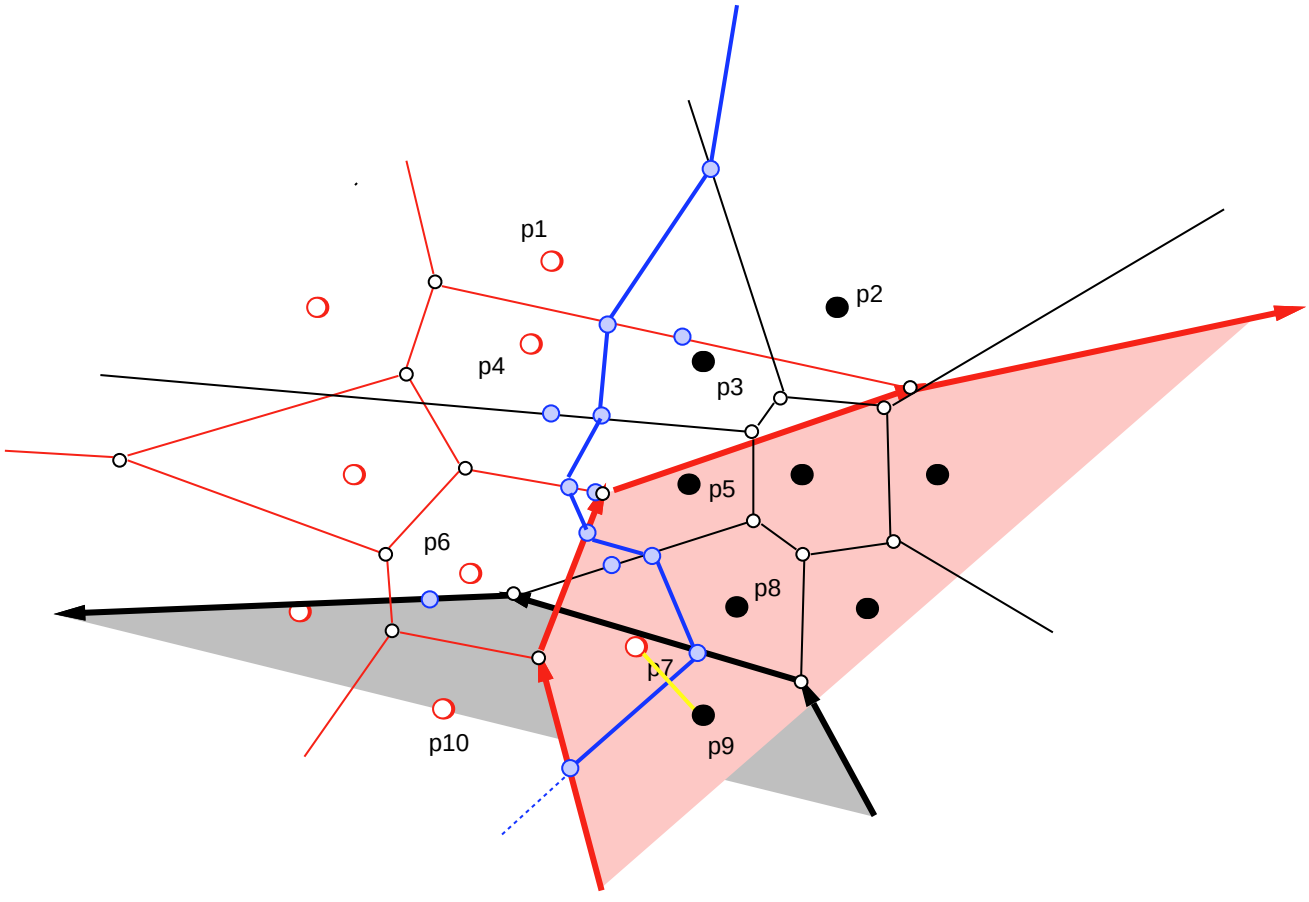
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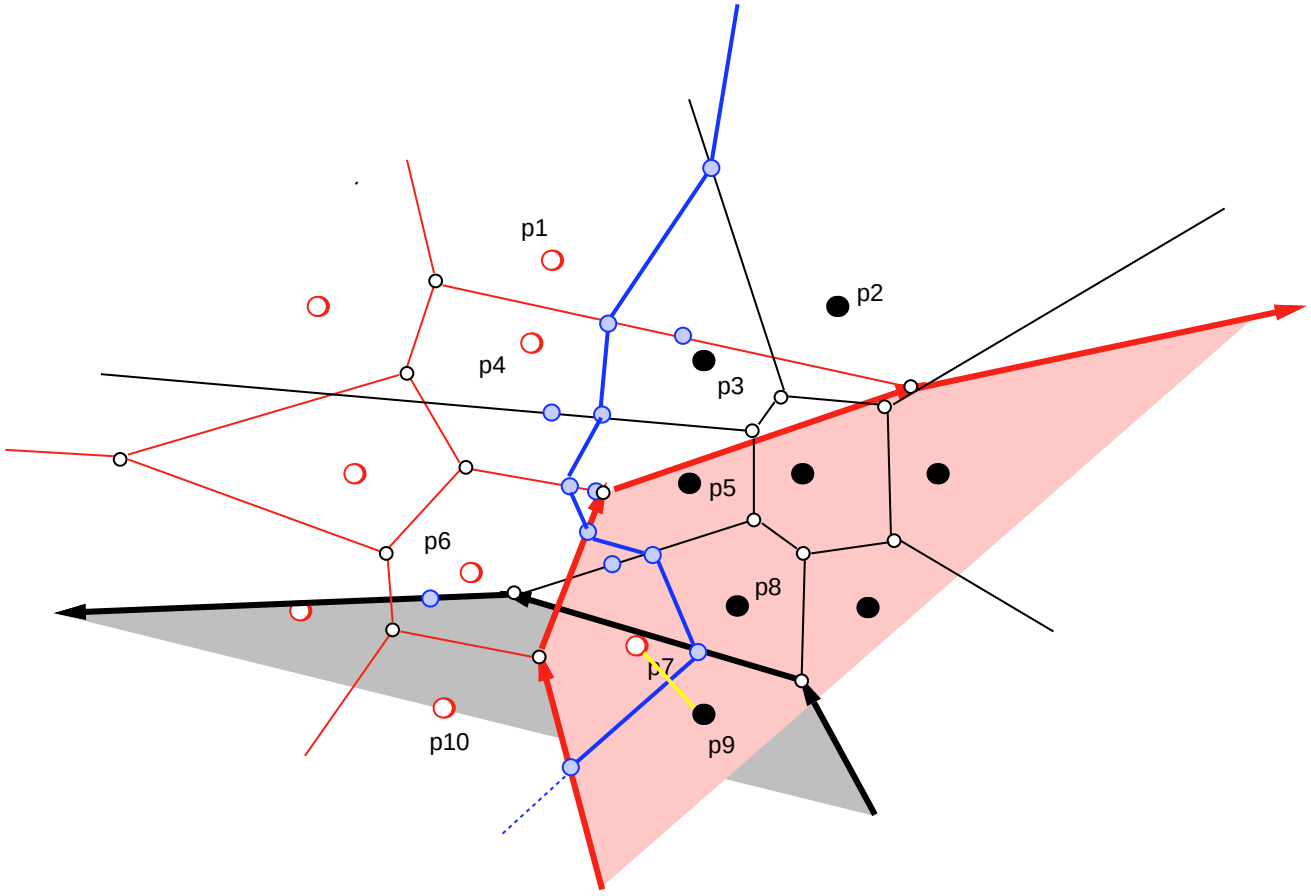
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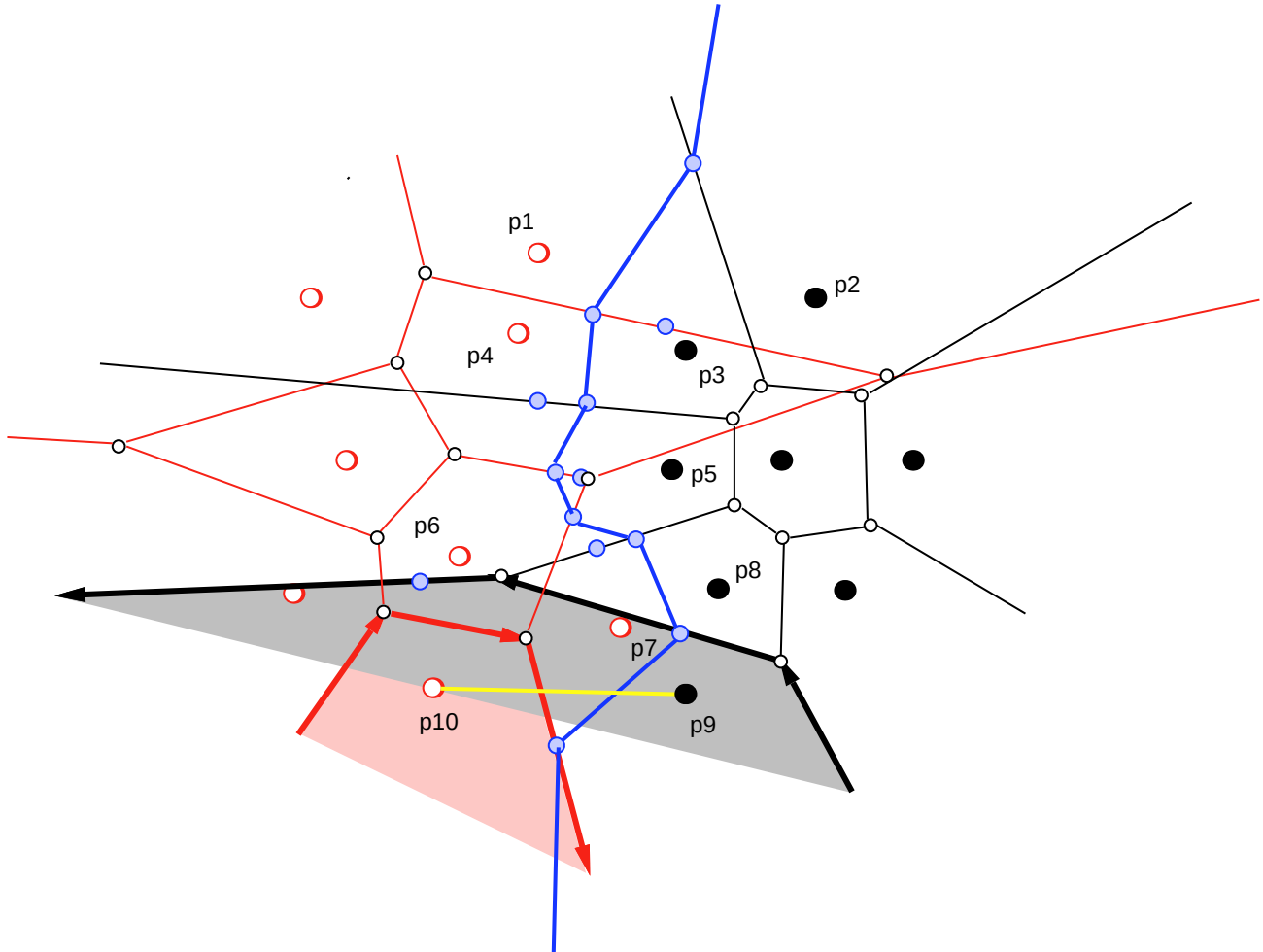
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