his independent 1971 t defined the P vs. NP By the time Levin's paper west, Pvs. NP had alread itself as computing tant question.

agliazzo, in a classic 1995 cribed five worlds with es of possibilities for the lem: nica: P = NP or something uivalent," such as fast algorithms for NP. ca: NP problems are hard case but easy on average. de We can easily create oblems, but not hard NP here we know the solution. orst of all possible worlds neither solve hard probrage nor do we get any apographic advantage from of these problems. t: Cryptographic one-way sist, but we do not have

syptography. ania: Public-key cryptogsible—that is, two parties e secret messages over

orlds are purposely not fored but rather suggest the possibilities given our of the P vs. NP problem. belief, though not univerelive in Cryptomania.

220 draws upon a "you tall" from P vs. NP theon either solve hard NP have cryptography, but lave both (you can have erhaps, though, we are a de facto Optiland. Admachine learning and opin both software and re allowing us to make problems long thought impossible—from voice to protein folding-and most part, our cryptoocols remain secure. n called "What if P=NP?" Survey,13 I wrote, "Learneasy by using the princia's razor—we simply find

program consistent with

ar-perfect vision recogni-

ge comprehension and

eranslation, and all other learning rasks become trivial. We will also have much better predictions of weather and earthquakes and other natural phenomenon."

Today, you can use face-scanning to unlock your smartphone, talk to the device to ask it a question and often get a reasonable answer, or have your question translated into a different language. Your phone receives alerts about weather and other climatic events, with far better predictions than we would have thought possible just a dozen vears ago. Meanwhile, cryptography has gone mostly unscathed beyond brute-force-like attacks on small key lengths. Now let's look at how recent advances in computing, optimization, and learning are leading us to Optiland.

Solving Hard Problems

In 2016, Bill Cook (no relation to Steve) and his colleagues decided to tackle the following challenge:9 How do you visit every pub in the U.K. in the shortest distance possible? They made a list of 24,727 pubs and created the ultimate pub crawl, a walking trip that spanned 45,495,239 meters approximately 28,269 miles—a bit longer than walking around the earth.

Cook had cheated a bit, eliminating some pubs to keep the size reasonable. After some press coverage in the U.K.,7 many complained about missing their favorite watering holes. Cook and company went back to work, building up the list to 49,687 pubs. The new tour length would be 63,739,687 meters, or about 39,606 miles (see Figure). One needs just a 40% longer walk to reach more than twice as many pubs. The pub crawl is just a traveling salesman problem, one of the most famous of the NPcomplete problems. The number of possible tours through all the 49,687 pubs is roughly three followed by 211,761 zeros. Of course, Cook's computers don't search the whole set of tours but use a variety of optimization techniques. Even more impressive, the tour comes with a proof of optimality based on linear program duality.

Taking on a larger task, Cook and company aimed to find the shortest tour through more than two million stars where distances could be computed. Their tour of 28,884,456 parsecs is within a mere 683 parsecs of optimal.

Beyond Traveling Salesman, we have seen major advances in solving satisfiability and mixed-integer programming-a variation of linear programming where some, but not necessarily all, of the variables are required to be integers. Using highly refined heuristics, fast processors, specialized hardware, and distributed cloud computing, one can often solve problems that arise in practice with tens of thousands of variables and hundreds of thousands or even millions of constraints.

Faced with an NP problem to solve, one can often formulate the problem as a satisfiability or mixed-integer programming question and throw it at one of the top solvers. These tools have been used successfully in verification and automated testing of circuits and code, computational biology, system security, product and packaging design, financial trading, and even to solve some difficult mathematical problems.

Data Science and Machine Learning

Any reader of Communications and most everyone else cannot dismiss the transformative effects of machine

Shortest route through 49,687 U.K. pubs. Used by permission. (http://www.math.uwaterloo. ca/tsp/uk).

