ICME 11 – TSG 22 Theme 3

WME: a Web-based Mathematics Education System for Teaching and Learning

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Abstract (Research-based paper)

WME is being developed as a modern Web-based system that fosters a new paradigm for creating, customizing, and sharing, interactive mathematics education materials online. By an innovative combination of standard Web technologies and by creating powerful Web-based tools, WME can deliver classroom-ready lessons that are interesting, inquiry-based, as well as interoperable. The WME system allows easy implementation and modification of lessons or manipulatives so that very little technical ability is required. To-date we are developing the following tools to create manipulatives within WME pages: GeoSVG is a Web-based tool to author and run SVG-based geometry, DMAS an assessment system that supports all types of assessments, and MathEdit to enable mathematics to be displayed and computed on the web. Modifications of manipulatives or new lessons can be published for other teachers using the WME system to view or incorporate into their own lessons and modules.

To date we have piloted several WME lessons in area middle schools. We have gathered both technical and educational data that we have used to guide the development and future directions of our work, but more trials and data are needed to measure the total effects of our system on the teaching and learning of mathematics.

1. Theoretical framework guiding the WME system development and use.

The WME work has the benefit of an interdisciplinary team at the Institute for Computational Mathematics (ICM/Kent State University). Our group consists of faculty and graduate students from Computer Science, Mathematics, Mathematics Education, Graphic Design, and Middle School Mathematics. The technical part of the system is being guided by the use of a combination of innovative open Internet technologies. Our diverse group of students and faculty have allowed us to work on complex problems and issues involved in teaching and learning mathematics, but it has also created the need to develop a common philosophy of learning and teaching mathematics among this group of individuals. Our philosophy has been evolving over the past 4 years and we have (over many iterations of our work) developed a consistent set of beliefs about what is important mathematics and how middle school students learn specific topics.

Our work is guided by current research in mathematics education and we have adopted the view that students need to make sense of mathematical ideas by building rich connections to their existing knowledge and exploring the limits of their mathematical thinking. Another belief of the group is that the context in which tasks are developed needs to be interesting to the students and contain significant mathematical ideas for students to explore. Our activities are constructed in many cases to confront the limitations of students' thinking. With this in mind, we have adopted the research framework outlined by Silver that theory, practice, and problems should mutually support one another (Silver, E. & A. Herbst, P.G., 2007). That is the manipulatives and lessons are designed consistent with recommendations from current research. Many topics have been identified initially by collaborating middle school teachers to help their students learn challenging mathematics concepts identified in their practice. This design provided not only a consistent theory for building the WME system, but also a specific process to guide the day-to-day work.

The mathematics education faculty met regularly with the middle school teachers to discuss where their students were having difficulty learning specific topics. The teachers would communicate both how the students encountered the topic (types of lessons and problems) and specific difficulties (sometimes referred to as errors) they observed in their students' thinking. This began a discussion about specific research studies (and their implications) for how teachers might provide different tasks for their students to facilitate them making sense of these ideas. Together the middle school teachers and mathematics educator sketched ideas for tasks and tools, in web-based active lessons that would be helpful in creating an environment for the middle school students to overcome their difficulties and make sense of the ideas. The mathematics educator then met with the larger group to discuss the specific needs for the lesson and tools. These interactions and discussions helped to guide the computer science part of the team to determine the most appropriate way to develop what the educators needed. A computer science student developing a specific tool or active lesson presented his/her work by first presenting the goals and programming techniques. These presentations by all members of the group served as a way to critique our work before, during, and after being developed. As tools and lessons were developed they would be made available to teachers for use with their students. When the teachers would implement these WME lessons with their classes at least one member of the WME team were present in the room with teachers and students. We assisted in technical, mathematical, and educational matters during these in-class trials. As these lessons were implemented we gathered observational data about both teacher and student use of the WME lessons, and informal survey data from students regarding how they liked using the system. This data then was used to update the technical aspects of the system and adjust specific educational aspects of the lessons. The next part of the paper will illustrate this process with an individual topic that we built early on in our WME project.

Example of initial topic module and lessons

The initial teachers for this project both taught 7th grade mathematics to 4 different classes of students each day. After initial discussions of our work the mathematics educator met with these teachers to discuss what topic needed additional support. Both teachers agreed initially that their students had very little understanding (procedural or conceptual) of percents. The teachers indicated that their students were initially taught percentages in 6th grade in a standard manner (introduced the definition, some examples of the 3 basic types of percent problems and then these same types embedded in contextual problems). In 7th grade they were given additional problems where students had to apply their knowledge of percents to a wider variety of problems (involved multi-step problems for example). The resource materials used (mostly the text) presented the data to students by illustrating common types of problems and standard techniques for solving the problems.

Teachers reported that their students (then in the 7th grade) scored average or better than average on the 6th grade tests regarding this topic. However, they noticed that a significant number of their students were struggling with this same topic in their 7th grade classes. What knowledge these students had acquired in the 6th grade either wasn't accessible or wasn't useful in solving the problems they were facing in the 7th grade. The mathematics educator introduced some research that was conducted regarding the importance of understanding the concept of percents and how even students without formal instruction in percentages had an informal understanding of standard benchmarks (50% for example means ½ of something)(Lembke & Reys, 1994). We discussed how we might take advantage of the informal notion of percents and try to build some proficiency in solving the 3 types of standard percentage problems mentioned above. We sketched out a set of tools that we thought would help to develop the concept of percent and a few activities with spreadsheet like tools for students to explore and build on their informal notion of percentages. Some of the initial tools developed a stronger notion of percent by presenting pictures of pizza arranged with 10 rows of 10 pepperoni when students clicked a pepperoni it would disappear and keep track of the percent "eaten" and the percent left. This was followed by other tools like the one below where the total number pieces were different factors of 100. These tools were available for students to use while answering questions like, how many different ways can you select 50%,? What about for 25%? How are these representations of 25% the same? How are they different?

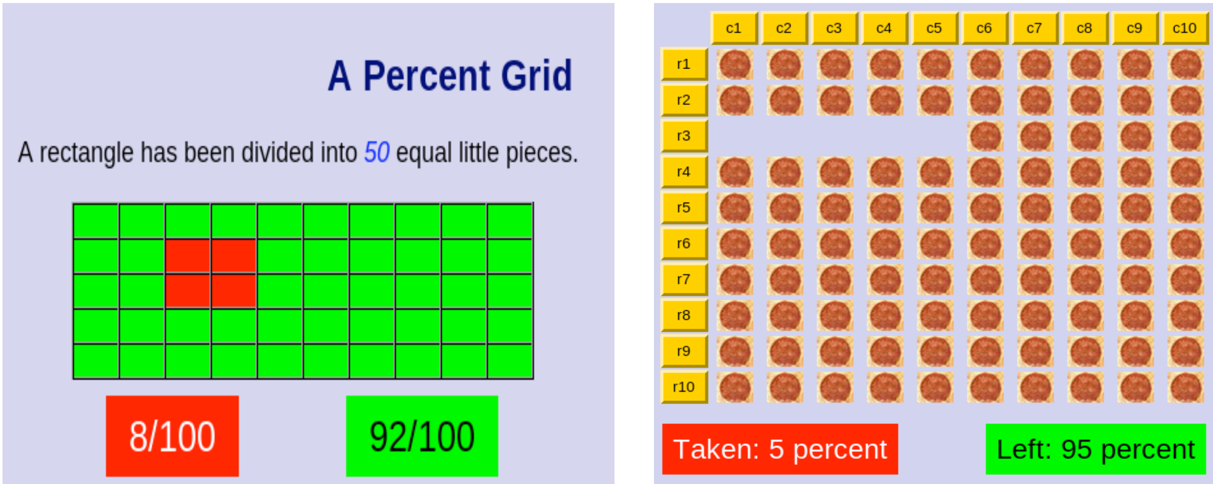


Figure 1: Example of tools that were developed to help students broaden their notion of percents.

Both teachers used WME lessons with groups of their students while part of the WME team observed. These trials lasted for three to five days. We helped with the technical issues that came up during the class as well as observed students using the web-based lessons and tools. We were encouraged by the student interest and their candid comments about the system and their using it to learn mathematics. Because the use of the WME system was intermittent and limited to remediation of topics where the teachers identified as needing supplement, we were unable to assess mathematics learning in any significant way. We did, however, continue this process to develop our lessons, tools, and technical parts of our WME system.

2 The technical aspects of the WME system

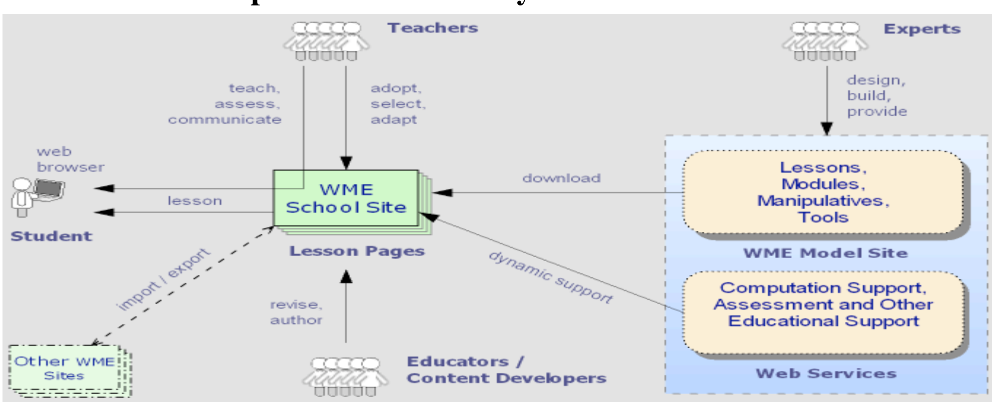
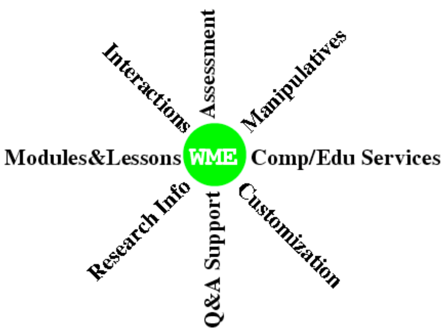


Figure 2: The WME Concept

WME can deliver, via the Internet or a LAN (wired or wireless), classroom-ready lessons that are well-prepared, mathematically and developmentally appropriate, interactive, and effective. In addition to multimedia content and hyperlinks, lesson pages feature interoperable and customizable manipulatives to help students understand and explore mathematical concepts through hands-on activities and a teacher guide that assists in the teaching of mathematics more effectively. We believe a system like WME can help teachers provide a significantly better mathematical experience for their students.

The WME system supports, among other features, mathematical formulas through MathML, interactive geometry objects through SVG (Scalable Vector Graphics), and a distributed mathematics assessment system called DMAS. In addition, WME also makes its educational components (manipulatives, lessons, modules, formulas, geometry objects and assessments) interoperable and customizable. Interoperable so these components can readily be combined to form different lessons and modules. In other words, these components are "plug-compatible." Customizable so teachers can easily edit any such component for their own purposes and not affect its use by others. Thus, WME is different and much more advanced than existing approaches. It aims to be a modern, practical, efficient and effective Web-based system to provide more comprehensive support for implementing research-based reforms called for in mathematics education. Our WME system conforms to open standards, works with regular browsers, provides systematic access to client-side and server-side support, and allows independently developed WME components to interoperate seamlessly. In short, WME is creating a Web for Mathematics Education.



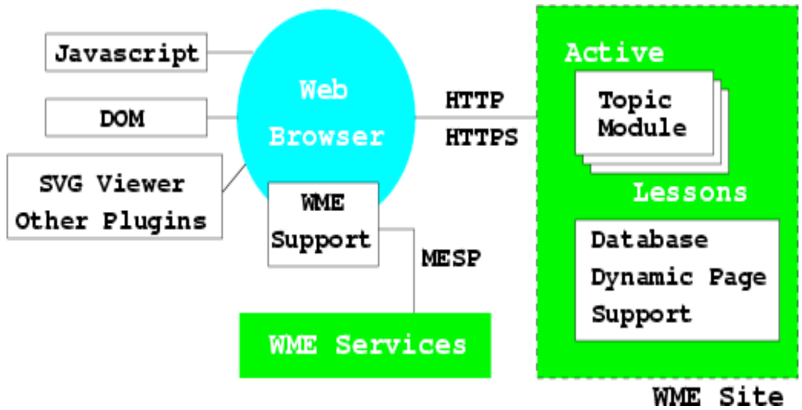


Figure 3: WME Integration (left) WME Architecture (right)

3 Results from school-based trials

WME delivers classroom-ready lessons for teachers to use in their classes that contain a variety of interactive-tools, for students to investigate a variety of mathematical concepts. Specially designed manipulatives help students explore mathematical concepts, encourage conjectures, and other WME tools provide teachers with a way to collect formative data to facilitate whole-class discussions.

Teachers can use existing lessons, modify lessons, customize manipulatives, or create a new lesson with minimal technical training. Any new object created in the WME system can be shared with other teachers at the school or the WME community at large. The days of cutting and pasting lessons have taken a quantum leap forward. The current curriculum hierarchical arrangement is by grade level, by topic module (percents, integers, etc.) and individual lessons within these modules. Teachers can modify both individual lessons to suit their student's needs and what specific lessons are in individual topic modules. Research-based teacher guides aligned with Ohio's academic content standards will be attached to lessons and modules to provide teachers with essential information for effective implementation of the lessons.

We have piloted several portions of the WME system in middle school classrooms using a prototype site, a step towards a model WME site that can be easily deployed to different schools. In our trials we have had a variety of teachers using WME to teach a variety of topics in grades 6, 7 and 8. Seven different teachers have used a variety of lessons with their classes to-date. The classes all have access to laptops with wireless access to the Internet. We worked through many technical and education issues in these trials. The preliminary results of these trials are that students and teachers liked using WME, they reported being engaged in thinking about the topics using the system, and that they were ready for more of our lessons. Each implementation of the test system was of a short duration (1 week or less) to gather initial data to determine where the system could be improved and how it might be affecting student learning.

During the first few implementations teachers tested lessons on percentages and fractions. These lessons were meant to help develop a better conceptual understanding. The manipulatives used ranged from inquiry-based area models (some that looked like pizzas and chocolate bars) to interactive restaurant menus. Students chose menu items and the total was calculated automatically for each student. Students then used percent to determine tax, tip, and labor costs. Answers by each student were checked immediately. However, because the students determined what food they would purchase (and changes for different problems could occur at any time they chose) these tools allowed students to determine (at least in part) what problem was to be studied. While students explored with WME the teachers were provided with formative information about how all students were doing with each lesson by allowing him/her to view individual responses to questions immediately after answers were submitted. This gave the teacher an opportunity to create new questions either for students to respond online or for a whole class discussion.

After each short trial of the WME system, students and teachers supplied the researchers with their comments and suggestions about the lessons and the tools. These suggestions were incorporated into the next revisions of both the content and technical aspects of the WME system. We are encouraged by the reaction of the students and teachers who have used our system and continue to increase the depth and breadth of its testing. Despite our enthusiasm for the WME system, more trials are needed to improve and refine the system and to determine the overall impact of WME on the teaching and learning of mathematics.

4 WME Advantages

Our experience with WME has provided indications that it can bring these advantages to teaching and learning of mathematics.

- Accessibility—Is accessible 24/7 by students and teachers with regular Web browsers, usable in classrooms, in computer labs, at home and on the road; makes mathematics education content widely accessible in and out of classrooms and schools.
- Compatibility and Interoperability—Follows modern Web standards and compatible with all XML tools; is easy to share, extend, and internationalize; allows easy incorporation of new Web and Internet tools and technology; provides interoperable contents and services (Chiu & Wang, 2006).
- Richness and Variety—Empowers school teachers, independent educators, mathematics experts, and computer professionals to contribute to mathematics education; forms a potentially unlimited set of mathematics education curricula, services and manipulatives provided by experts; and provides access to all lessons, modules, assessment items, & tools made available by those using or contributing to the WME system.
- Integrated, Dynamic, and Classroom Ready—Integrates text, graphics, interactions, explorations, and assessment in concept-centered lesson pages that are generated dynamically; provides page modification, question posing, and answer collecting capabilities inside individual lessons; allows teachers to import, select, and customize as well as mix-and-match ready-made contents for individual classes; supplies easily customizable model WME sites to schools.
- Efficient Communication—Maintains a flexible and effective channel of online communication between teachers and students; supplies math-capable chat and message boards; provides much-needed opportunities for written communication by students; enables teachers to interact with each student in the entire class at once, and allows students to give answers privately or as part of a class discussion.
- Concepts not Steps—Lessons encourage students to explore and make conjectures through using easy to use manipulatives; supports automatic plotting and diagramming; accessing mathematical algorithms as on-Web tools; frees teachers and students from tedious and time consuming work such as long calculations so more time can be spent on collecting data, making conjectures and creating arguments which support their thinking.
- Educator Support, Convenience, and Control—Allows teachers to control who may access the Web-based contents; provides real-time control of the visibility of parts of lesson pages to better focus student attention; helps teachers organize questions, answers, tests and grades; provides assessment databases with modifiable questions; enables teachers to pose their own questions to

assess each and every individual student in the class at once.

- Real-world Motivations—Makes it easy to put up good timely and topical examples to motivate the learning of mathematics; allows easy access to interesting and current examples of real applications on the Web;

5 WME Architecture and Components

We hope to achieve Web-based mathematical education through an innovative combination of standard technologies and interoperable components.

- Interoperable Manipulatives—In-page objects for hands-on experiments designed to support the teaching of specific concepts and skills. We currently have a small set of hard-coded manipulatives (not customizable) and three other customizable tools (GeoSVG, DMAS and MathEdit).
- Interoperable Topic Lesson Pages (TLP), and Topic Modules (TM) -- Complete lessons with identifiable learning goals that integrate motivations, real-life applications, introductions, visuals, examples, hands-on exploration, assessment, guide for teachers and so on. These lessons can be used by the same teacher in different classes or exported to a central WME database and shared with all users of the WME system. New users can use these lessons in their classes or used modified versions of these lessons without "harming" the creator's lesson. The same can be done with Topic Modules (or units) that are created by individuals using the WME system.
- Assessment Support—A well-organized and searchable database of assessment questions for each WME site with support for grading, storing, and retrieving answers and scores. Teachers can import and modify existing questions to use in tests, contribute questions to the database, and easily manage tests online. Databases in different schools form a distributed system to support all WME sites.
- Client-side Support—On the user side, common Web browsers can be used. The browser provides support for Standard ECMAScript (Javascript), Document Object Model (DOM), Scalable Vector Graphics (SVG), and preferably other plug-ins.
- Server-side Support—On the Web server side, regular active page and database capabilities support site operation, administration, customization, and configuration. Other services such as MathChat (Chiu, 2004) can also be included.
- Support for Mathematical Formulas—The MathEdit tool (Su, Wang, Li, Li, & Zou, 2006) supports interactive formula input and editing. The resulting formulas entered by the user can be shown in textbook style display. Infix and MathML representations of the formulas can then be used for processing.
- Geometry Manipulative Support—The GeoSVG (Lai & Wang, 2006) tool supports Web-based geometry manipulatives implemented in SVG and Javascript. Interactive authoring, learning, and sharing of these manipulatives are supported. The manipulatives follow the WME manipulative API for easy interoperability.
- WME Services—WME compliant Web services can supply a variety of useful functions such as plotting, expression simplification, answer checking, specific mathematical computations, terminology dictionaries, and assessment databases. This way, computations for different areas of mathematics can be supplied to MeML pages from anywhere on the Internet.

6. Summary

A WME website organizes the mathematics curricula for a particular school by teacher per class. Topic modules and lesson pages are selectable and customizable by individual teachers for each different class he/she teaches. The lesson pages, topic modules, interactive manipulatives, and other lesson elements are interoperable in a plug-and-play fashion among all WME sites, however all modifications are done to specific modules or lessons for specific classes. These modifications do not affect other classes or other teachers using the WME system.

WME is being created with teachers using the WME system in classrooms with their students. The main features of the system (the interesting web-based topics, easy to use and customizable manipulatives and assessment system) are provided to help teachers create an appropriate and challenging mathematical environment that will ultimately help more students make sense of significant mathematical ideas. The WME system could also be used in different settings as well. Examples of these are: specifically designed tutorial programs for assisting students in reviewing mathematical procedures or concepts and creating investigations for students to explore gather data for projects or homework, and to interact with real data to collect, organize, analyze, and explain their mathematical conjectures.

One of the major obstacles to implementing reform in mathematics education is the lack of an easily accessible context in which to capture learner interest and to challenge students with different mathematical abilities. Research indicates that technology has the capability to help all students learn much more significant mathematics than they might otherwise be able to learn without its use (National Council of Teachers of Mathematics, 2000). We are attempting to use the research on how students learn mathematics with the available web resources to create an exciting and effective environment to teach and learn mathematics.

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