PhD Preliminary Examination April 2019

Design and Analysis of Algorithms

Question #1:
Let $A[1...n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an *inversion* of A.

- List the five inversions of the array $A[2, 3, 8, 6, 1]$.
- Give an algorithm that determines the number of inversions in any permutations of n elements in A in $O(n \log n)$ time. Explain the analysis of runtime.

Question #2:
Suppose you have one machine and n jobs, $a_1, \ldots, a_n$. Each job a_j has processing time t_j, profit p_j, and deadline d_j. The machine can only process one job at a time and that job must run uninterruptedly until completion. If job a_j is completed by deadline d_j, you receive profit p_j, but if it is completed after, you receive nothing. Assuming all processing times are integers between 1 and n and $d_j \geq t_j$ for all jobs, give an algorithm for computing the maximum profit you can make. What is the run time of your algorithm? Justify the run time.

Question #3:
Suppose that a graph G has a minimum spanning tree (MST) already computed. What is the runtime complexity of updating the MST if you add a new vertex and incident edges to G? Justify the complexity.