Geometrical Manipulatives in WME

Paul Wang
Xun Lai
Department of Computer Science
Institute for Computational Mathematics
Kent State University

Topics

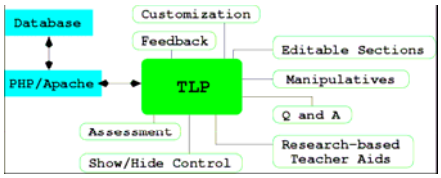
- What you will learn
- Background
- Warm-up: Manipulative you can create by yourself
- Need for Manipulatives
- Manipulative Examples in WME Pages
- Authoring Manipulatives
- Deploying Manipulatives in WME pages
- Customization of Manipulatives
- How to contact us for further collaboration

What you will learn

- After the tutorial, you will know
  - The role manipulatives play in WME
  - How to use manipulatives in WME topic lesson pages
  - How to author geometrical manipulatives by using the authoring tool *GeoSVG*
  - How to deploy manipulatives you create
  - How to customize manipulatives

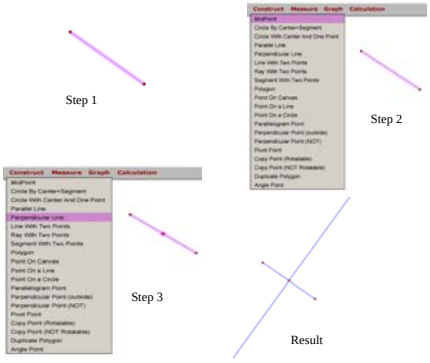
Background

- WME offers classroom-ready lessons (topic lesson pages) that host the manipulatives



Warm-up: Manipulative you can create by yourself

- Experiment 0: construct a perpendicular line of a segment through its mid-point
- Steps
  1. Draw a segment
  2. With the segment selected, construct its midpoint
  3. With the segment and the midpoint selected, construct a perpendicular line
- Test: drag any object around to see the mathematical relations are maintained



Need for Manipulatives

- Manipulatives enable students to have active involvement to enhance learning
- Manipulatives can make students to move to higher levels of conceptualizing and begin to integrate spatial and symbolic mental representations
  - Success of Logo, one of the earliest interactive program
- Geometric manipulatives covers important areas in math teaching
  - plane geometry, algebra, trigonometry, calculus, statistics, and so on (snapshots in next slides)

Need for Manipulatives (cont.)

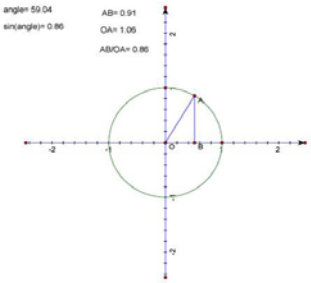


Figure 1: Geometric Manipulative in Trigonometry
- Definition of sine function (opposite side/hypotenuse)

Need for Manipulatives (cont.)

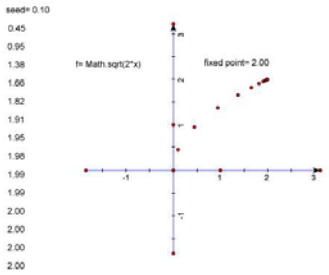


Figure 2: Geometric Manipulative in Algebra
- Fixed point of function $f(x) = \sqrt{2x}$

Need for Manipulatives (cont.)

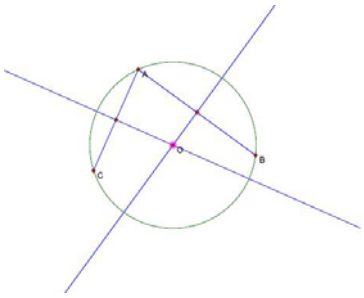


Figure 3: Geometric Manipulative in Plane Geometry
- How to construct a circle through three points

Need for Manipulatives (cont.)

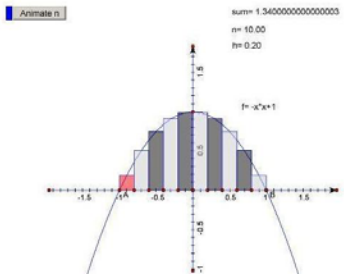


Figure 4: Geometric Manipulative in Calculus
- Approximate the integral of a function

Need for Manipulatives (cont.)

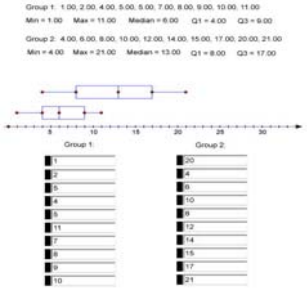


Figure 5: Geometric Manipulative in Statistics
- Box and Whisker

Need for Manipulatives (cont.)

- Advantages of manipulatives over physical drawings
  - able to draw an entire set of similar cases in seconds
    - Example: Construct a circle through three points.
    - Circle is larger than a scratch paper when three points are almost co-linear
  - able to visualize the transitions between cases
    - How does the circle change when points are moving
  - easy to repeat
    - Approximate the area under a curve by drawing rectangles
  - easy to represent algebraic relations between geometrical objects
    - Measurements can be updated dynamically

KENT STATE UNIVERSITY

Manipulative Examples in WME Pages

- Components of a WME Topic Lesson Page
  - Text
  - Manipulative(s)
  - Question set(s)
- Examples (snapshots: next a few slides)
  - Plane Geometry example
    - Area of a Triangle (link) (generated by GeoSVG)
  - Algebra example
    - Walking wolf over a number line (link) (ad hoc program)
  - Statistics example
    - Box-and-Whisker (link) (generated by GeoSVG)
  - Number relation example
    - Meaning of equal (link) (ad hoc program)

Manipulative Examples in WME Pages (cont.)

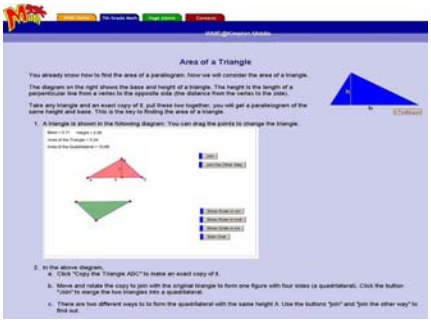


Figure 6: WME page: Area of a Triangle

Manipulative Examples in WME Pages (cont.)

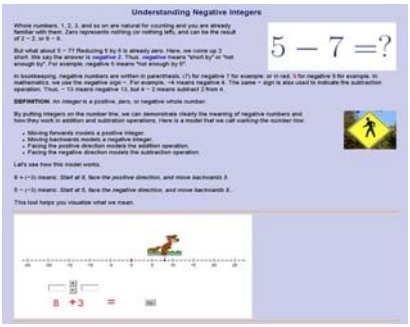


Figure 7: WME page: Understanding Negative Integers

Manipulative Examples in WME Pages (cont.)

Figure 8: WME page: Box-And-Whisker Plots

Manipulative Examples in WME Pages (cont.)

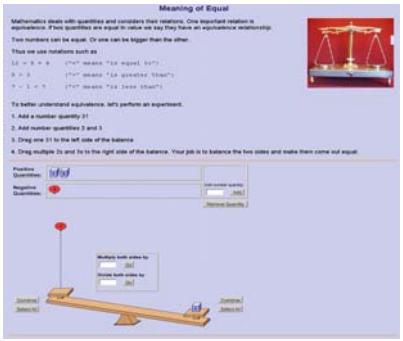


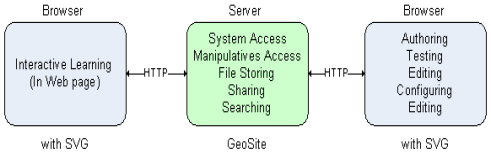
Figure 9: WME page: Meaning of Equal

Authoring Manipulatives

- Two ways to create manipulatives
  - By writing ad hoc programs by software developers
  - By using authoring tool
- A large category of manipulatives, especially those related to geometry, can be authored by using *Dynamic Geometry Software* (DGS)
  - A DGS system supports diagramming, interactive learning, and easy authoring of geometrical manipulatives
  - Successful DGS systems: *Geometer SketchPad*, *Cabri Geometry II*, and *Cinderella*
- We are developing our own DGS system *GeoSVG*
  - Reason to develop our own DGS system: its complete Web-orientation

Authoring Manipulatives (cont.)

- GeoSVG Architecture and Components
  - The GeoSVG toolkit --- an SVG-coded library for authoring (creating, editing, publishing plane geometry manipulatives and for their display, animation, and user interaction).
  - The GeoSite --- a Web site that makes the GeoSVG toolkit available and stores manipulatives for access, searching, and sharing.



Authoring Manipulatives (cont.)

- Steps to access GeoSite and the authoring environment
  - Open page http://wme.cs.kent.edu/geosvg/software.html
  - Follow the link in the page to GeoSite
  - Log into GeoSite (30 accounts are already pre-registered and will be assigned to the participants of the tutorial)
  - Now you are at your home folder
  - Next slide shows possible operations on GeoSite
  - Either authoring a new manipulative or editing an existing manipulative will open the authoring environment

Authoring Manipulatives (cont.)

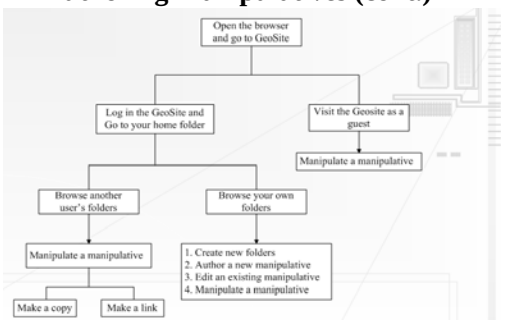
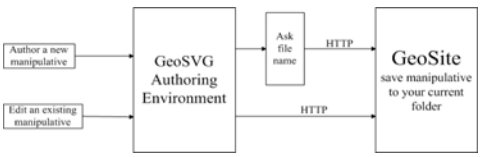


Figure 10: GeoSite Operations

Authoring Manipulatives (cont.)

- GeoSVG authoring environment
  - Canvas: a bounded area in which to create manipulative
  - Toolbar: basic drawing tools and property dialog box
  - Menu: providing various authoring supports listed in next slides
- Authoring procedures



Authoring Manipulatives (cont.)

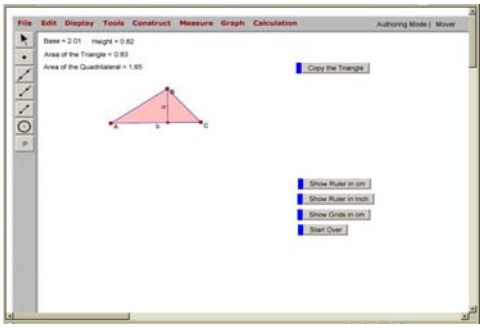


Figure 11: The GeoSVG Authoring Environment

Authoring of Manipulatives (cont.)

- **Authoring Supports**
  - **Drawing primitives**: Making it simple to create basic geometric shapes such as points, lines (segments, rays and vectors), circles (ellipses and arcs), polygons, etc.
  - **Geometric object construction**: Constructing a new geometric object by combining existing objects that are subject to user-specified rules and constraints. For example, picking a point and a line to construct a new line through the point parallel to the existing line.
  - **Measurement**: Measuring length, slope, radius, distance, area, circumference, perimeter, angle, coordinate positions.
  - **Loci and Envelops**: Constructing loci of moving points and envelops of moving lines.
  - **Animation**: Visualizing the movement of objects to illustrate concepts much better than still pictures.

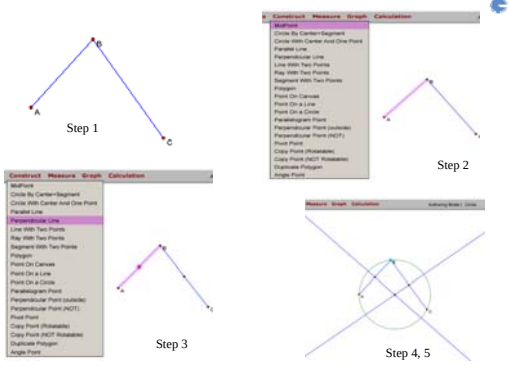
Authoring of Manipulatives (cont.)

- **Authoring Supports (cont.)**
  - **Iteration**: Generating a sequence of objects according to iteration rules defined by a user.
  - **Calculation**: A dynamic calculator updates results when statuses of dependent objects change.
  - **Graphing**: Plotting points and function graph in coordinate systems.
  - **Geometric transforms**: Allowing users to define center and mirror for translation, reflection, dilation, and rotation of objects.
  - **Defining Macros**: Condensing a series of steps in to one software command.
  - **Graphical User Interface(GUI)**: Combining menus, buttons, mouse actions, and keyboard inputs to provide an intuitive and convient authoring environment.

KENT STATE

Authoring hands-on Experiments

- Now I will guide you through how to author manipulatives
- Experiment 1: construct a circle through three points
  - Steps:
    1. Draw three points and connect them by two segments
    2. Select one segment and construct its midpoint from the construct menu; do the same thing for another segment
    3. Select one segment and its midpoint, construct a perpendicular line to this segment and through the midpoint; do the same thing for another segment
    4. Draw the intersection point of the two perpendicular lines
    5. Draw a circle with the intersection point as the center and through any of the three points
  - Objects can be labeled via the property dialog box
  - Drag any object to move around to see how mathematical relations are maintained

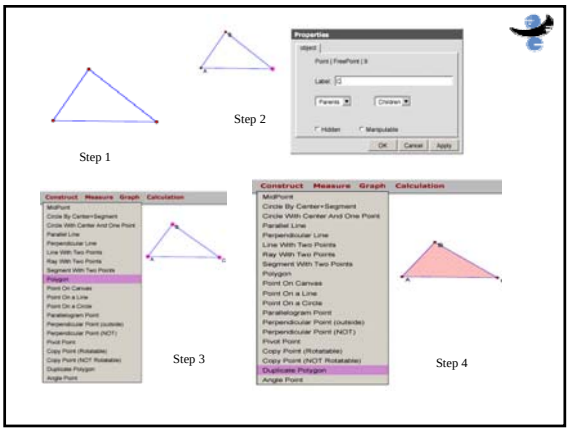


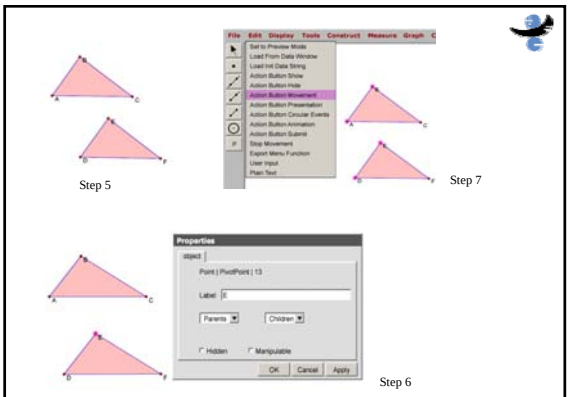
Authoring hands-on Experiments (cont.)

- Experiment 2: Area of a Triangle (link)
  - Steps
    1. Draw three connected segments
    2. Label the three vertices *A*, *B*, and *C* via the property dialog box
    3. With only the vertices *B*, *A*, and *C* selected in order, construct a shaded triangle
    4. With only the shaded triangle ABC selected, duplicate the triangle
    5. Label the three vertices of the new triangle *D*, *E*, and *F*, which correspond to the vertices *A*, *B*, *C* of the original triangle
    6. Via the property dialog box, identify which one of the points *D*, *E*, *F* is of type PivotPoint; in my example, *E* is of type PivotPoint, and *D*, *F* are of type CopyPoint. Dragging a PivotPoint will translate the duplicated shape, while dragging a CopyPoint will rotate the shape around the PivotPoint.

Authoring hands-on Experiments (cont.)

- Experiment 2: Area of a Triangle (link)
  - Steps (continued)
    7. Construct a movement button for merging two triangles into a parallelogram
       - Even number of points must be selected when movement button is to be created. A movement button moves the first point of each selected pair toward the second.
       - In order to move one side of a triangle to one side of another triangle, there must be two source points and two destination points
       - The point of type PivotPoint must be one of the source points because moving it will translate the triangle. In the example, *E* is the point of type PivotPoint.
       - In the example, four vertices selected in order are *D*, *B*, *E*, *A*, where *D*, *E* are the source points, and *B*, *A* are the destination points.





KENT STATE

Deploying Manipulatives in WME pages

- Let's use one manipulative you just created to deploy it in a WME page
- Page editing allows you to enter the URL of the manipulative located in GeoSite
- Test the page
  - Great! You have made contribution to the WME system.

KENT STATE

Customization of Manipulatives

- Manipulatives are customizable
  - Like other components in a TLP, a manipulative can also be tailored to a desirable configuration by teachers
  - This is significantly different from other computer games or software for math teaching, which are fixed.
- Each manipulative is associated with an xml file, which saves all the configurable parameters
- Customization example (next slide)

Customization of Manipulatives (cont.)

KENT STATE

How to contact us for further collaboration

- **WME Home:** http://wme.cs.kent.edu
- **Project Personnel with Contact information**
  - Prof. Paul S. Wang (ICM/Kent) pwang@cs.kent.edu
  - Prof. Michael Mikusa (College of Education/Kent) mmikusa@kent.edu
  - Mr. David Chiu (ICM/Kent) dchiu@cs.kent.edu
  - Mr. Kahraman Cem Karadeniz (ICM/Kent) kkaraden@cs.kent.edu
  - Mr. Saleh Al-shomrani (ICM/Kent) salshomr@cs.kent.edu
  - Mr. Xun Lai (ICM/Kent) xlai@cs.kent.edu
  - Dr. Xiao Zou (ICM/Kent) xzou@cs.kent.edu
- **Mailing List**

  wme@cs.kent.edu, which can be used to reach all project personnel