Game Mathematics

- Inner product
- Cross product
- Matrices
- Quaterions
- Convex Hull

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Dot Product

- Vector A and B

$$A \cdot B = AB^T = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix} = ad + be + cf$$

Think of the dot product as a matrix multiplication

$$\|A\|^2 = AA^T = aa + bb + cc$$

The magnitude is the square root of the dot product of a vector with itself

$$A \cdot B = \|A\| \, \|B\| \cos(\theta)$$

The dot product is also related to the angle between the two vectors

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Perpendicular Vectors

- Inner Product of two vectors w and v, $w^T v$.

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}, v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, w^T v = w_1 v_1 + w_2 v_2 + w_3 v_3 = v^T w = |w||v|\cos\theta,$$

$$where\ \theta = angle\ between\ vectors\ v, w,$$

$$where\ |v| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{v^t v}.$$

- Two vectors w and v are perpendicular (orthogonal) if and only if $w^T v = 0$.

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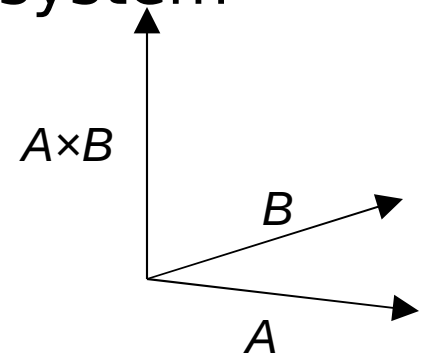
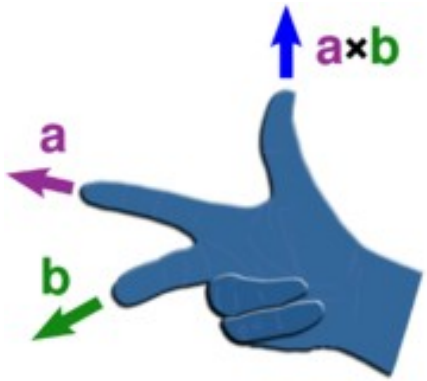
Cross Product

- The cross product of vectors A and B is a vector C which is perpendicular to A and B
- The magnitude of C is proportional to the sin of the angle between A and B
- The direction of C follows the **right hand rule** if we are working in a right-handed coordinate system

$$\|A \times B\| = \|A\| \ \|B\| \sin(\theta)$$





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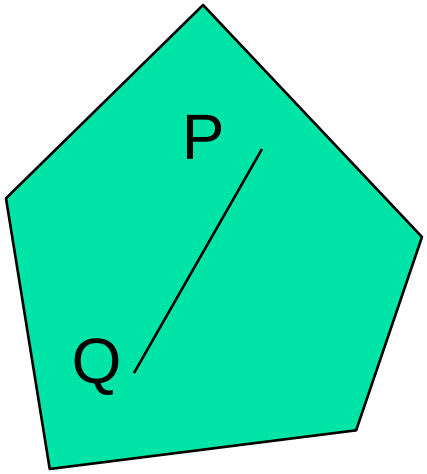
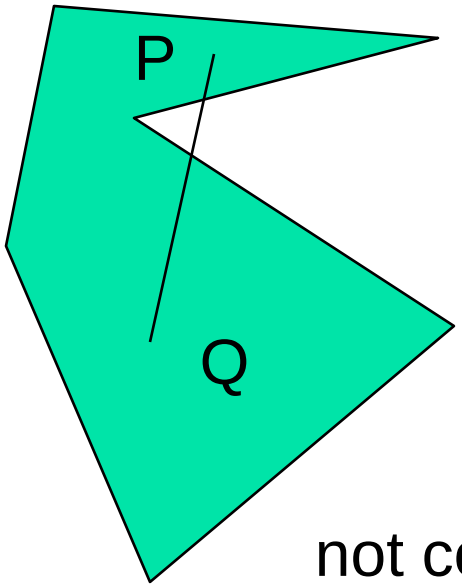
Convexity

- An object is *convex* iff for any two points in the object all points on the line segment between these points are also in the object



convex



not convex

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Matrices

- 4x4 matrices are used to compute the new location of the coordinates of an object that is subjected to a rotation, reflection, or transformation.
- A rotation matrix

$$\begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Matrices

- A Translation Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- A tranlation followed by a rotation

$$\begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Quaternions

- In mathematics, the quaternions are a number system that extends the complex numbers
- Quaternions find uses in both theoretical and applied mathematics, in particular for calculations involving three-dimensional rotations such as in three-dimensional computer graphics and computer vision.

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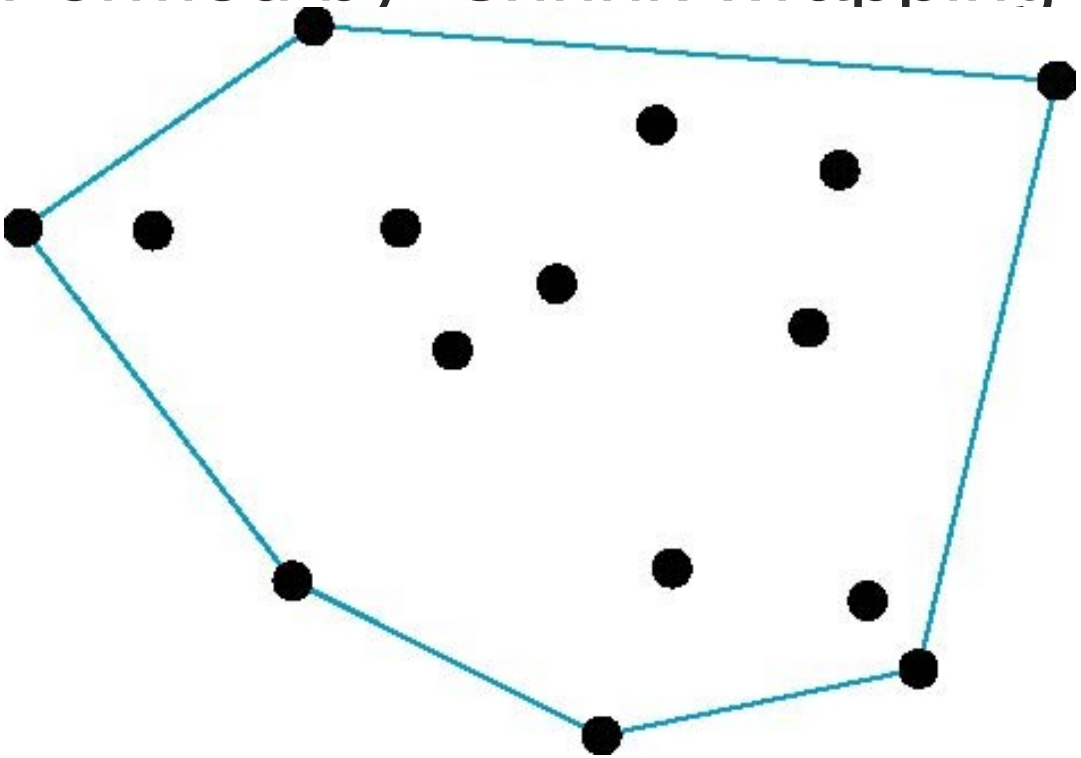
Convex Hull

- Smallest convex object containing $P_1, P_2, \ldots P_n$
- Formed by “shrink wrapping” points



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