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WME and Automatic Mathematical Answer Checking

Paul S. Wang 王士弘
Institute for Computational Mathematics
Kent State University
pwang@cs.kent.edu

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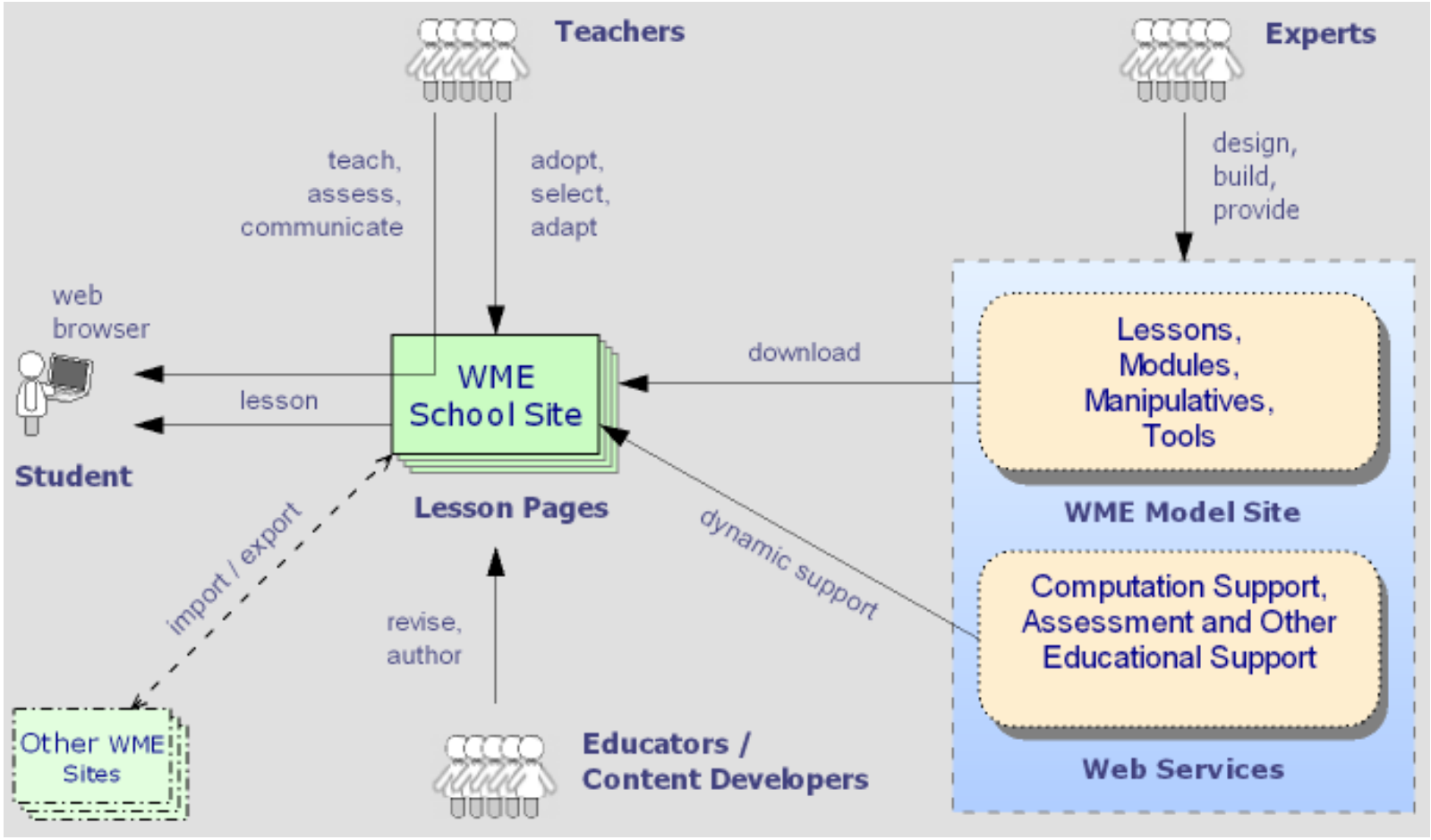
The WME Concept



The WME Project Web Site.

An Idea Whose Time Has Come

- Symbolic and numerical computation systems, have matured and become *Internet Accessible*.
- Mathematics teachers and students need help especially in the US.
- Availability and standardization of the Web and the Internet have grown and evolved sufficiently.
- Maturing technologies: MathML, ECMAScript, DOM, SVG, XML, CSS, Web Services, ...
- Increasing number of school districts have already deployed Internet/Web in classrooms.
- Web has begun to offer helpful materials for Mathematics teaching/learning.

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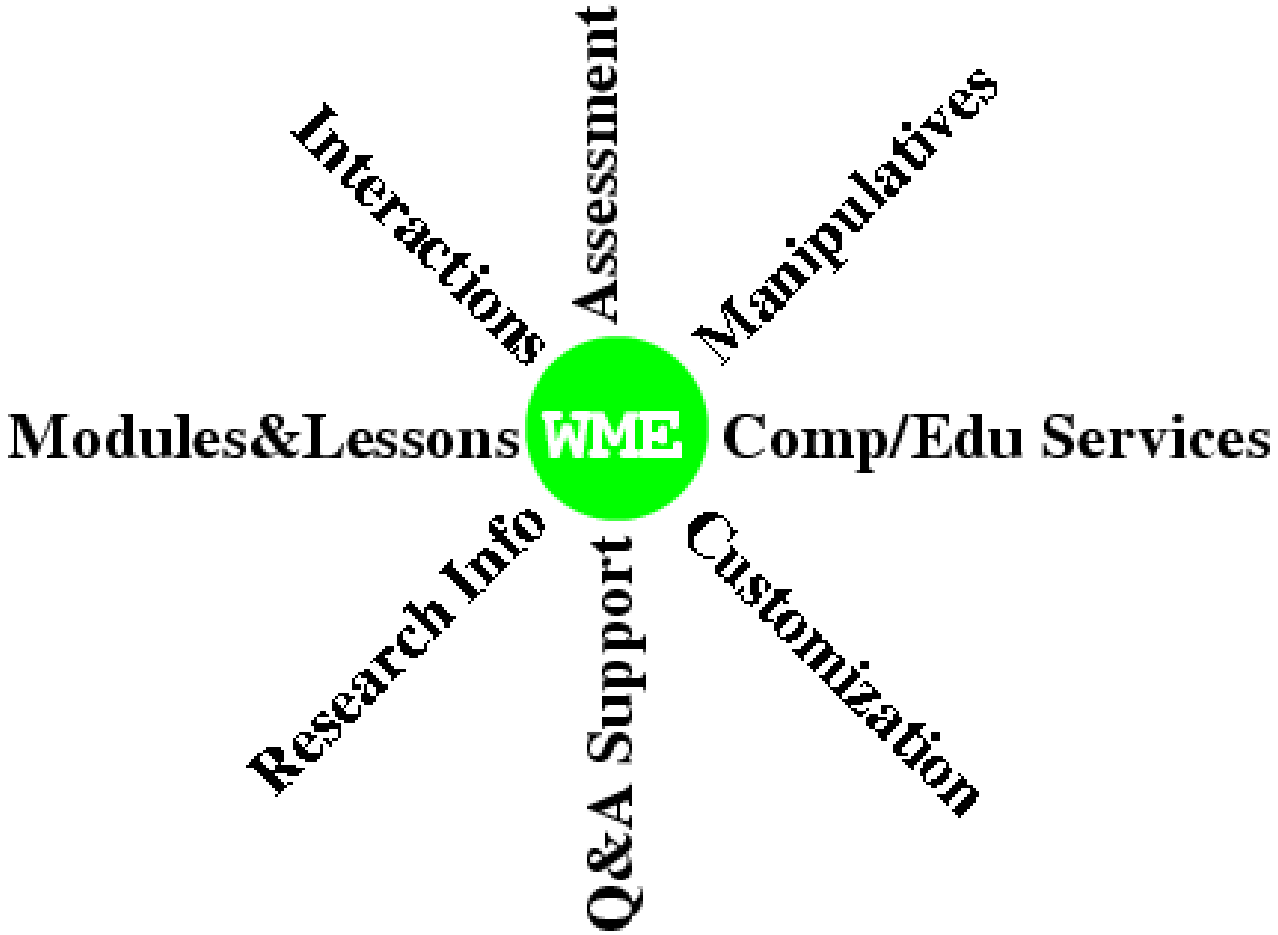
The WME Integration



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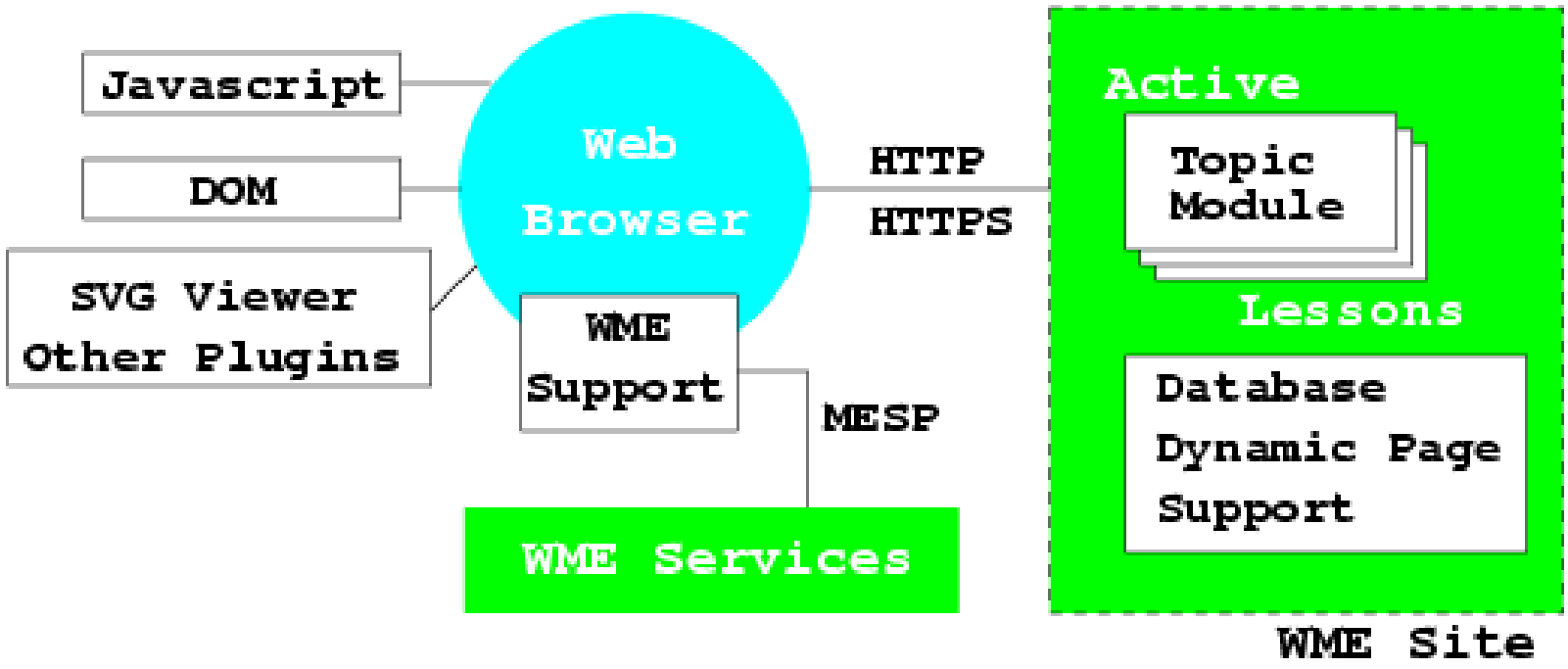
The WME Architecture

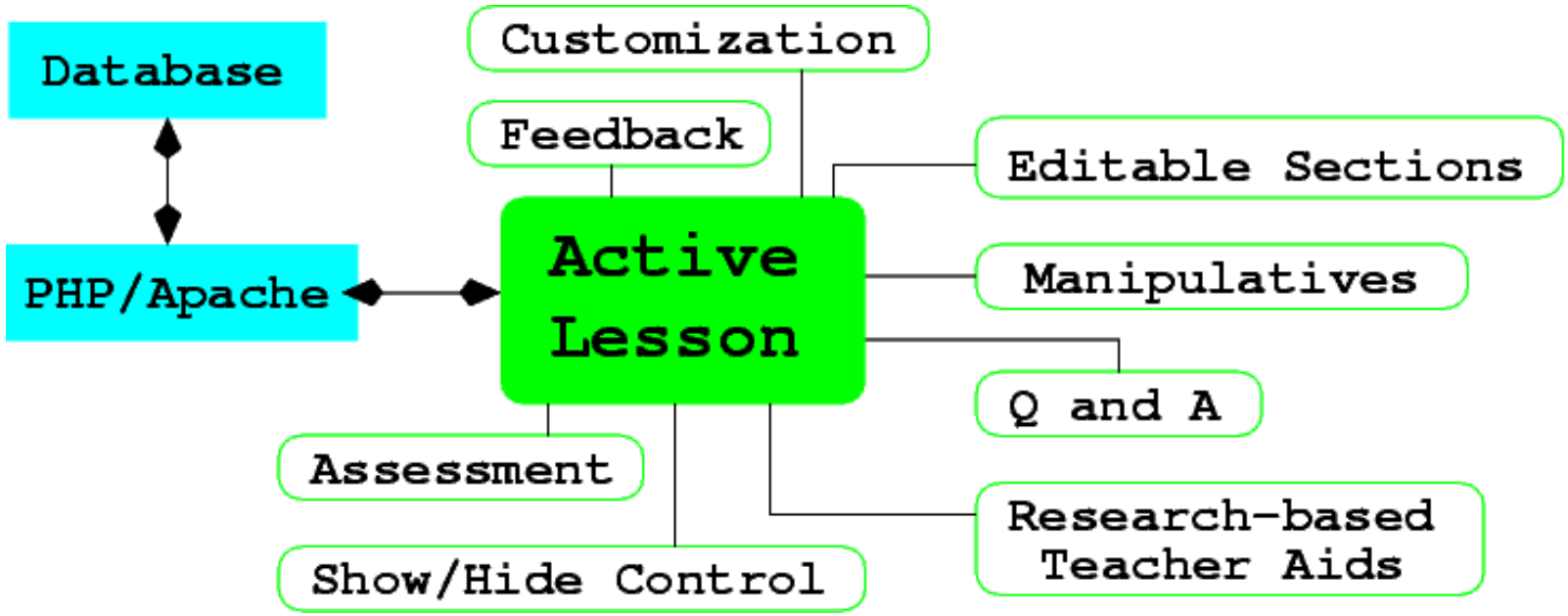


WME Components

- On-Web educational content—*Active Lesson Pages* (ALs) and *Topic Modules* (TMs)
- In-lesson *Manipulatives*—Interoperable, reusable, and user customizable objects.
- Assessment Support—assessment question database, test construction, grading, evaluation, and online tests.
- Client-side Support—regular browsers, javascript, SVG viewer, DOM, browser plug-in.

- Server-side Support—using active pages (PHP) and database (SQLite or MySQL).
- Content-markup Support—MeML and MeML processor, MathGraph.
- WME Services and Tools—GeoSVG, MathEdit, MathPlot, MathGlossary, MathChat, MathBoard, TISM and more.

Kimpton Pilot Project



The Kimpton Site.

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Manipulatives

Roll Start Over

Roll count (the number of rolls you made): 0.

Sum	2	3	4	5	6	7	8	9	10	11	12
Count	0	0	0	0	0	0	0	0	0	0	0

Dice. Equality. Straight Lines.

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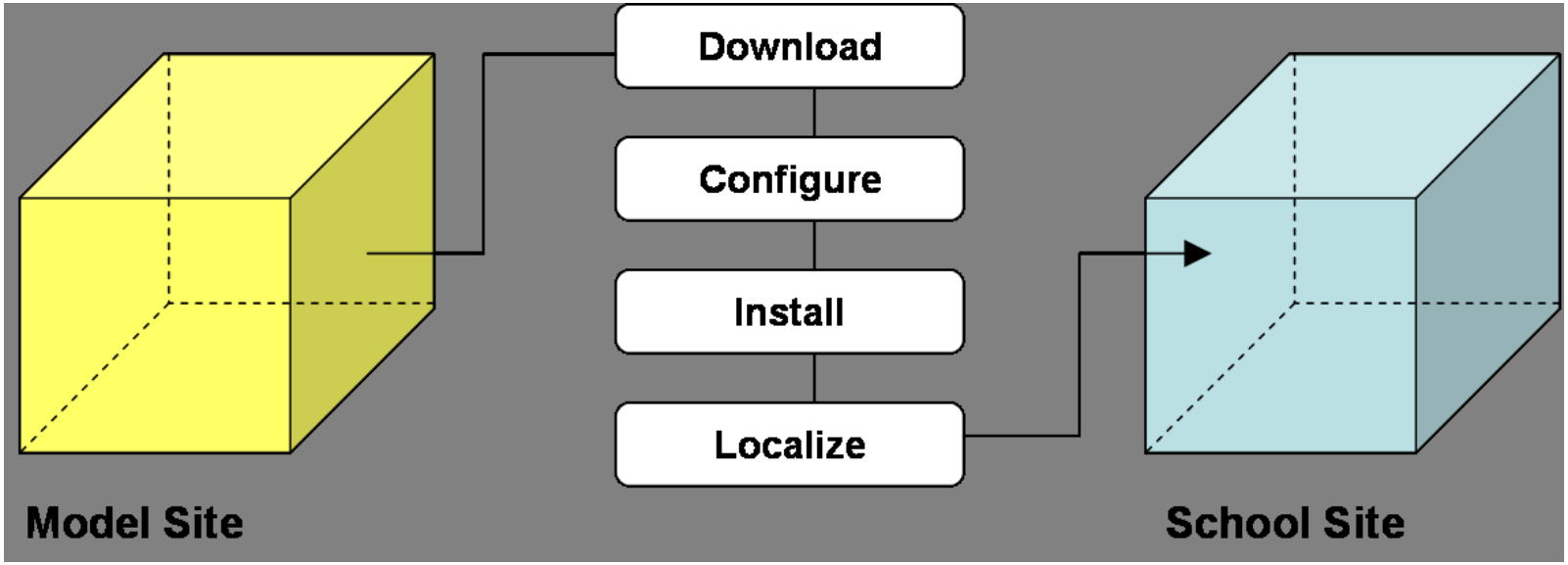
WME Model Site Download



WME Customizations

- For each school—user accounts, grade levels, course listings, course sections.
- For each course—TM and AL selection, student list.
- For each lesson—manipulatives editing: including text, presentation, and functionality, assessment and challenge questions.

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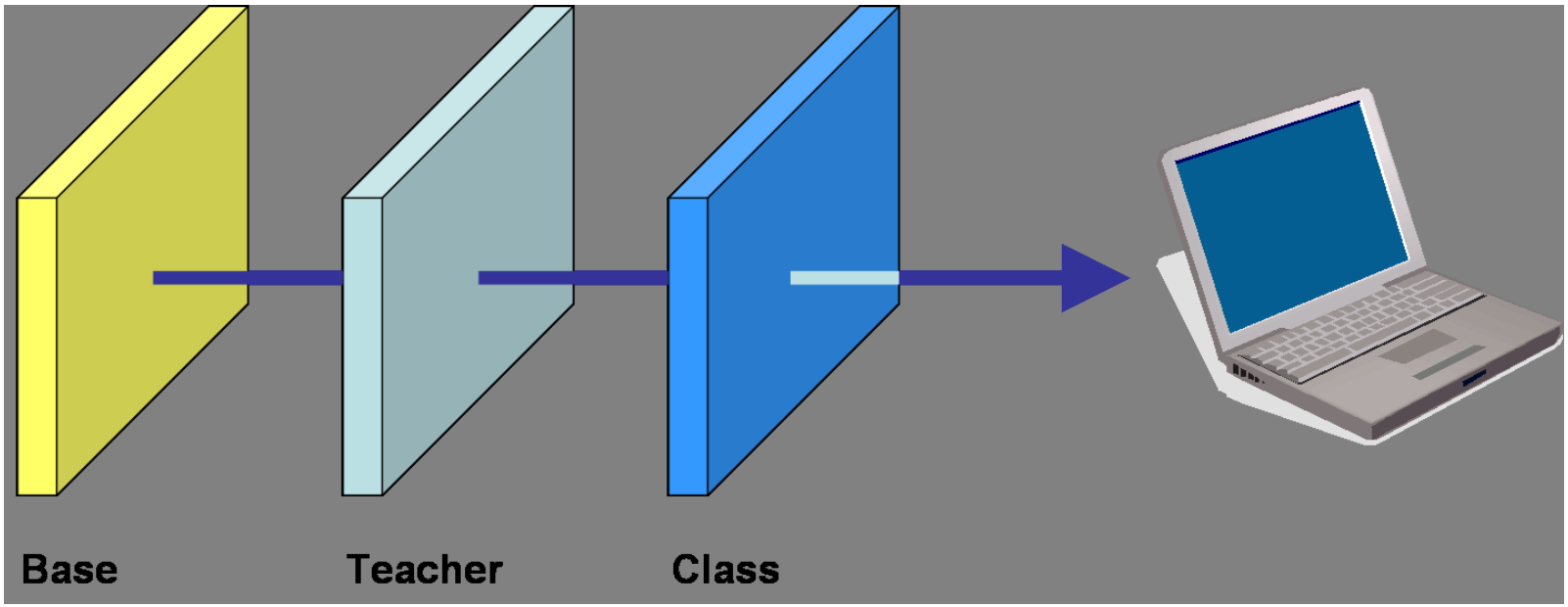
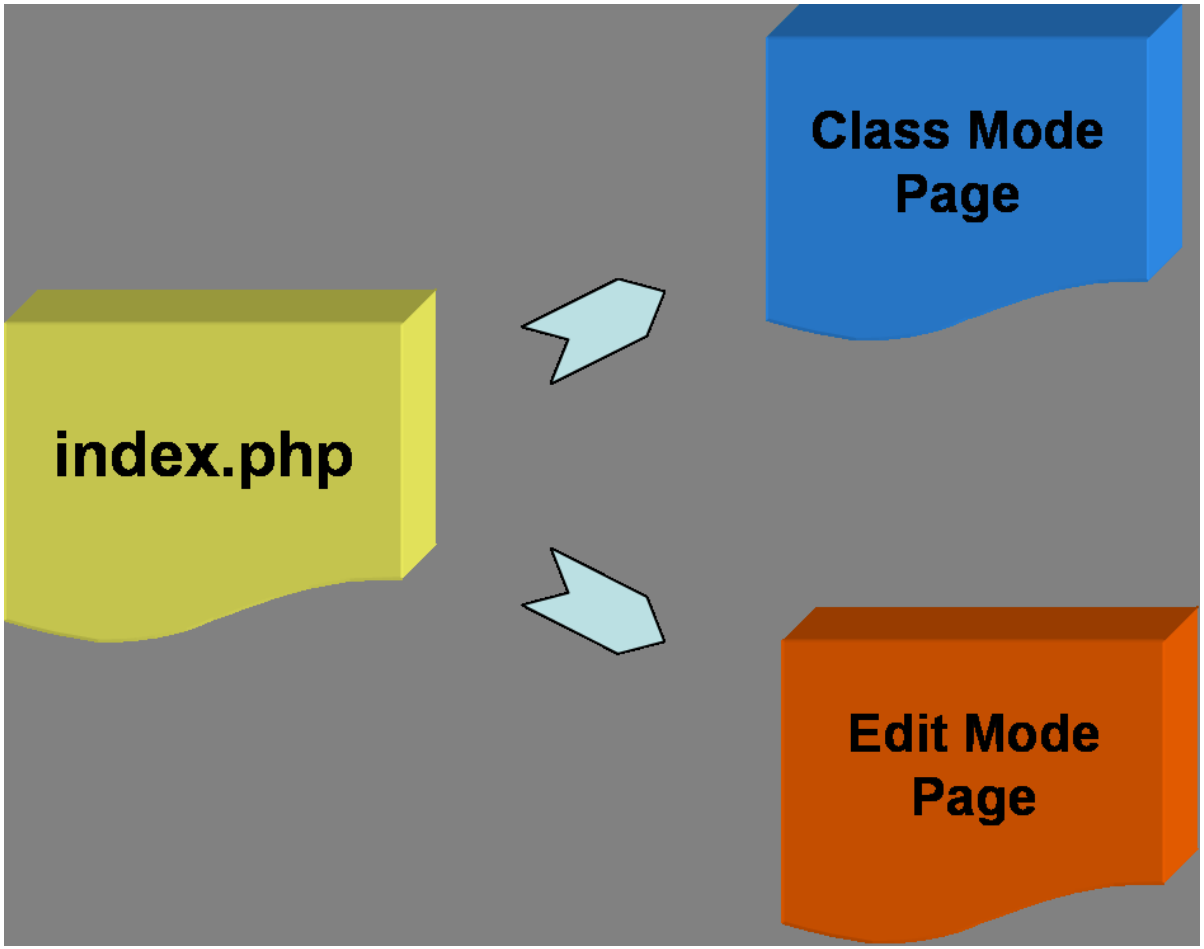
Page Customization Layers

Customizations are per-teacher and per-class.



Dynamic Page Generation



Assessment Help and Automation

- Test authoring, construction, and editing
- Online test taking
- Importing and exporting test questions
- Automatic grading and test data management
- Results evaluation/grading, diagnoses and suggested interventions

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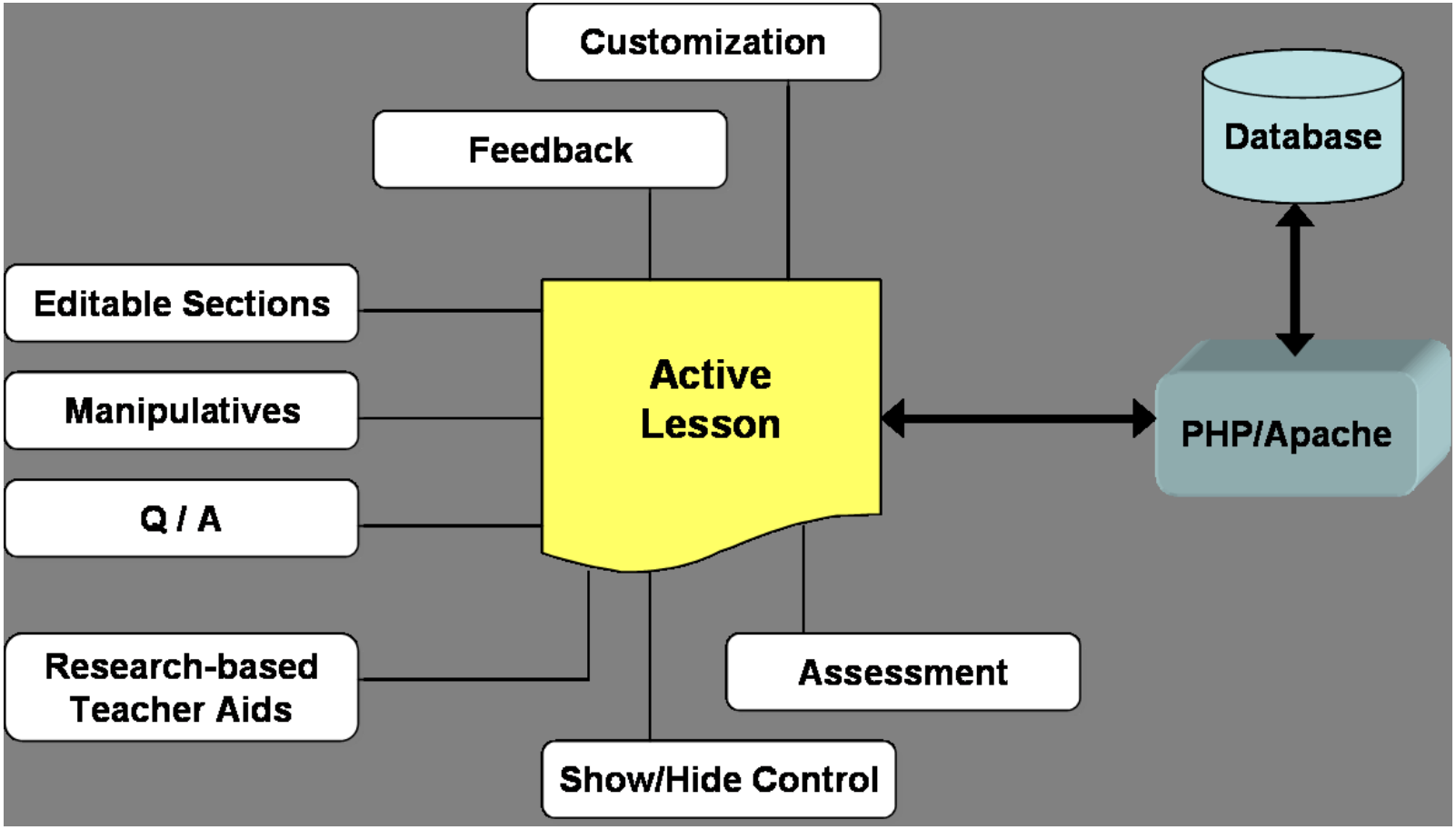
Interoperable Modules and Lessons



Answer Checking

- The usefulness and challenges of automatic checking (grading/marking) of mathematical answers have been investigated before.
- *SAGE: a Homework on the Web System* by Brad Lucier, Purdue University, USA, 2005
- *Assessing mathematics automatically using computer algebra and the internet* by C. J. Sangwin, University of Birmingham, UK, 2003

Answer Checking Aspects

- Problem types, answer collection forms.
- Answer input by user.
- Problem authoring/editing, problem generation, problem database.
- Answer grading, common mistakes.
- Feedback to learner.

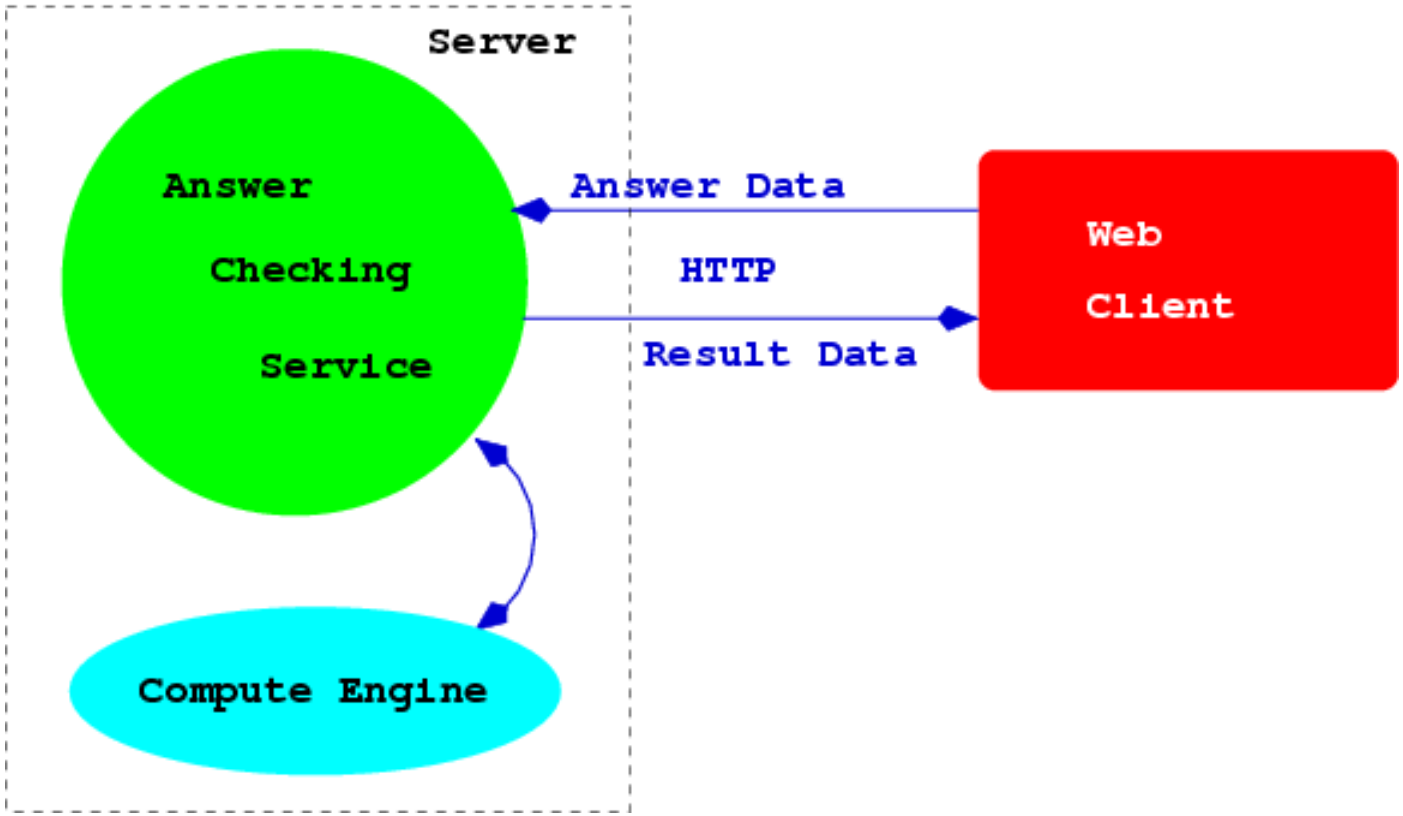
Problem and Answer Types

- True/False and multiple choice problems make answer checking trivial.
- Essay (short and long answer) questions are next to impossible to check automatically.
- What if the answer is a number or a mathematical expression?
- What if the answer is several expressions (such as in factors of a polynomial)
- What if the answer is not unique?
- What if we wish to accept "close" or "almost right" answers?
- What if the problems are automatically generated and the correct answers need to be derived?

Client-Side and Server-Side Answer Checking

- To provide quick feedback to users trying new concepts or skills, client-side answer checking is quick and responsive.
- To support complicate logic, manipulation of mathematical quantities, and to assign grades, server-side answer check is more suitable.
- Answer checking services: support wide variety of problem types, invoked by HTTP `POST/GET` requests and returning `true/false` and perhaps additional results.

Answer Checking Service



Answer Checking Example

Checking the derivative of a mathematical expression.

- Try this example On IE, On Firefox.
- It involves displaying mathematical formulas (MathML), receiving user entered formula (MathEdit tool), an ad-hoc HTML form to post to a server-side program that checks the answer and provides a response page.
- The user input is displayed using MathML and transmitted to the answer checker in infix notation.
- The correct answer, the derivative, is computed by the checking program rather than given as part of the *answer checking data* posted from the client side.

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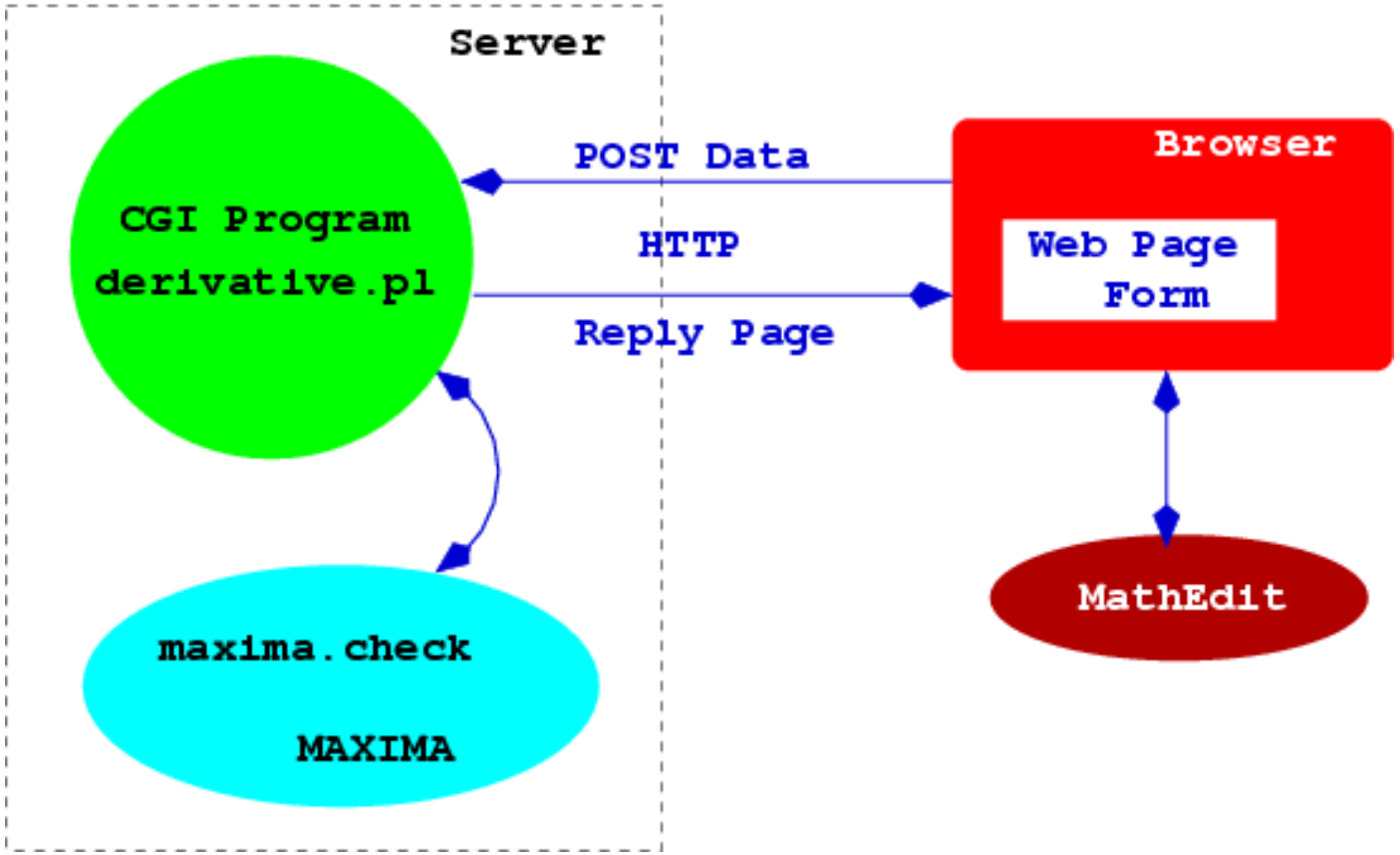
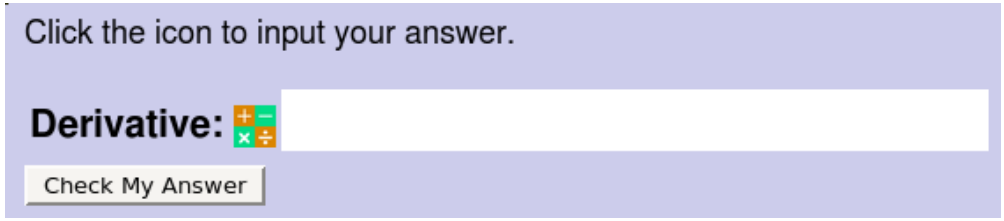
Derivative Checking Service



Collecting Answer from User

```
Derivative:
<img onclick="mathedit.newMath('e1', 'der');"
     style="vertical-align: middle"
     src="img/mewhite24.png" />
<div id="der" style="width: 400px">
 </div>
```



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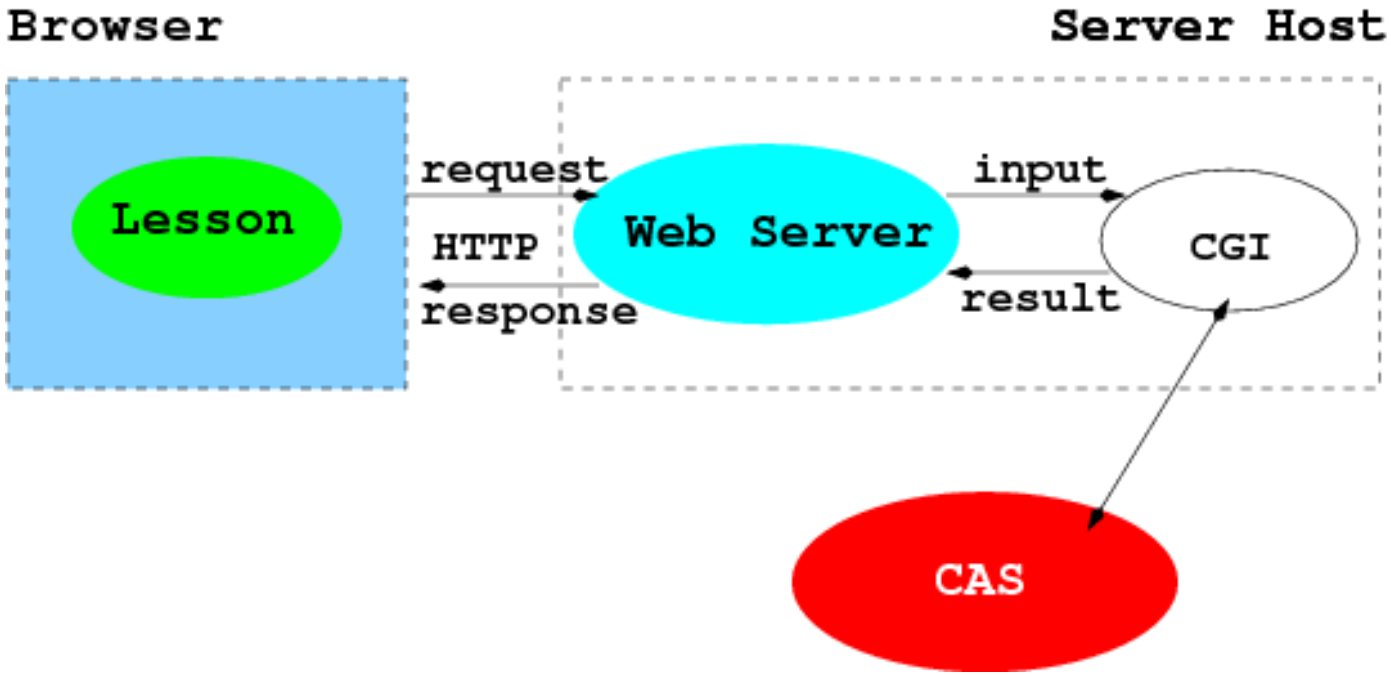
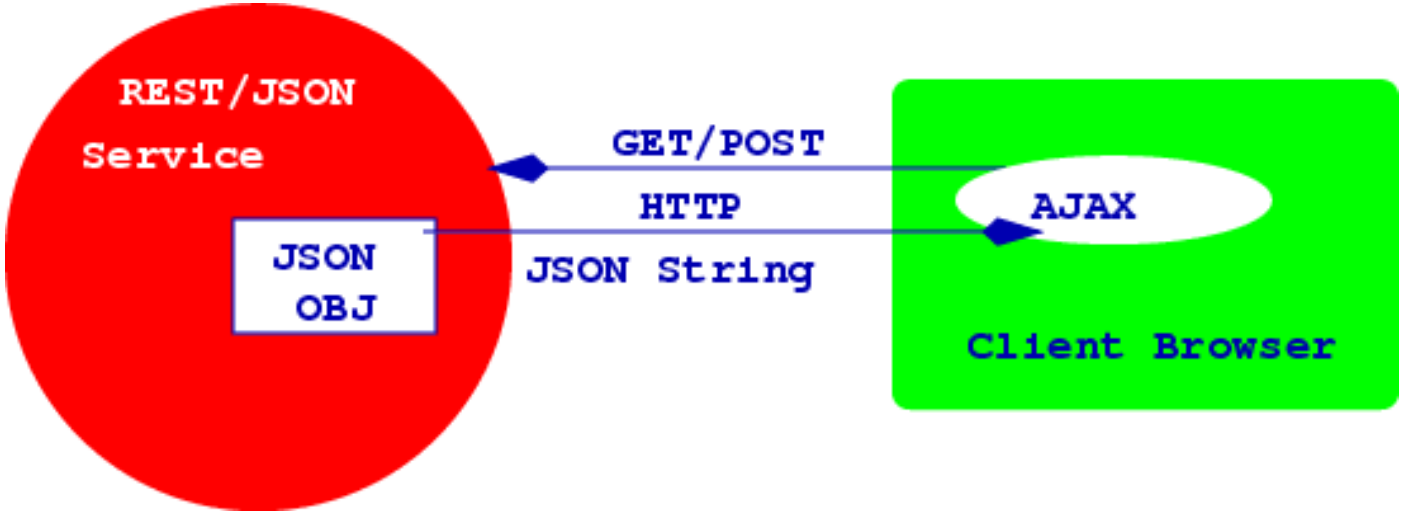
Using CAS for Checking



Decoupling Client and Server

- The above approach tightly couples the client side and server side for any particular answer checking application.
- We can break this tight coupling by client-side AJAX (Asynchronous Javascript And XML) and by establishing an *answer checking protocol* between the client and the answerch checking service.
- We will consider a REST/JSON based answer checking Web service.
- REST = Representational State Transfer
- JSON = Son of Javascript, a data format

Browser Access to JSON Service



Service Returning JSON

- Made easier by using JSON, a Javascript-defined data representation format.
- The server returns a JSON string which can usually be evaluated directly by Javascript by calling `eval()`.
- The returned MIME type: `application/json`
- The returned data in UTF-8 consists of one or more lines of correct Javascript code. Strict JSON requires returning a *JSON object*.

Client Receiving JSON

- Use an AJAX object to send GET or POST request to the service.
- Use `encodeURIComponent()` to construct query string.
- Service must be on the save server host.
- Because of the async nature of the http request, processing of incoming data is via a *callback function*.
- The results returned are directly accessible in Javascript and can be used to immediately update the host page.

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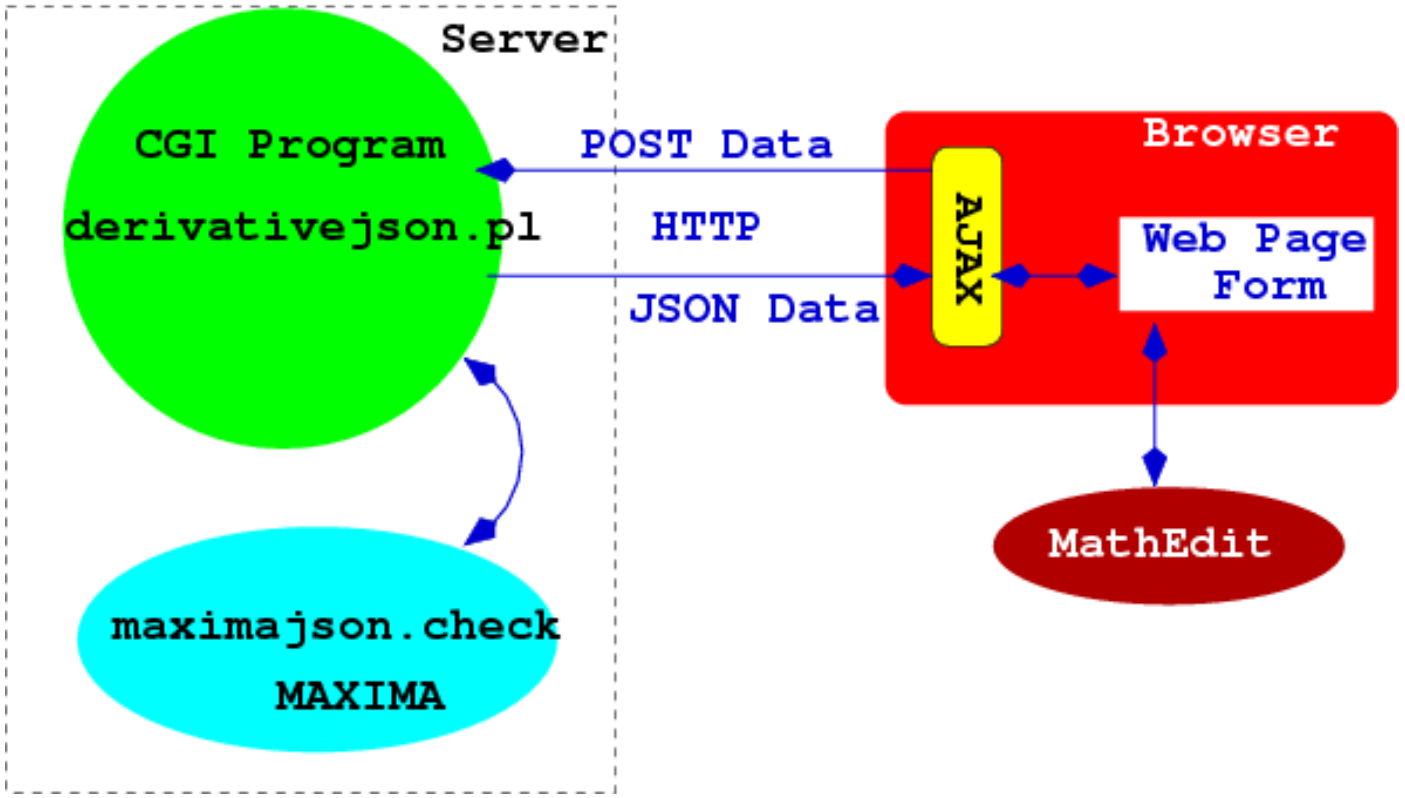
Derivative Checking Service



Derivative Checking Example

Client Side Code for Javascript files

```
<script type="text/javascript" src="ajaxobj.js"></script>
<script type="text/javascript" src="derivativejson.js">
</script> <script type="text/javascript"
                  src="changejson.js"></script>
```

Client Side Code for collecting and checking answer.

```
<p><input type="submit" name="check" value=
    "Check My Answer" onclick="doCheck(theExp)" /></p>
<div style="color: #00c; font-weight: bold"
     id="checkResult"> </div>
```

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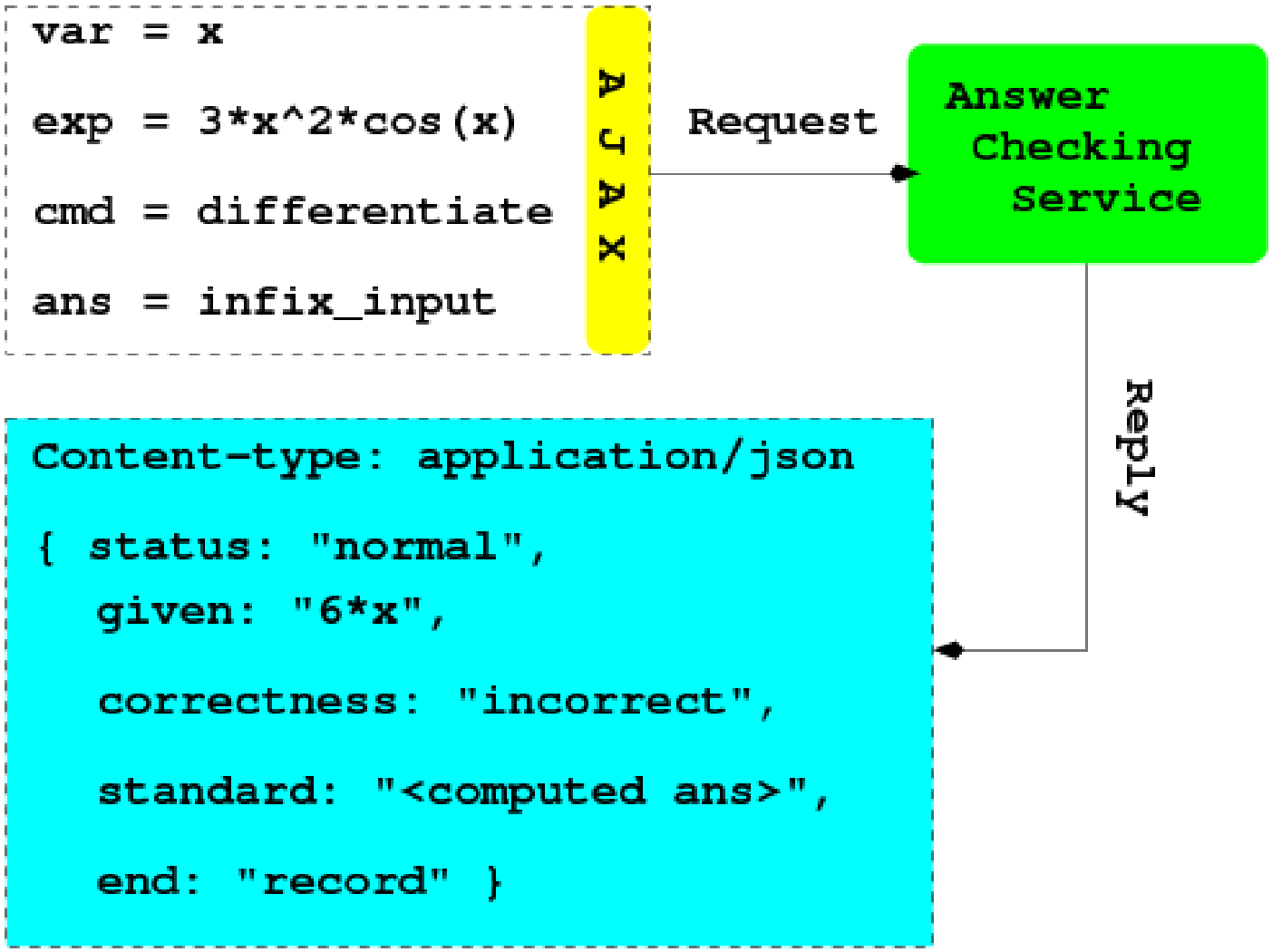
Checking Data Flow



Javascript Code checkResult

```
var answer;

function doCheck(e)
{ var ex=encodeURIComponent(e);
  var infix=encodeURIComponent(
       mathedit.getInfix("e1"));
  var ct=encodeURIComponent(
       mathedit.getContentMathML("e1"));
  var qr = "var=x&expr=" +ex+ "&answerct="
      +ct+ "&answer=" +infix;
  jsonCheck("http://wme.cs.kent.edu/cgi-bin"
      + "/derivativejson.pl", qr);
}
```

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```
function jsonCheck(url, query)
{  var ajaxRequest = new ajaxObject(url);
   ajaxRequest.callback = processResponse;
   // sending the post request
   ajaxRequest.update(query, "POST");
}
```

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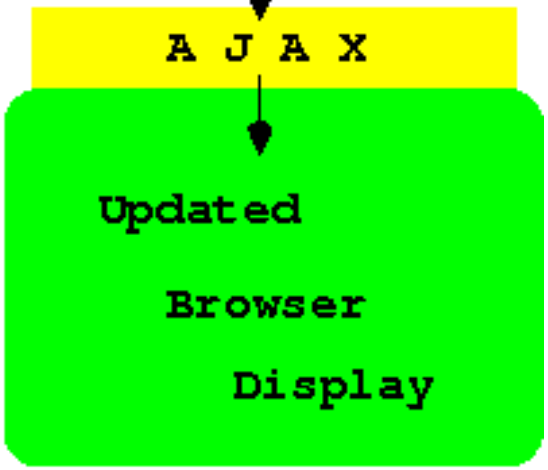
Browser Display Update

```
Content-type: application/json

{ status: "normal",
  given: "6*x",
  correctness: "incorrect",
  standard: "<computed ans>",
  end: "record" }
```



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```
function processResponse(responseText)
{ eval("answer=" + responseText + ";");
  var node=document.getElementById(
                          'checkResult');
  if ( answer.correctness=="incorrect" )
  {  node.innerHTML="Your answer is incorrect."+
     "  The correct answer is "+answer.standard;
  }
  else
  {  node.innerHTML = "Congratulations, " +
        "your answer is correct.";
  }
}
```

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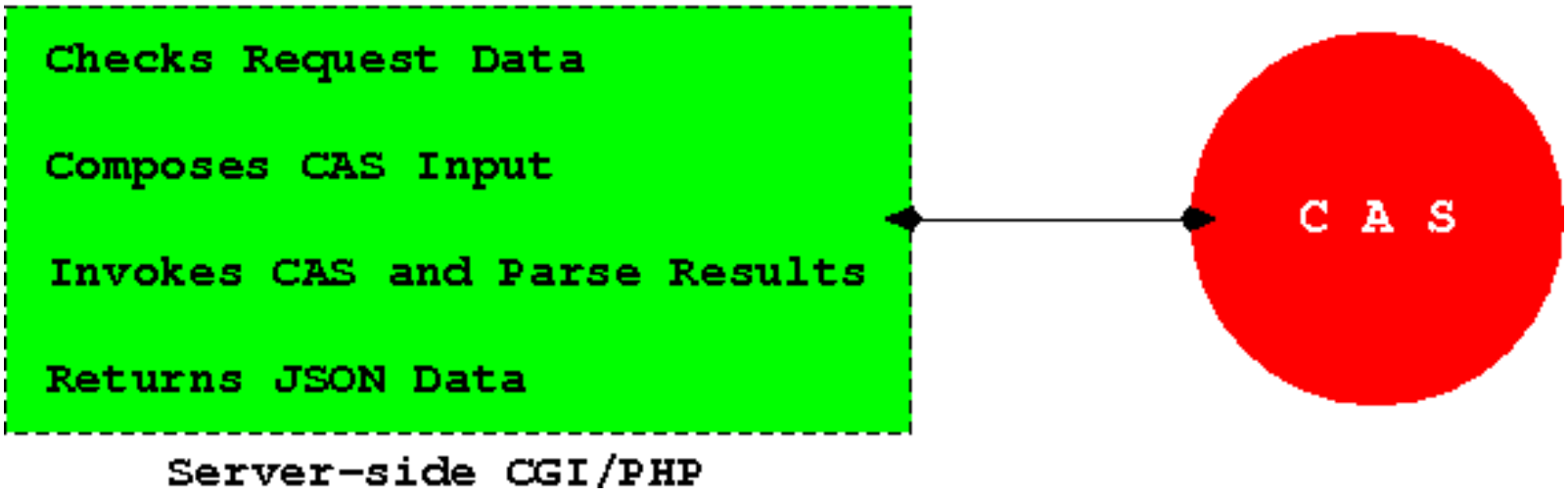
Interfacing to A CAS



Answer Checking Service Protocol

- Try This Example.
- Use REST for service request
- Use JSON for service reply
- Need to discuss and formulate the request and the reply formats.

Proposed Answer Checking Reply Format

Use JSON as reply format. Only `status` and `correctness` are required. Other values are optional.

- `status:` `normal` or `error`
- `correctness:` `yes`, `no`, `acceptable`, `approx`, or `unknown` (can be an array of such values)
- `yourans_ct:` the mathml content expression of user's answer (or array)
- `yourans_pr:` the mathml presentation expression of user's answer (or array)
- `yourans_infix:` the infix expression of user's answer (or array)
- `ans_ct:` mathml content of standard answer (or array)

- `ans_pr:` mathml presentation of standard answer (or array)
- `ans_infix:` infix notation of standard answer (or array)

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Example JSON Data

```
{  status: "normal",
   yourans_ct: '<mathml content>',
   correctness: 'no',
   ans_pr: "<mathml presentation>"
}
```

Proposed Answer Checking Request Format

Use REST format (query strings):

- `var=` the variable(s) involved
- `qans_in=`, `qans_ct=` infix notation, mathml content, of answer(s) to check
- `sans_in=`, `sans_ct=` infix notation, mathml content, of standard answer(s)
- `cmdexp=` command string to be evaluated to produce answer(s)
- `tolerance=` for floating-point answers
- `accept=` a percentage (floating-point number), `PlusC`, or `TimesC`, for acceptable answers

List items are "`,` " comma and space separated.

Infix Notations Used in Answer Checking

The infix notations need to be standardized and please

see this file for a proposal.

Answer Checking Services

- Numerical value checking (possibly with tolerance)
- Expression equality checking
  - rational simplification
  - canonical forms (trig transforms for example)
  - random evaluations points
- Checking for different problem types: integrals (antiderivatives), factoring, polynomial gcd, partial fractions, ...

Simplification VS. Evaluation

- Automatically verifying expression equivalence is *undecidable* in general.
- But in practice, algebraic simplification can often do the job for polynomial and rational expressions.
- Another technique is to test equivalence of expressions at randomly selected points. This is a probabilistic algorithm.
- We can introduce into the Request Format:

```
EvalCheck=true
points=0.0,1.75,-2.5
points=random,6
interval=0.1,9.9
```

- Simple EvalCheck can be done by Javascript on the client side.

Computing Standard Answers

- It is convenient if the correct answer is generated or computed by the answer checking service.
- This can be done by presenting the problem in an *executable* way to the service.
- Examples:

```
derivative(exp,var,n)
factor(polynomial)
gcd(p1, p2)
lcf(p1, p2)
simplify(exp)
```

A Loophole

- A sneaky user may send the problem statement as the answer.
- Any simplifications and transformations may destroy the form of the correct answer or transform the wrong form into the correct form.
- We need a way to specify the nature of the answer. For example we expect the answer given to be an integer, a real, a complex, a monomial, a sum, a product, a power, a polynomial with degree 2, etc.
- This means some problem-specific pattern checking must be performed. A general answer checker may provide some common patterns.
- Some of this can be readily checked on the client side.